1.138J/2.062J, WAVE PROPAGATION

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Homework no. 2

Due September 28, 2000

In all exercises, please describe the physical meaning of your mathematical results. Use graphics if it can help the explanation. Use of Matlab and/or Maple is uncouraged.

1. Read §1. Chapter one, Notes.

Consider an infinitely long string taut with tension T, $-\infty < x < \infty$ free from any lateral support. A mass M is attached to the string at the origin. Show first that Newton's law for the mass requires that

$$M\frac{\partial^2}{\partial t^2}V(0,t) = -T\frac{\partial}{\partial x}V_-(0-,t) + T\frac{\partial}{\partial x}V_+(0+,t), \quad t > 0. \tag{H.2.1}$$

where V_- represents the displacement on the left side ($x < 0$) and V_+ on the right ($x > 0$). An incident pulse with duration T and length $L = cT$ arrives from $x \sim -\infty$. The incident wave is prescribed by

$$V_i(x,t) = \begin{cases} \sin(\pi(t - x/c)/T), & t < 0, -L < x < 0, \\ 0, & t < 0; -\infty < x < -L, x > 0 \end{cases} \tag{H.2.2}$$

Note that the front of the pulse arrives at the origin just at $t = 0$, i.e., $V_i(0,0) = 0$. Find the reflected and the transmitted waves and the motion of the mass.

2. A semi-infinite cylindrical rod of uniform cross section S is made of two materials. The elastic constant is E_1 in $0 < x < L$ and E_2 in $x > L$. Before $t = 0$ the rod is free of any loading. After $t = 0$ a pulse-like force is applied at the left end $x = 0$. Specifically, the total applied force is

$$F(0,t) = \begin{cases} F_o \sin(\pi t/T), & 0 < t < T, \\ 0, & t < 0, t > T \end{cases} \tag{H.2.3}$$

Find the displacment $u(x,t)$ everywhere after $t > 0$. Assume that $L > cT$.

3. Derive the linearized equation for wave propagation in an artery where there is a steady and uniform flow of velocty U. You can start from eqs(5.1), (5.5) and (5.6), Chapter 1, let $u = U + u'$ with $u' \ll U$ and assume that $U = O(c_o)$. Heuristically you can drop products like $O((a')^2, a'u', (u')^2)$, but you must retain terms like $O(Uu', Ua')$. Use scaling arguments to find the condition for linearization.

4. Consider forced waves governed by

$$\frac{\partial^2 u}{\partial t^2} - c^2 \frac{\partial^2 u}{\partial x^2} = h(x,t) \tag{H.2.4}$$

where the forcing is limited in range and duration so that h is constant h_o in a triangular region in the $x - t$ plane and zero elsewhere. The triangle has the base $-L < x < L$ along $t = 0$ and the apex at $t = L/c$. Find the wave for all $t > 0$ and $|x| < \infty$.