

1.138J/2.062J, WAVE PROPAGATION

Fall, 2000 MIT

Notes by C. C. Mei

Homework set No. 1

1. A membrane is kept taut over an area S . The membrane has a constant density ρ per unit area and is under uniform tension in all directions. Derive the governing equation for the lateral displacement $u(x, y, t)$ of the membrane vibrating under distributed loading of $p(x, t)$ per unit area.

2. Consider the longitudinal vibration of a cylindrical rod with one end at $x = 0$ fixed and the other end at $x = L$ attached to a mass M . Before $t = 0$ the rod is compressed by the length ϵL with $\epsilon \ll 1$. At $t = 0$ the compression is released. State the governing equation and all boundary and initial conditions.

3. Consider the torsional vibration of a cylindrical rod of circular cross section of radius a . Let $\theta(x, t)$ = angular displacement of the cross section at x , $d\sigma$ = area element in the cross section and located at the distance r from the axis, see figure (). Let τ be the shear stress, G the shear modulus of elasticity, and ϕ angular displacement of a line originally parallel to the axis. Show that

$$\phi = r \frac{\partial \theta}{\partial x} \quad (\text{H.1.1})$$

Invoke Hooke's law $\tau = G\phi$ and show that the total torque applied to the cross section at x is

$$M = G \frac{\partial \theta}{\partial x} \iint_S r^2 d\sigma = GJ \frac{\partial \theta}{\partial x} \quad (\text{H.1.2})$$

where

$$J = \iint_S r^2 d\sigma \quad (\text{H.1.3})$$

is the polar moment of inertia of the cross section. Let I be the moment of inertia per unit length of the rod. Show that

$$\frac{\partial^2 \theta}{\partial t^2} = \frac{GJ}{I} \frac{\partial^2 \theta}{\partial x^2} \quad (\text{H.1.4})$$

4. During an earthquake, water in a reservoir exerts hydrodynamic pressure on a dam that may fail. Formulate the dam-reservoir interaction problem under the following

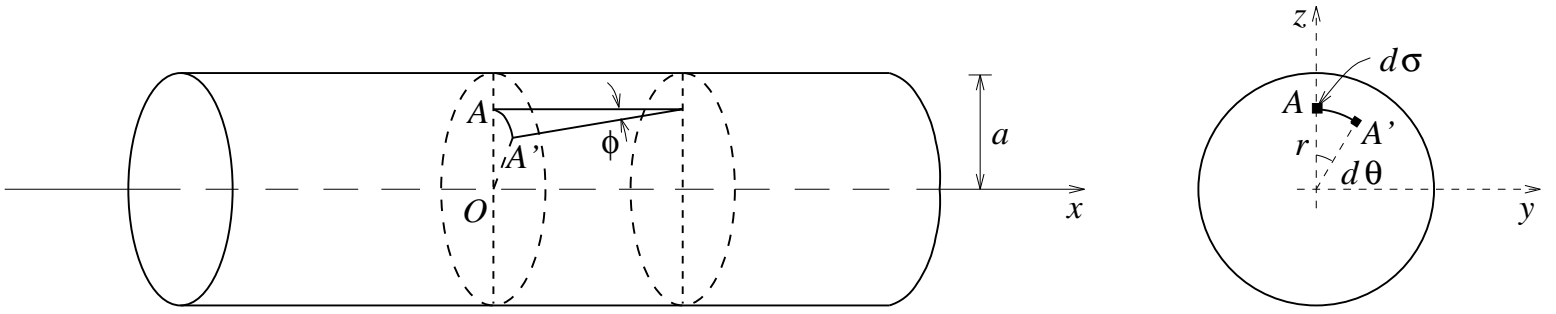


Figure 1: Torsion of a circular cylinder

idealizations. The reservoir is infinitely long and has a uniform rectangular cross section. Water is present only on one side of the dam ($x > 0$) and has the constant depth h . Before $t = 0$, all is calm. After $t = 0$ the dam is forced to vibrate horizontally so that

$$u(0, y, z, t) = \begin{cases} u_o(x, y, t) = \text{prescribed}, & 0 < t < T, \\ 0, & t > T. \end{cases} \quad (\text{H.1.5})$$

The free surface is exposed to constant atmospheric pressure. The reservoir bottom is rigid and does not vibrate vertically (What an idealization!!!). Neglect gravity but consider compressibility of water because of the high frequency ($\sim O(100)\text{Hz}$). Express all governing equations including the boundary conditions in term of the velocity potential ϕ defined by $(u, v, w) = \nabla\phi$. Show in particular that

$$\frac{1}{C^2} \frac{\partial^2 \phi}{\partial t^2} = \nabla^2 \phi \quad (\text{H.1.6})$$