Notes on
1.63 Advanced Environmental Fluid Mechanics
Instructor: C. C. Mei, 2002
ccmei@mit.edu, 1 617 253 2994

November 6, 2002

6.4 Geothermal convection in porous media

Motivations: Geothermal Energy, Pipe line in permafrost, thermal stress around buried nuclear waste.

S. Chandrasekhar: *Hydrodynamic and Hydromagnetic Stabililty*, Oxford, 1961.

D. A. Nield and A. Bejan, *Convection in porous media* , Springer-Verlag, 1992.

Let us begin with the effective equations over macroscales.

Recall the definitions of seepage velocity,

$$\mathbf{u} = \frac{1}{\Omega}\iiint_{\Omega_f} \mathbf{q}\, dV \tag{6.4.1}$$

and of porosity:

$$n = \frac{\Omega_f}{\Omega} \tag{6.4.2}$$

Mass conservation for an incompressible fluid:

$$\nabla \cdot \mathbf{u} = 0 \tag{6.4.3}$$

Momentum conservation (Darcy 's law)

$$0 = -\nabla p - \frac{\mu}{k}\mathbf{u} + \rho_f \mathbf{g} \tag{6.4.4}$$

Inview of the small size of the solid grains heat diffuses by conduction almost instantly in time scales of geothermal interest. We therefore assume the temperature in both phases to be equal,

$$T_s = T_f = T \tag{6.4.5}$$

The energy conservation law in each phase is :
Solid

$$(1-n)(\rho C)_s \frac{\partial T}{\partial t} = (1-n)K_s \nabla^2 T \tag{6.4.6}$$

fluid:

$$n(\rho C)_f \frac{\partial T}{\partial t} + (\rho C)_f \mathbf{u}\cdot\nabla T = nK_f \nabla^2 T \tag{6.4.7}$$

Adding the two we get the energy equation for the mixture

$$(\rho C)_m \frac{\partial T}{\partial t} + (\rho C)_f \mathbf{u} \cdot \nabla T = K_m \nabla^2 T \tag{6.4.8}$$

where

$$(\rho C)_m = (1-n)(\rho C)_s + n(\rho C)_f, \quad K_m = (1-n)K_s + nK_f \tag{6.4.9}$$

Defining

$$\sigma = \frac{(\rho C)_m}{(\rho C)_f} \tag{6.4.10}$$

and

$$\kappa_m = \frac{K_m}{(\rho C)_f} \tag{6.4.11}$$

as the mixture diffusivity, we can rewrite (6.4.8) as

$$\sigma \frac{\partial T}{\partial t} + \mathbf{u} \cdot \nabla T = \kappa_m \nabla^2 T \tag{6.4.12}$$

Note: Strictly speaking, the above equation ought to be derived from the microscale considerations, i.e., by homogenization analysis.

Equation of state:

$$\rho_f = \rho_o[1 - \beta(T - T_o)] \tag{6.4.13}$$

In summary the governing eqautions are (6.4.3), (6.4.4), (6.4.12) and 6.4.13).