1.138J/2.062J, WAVE PROPAGATION

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Please abide by the following rules of this exam, strictly

- You can use handouts, your own notes and homework and mathematical handbooks. Do not use any other references, printed or handwritten.
- If you have any question regarding the questions and the exam, ask me only . Do not ask others even just for clarification of the exam. In case the questions are of general interest, I will inform all of you by E-mail.

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1. A straight elastic tube of mean radius a containing incompressible fluid communicates with a branch tube of equal radius. The branch has a finite length $L \gg a$ and is closed at the end. A monochromatic incident waves of frequency ω approaches from $x \sim -\infty$. Find the reflected and transmitted waves and the waves in the branch. Examine the physical effects of different frequencies.

2. A long and taut string with uniform tension T is laterally supported on a nonuniform elastic foundation. The foundation elasticities are constants in Region 1 (K_1 in $-\infty < x < -a$) and Region 3 (K_3 in $a < x < \infty$) with $K_1 \neq K_3$, and varying in the middle $K(x)$ in $(-a < x < a)$. Examine the scattering of a monochromatic incident wave train of frquency ω from $x \sim -\infty$.

i Derive the explicit solution for the simple case where $K = K_2$ =constant in the middle section $-a < x < a$.

ii For arbitrary $K(x)$, derive a general identity relating the energy fluxes of the reflected, transmitted, and incident waves.

iii Check whether your explicit solution for the special case satisfies this identity.

3. Recall the dam-reservoir exercise of HW no. 1. An infinite channel of rectangular cross section is an acoustic duct. The three dimensional velocity potential $\phi(x, y, z)$ is governed by

$$\frac{1}{c^2}\frac{\partial^2\phi}{\partial t^2} = \frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial y^2} + \frac{\partial^2\phi}{\partial z^2} \tag{1}$$

with the boundary conditions on the lateral boundaries.

$$\phi = 0, \quad y = 0 \tag{2}$$

$$\frac{\partial\phi}{\partial y} = 0, \quad y = -h, \tag{3}$$

$$\frac{\partial\phi}{\partial z} = 0, \quad z = \pm a. \tag{4}$$

Consider two-dimensional propagating waves of the form:

$$\phi = f(y)e^{i\alpha x - i\omega t} \tag{5}$$

Find all possible solutions of $f(y)$ and the corresponding values of α, i.e., f_n, α_n with $n = 0, 1, 2, \cdots$. Discuss the dispersion relations between ω and α_n for all n and the corresponding solutions.

4. *Sound below the sea surface.* Sound signals from a submarine object (fish, submarine) must often be analyzed by accounting for environmental factors such as surface waves, sandy seabed, ice cover, etc. Consider a plane monochromatic sound wave incident on the sea surface from below. The sea surface is covered by gravity waves of a single wave length.

$$\zeta = \frac{A}{2}\left(e^{iKx} + e^{-iKx}\right) \tag{6}$$

Because of its much lower speed, the sea wave appears as if stationary relative to sound. Impose zero pressure condition at the sea surface,

$$p(x, z, t) = 0, \quad z = \zeta(x) \tag{7}$$

and assume the amplitude of surface waves to be very small. $KA \ll 1$.

i Approximate the boundary condition (7) by Taylor expansion abvout the mean sea surface :

$$p(x, \zeta, t) = p(x, 0, t) + \zeta\left.\frac{\partial p}{\partial z}\right|_{z=0} + \left.\frac{\zeta^2}{2}\frac{\partial^2 p}{\partial z^2}\right|_{z=0} + \cdots \tag{8}$$

ii Try a perturbation solution

$$\phi = \phi_0 + \phi_1 + \phi_2 \cdots \tag{9}$$

$$p = p_0 + p_1 + p_2 \cdots \tag{10}$$

$$\vec{u} = \vec{u}_0 + \vec{u}_1 + \vec{u}_2 \cdots \tag{11}$$

in ascending orders of $O(KA)$. At the leading order the solution should be that of reflection from a plane surface. Find the solution.

iii Find at the next order the scattered sound by the sea surface roughness. What are the properties of the scattered waves? Examine all possible directions, wavenumbers and amplitudes of the scattered waves.