1.138J/2.062J, WAVE PROPAGATION
Fall, 2000 MIT
Final exam, Dec. 1-7, 2000

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You are asked to abide by the following rules of this exam, strictly

- *You can use handouts, YOUR OWN notes and homework. Do not use any other references, printed or handwritten from anyone and anywhere.*
- *If you have any question regarding the questions and the exam, ask ME only . Do not ask others even just for clarification of the exam. In case the questions are of general interest, I will inform all of you by E-mail.*

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1. Refering to figure , a lighter fluid of density ρ' rests on top a heavier fluid of density ρ. Let the mean position of the interface be $z = 0$ and th etow fluid be infinitely deep. Ignore viscosity and compressibility so that both air and water can be described by velocity potentials which satisfy the Laplace equation. Now consider a spatially sinusoidal wave train on the interface so that the vertical displacement is

$$\zeta = \Re A e^{i(kx-\omega t)} \tag{1}$$

(i) Ignore surface tension, solve the potentials in water and in air ϕ and ϕ', and find the dispersion relation between ω and k. (ii). If at time $t = 0$ the interface is displaced to

$$\zeta(x,0) = \frac{Sb}{\pi(x^2+b^2)} \tag{2}$$

How does the interface vary in space and time for fixed x/t and large t? Use your approximate formula to describe snap shots at large and different times.

2. A pressure distribution moves from right to left at constant speed on the flat ground surface $y = 0$. The ground is an infinitely deep elastic half space $y < 0$. In the coordinate system where the loading is stationary, the wave equations for the elastic potentials ϕ and H can be formally solved in terms of characteristics

$$x + \beta_i y, \quad \text{and} \quad x - \beta_i y, \quad i = 1, 2 \tag{3}$$

Apply the boundary conditions to solve for ALL the stress components in the elastic half space, and sketch your anwers for a rectangular load:

$$P(x) = \begin{cases} P_o, & |x| < L, \\ 0, & |x| > L \end{cases} \tag{4}$$

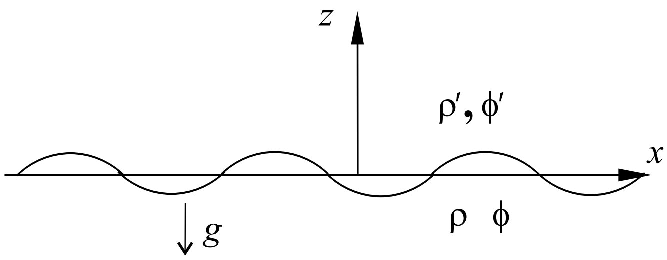

Figure 1:

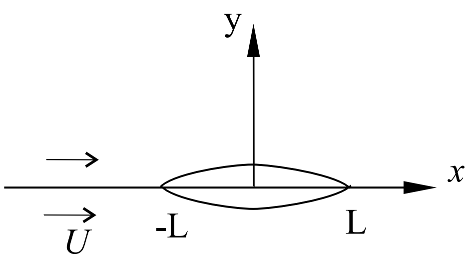

Figure 2:

(If your solution differs from those in the handout, just make sure of you own.) 3. Refering to figure , a thin airfoil of symmetrical cross section moves at the constant speed U along its axis (x axis) in a compressible air. Sound waves can be generated. For sufficiently slender airfoil linear equations suffice, so that in the nmoving coordinates foixed on the airfoil, the steady-state velocity potential satisfies

$$U^2 \frac{\partial^2 \phi}{\partial x^2} = c^2 \left(\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} \right) \tag{5}$$

Let the cross section be given by

$$y = \pm h(x), \quad \text{for} - L < x < L \;\; \text{where} \;\; h/L \ll 1 \tag{6}$$

(i). Ignore viscosity and derive first the approximate boundary condition represent no flux on the airfoil. (ii). Solve for the steady wave generated when $U > c$ (supersonic). (iii) For a parablolic profile $h(x) = h_o(1 - x^2/L^2)$, find the wave drag in the x direction by somehow integrating the pressure on the airfoil.

4. Ocean or atmospheric wave guide. Due to variations of temperature (or salinity in oceans), the fluid density and sound speed can be nonuniform in depth. Under certain

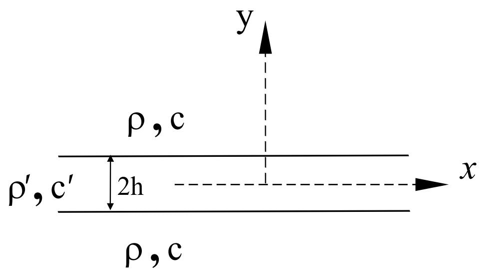

Figure 3:

conditions waves can propagate only in and near a part of the sea depth, hence they are are trapped in side a layer which then serves as a waveguide. Guided waves can travel a long horizontal distance before losing energy to dissipation.

Refering to figure , study a model ocean with three layers. The depth of the middle layer is $2h$. In the top and bottom layers the densities are ρ, c and in the middle layer ρ', c'. Study waves modes that propagate in the x direction with frequency ω and wave number α, but exponentially attenuating as $y \to \pm\infty$. Specifically (i) find the condition under wihc trappewaves gudes are possible e.g. should $c > c'$ or $c < c'$?. (ii) How many modes are possible modes for given $\omega, c, c', \rho, \rho'$ and h. (iii) What is the dispersion relation between ω and α for each mode? (iv) What is the vertical variation of pressure or velocity of each mode?

5. Refering to figure . A sinusoidal P-wave train is incident obliquely towards a semi-infinite crack along the positive $x-$axis. Find by the parabolic approximtion waves near the boundary rays of the reflected P and SV waves.

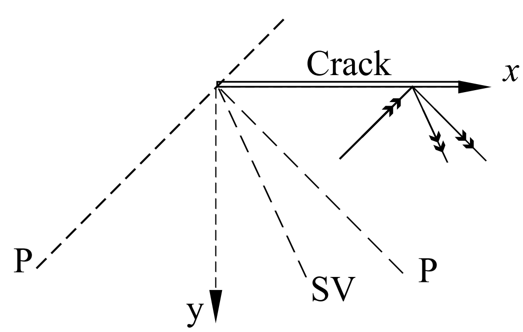

Figure 4: