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Abstract

This paper investigates the opportunities for risk hedging available to competitive electric power suppliers through the use of forward contracts. We formulate the production- and marketing-decision process of suppliers as a two-stage optimization problem. This optimization problem is solved employing the dynamic programming technique given the mean-variance cost function. Due to the unique characteristics of uncertainties in electricity markets, it is shown that the production decisions and the marketing decisions are interrelated, dissimilar to the earlier results. This is the direct consequence of using the two-stage model, which explicitly considers the inter-temporal effects. A more general formulation over many time periods is also presented; however, its complexity renders it difficult to solve.

Keywords

Forward contracts, Futures market, Mean-variance cost function, Unit commitment, Risk management

I. Introduction

The production- and marketing-decisions by a supplier whose product is subject to a price uncertainty has been a topic of extensive studies in the literature. This problem is of a particular interest in commodity markets where there may be a significant volatility in spot prices. A spot price is the price at which a commodity is traded for immediate delivery. We refer the marketplace where the spot prices prevail as a spot market.

One of the most common methods used to deal with this spot price uncertainty is the risk hedging through forward (delivery) contracts which are the contracts to buy or sell the commodity at a fixed time in the future at a pre-specified price. We call this pre-specified price, a forward price, and the marketplace where the commodity is traded based on forward contracts, a futures market. In a futures market, suppliers can commit some or all of their outputs at the forward price before the actual production. By entering into forward contracts, the risks on profit stemming from the uncertainty in spot prices can be eliminated for the amount of output committed in the contracts.

The purpose of this paper is to investigate the opportunities for risk hedging available to competitive electric power suppliers through the use of forward contracts. We focus on the decisions by an individual supplier for the prevailing forward and spot prices, and derive an explicit decision rule that incorporates the attitude towards the risks (on her income).

This paper is organized as follows:

In Section II we define a forward contract and a futures market for electricity and give interpretations of the forward price of the commodity. Section III presents the two-stage model for profit of electricity suppliers, as a function of generation cost, revenues from sales, and the gain or loss due to forward contract commitments. We solve for the production- and marketing-decision rules using the dynamic programming technique given the mean-variance cost function in Section IV. Concluding remarks are made in Section V.

II. Forward Contracts and Futures Market

The merit of a futures market lies on its ability to provide a risk management tool called forward contracts through which buyers and sellers can reduce parts of their profit that is exposed to the risks from the volatility in spot prices. For example, suppose an agricultural supplier, i faces a price uncertainty in the spot market for the crop that he plants in the spring and sells in the following fall. We assume that his only production cost is the one time investment he makes for planting in the spring, C_i:

$$C_i(Q_i) = a_i Q_i^2 + b_i Q_i + c_i \tag{1}$$

where Q_i is the quantity of the crop he produces in the fall as the result of his investment, C_i. Not surprisingly the cost of investment is a function of the output. Suppose the spot market for this particular crop is perfectly competitive; i.e., no one supplier can alter the spot prices by changing his output.

Without the presence of futures market, the amount of crop the farmer, i, sells in the spot market, Q_s, is equal to his entire output, Q_i. For the profit-maximization cost function, his optimal output may be derived by solving

$$\min_{Q_i} \mathcal{E} \left\langle C_i(Q_i) - p_s Q_i \right\rangle, \tag{2}$$

which yields

$$Q_i^\star = \frac{1}{a_i} \left[\mathcal{E} \left\langle p_s \right\rangle - b_i \right] \tag{3}$$

where $\mathcal{E} \langle \cdot \rangle$ denotes the expected value operator.

Suppose the spot market price has the log-normal probability distribution, i.e.

$$f_{\mathbf{p_s}}(p_s) = \begin{cases} \frac{1}{\sqrt{2\pi}\sigma p_s} \exp\left[- \left(\ln p_s - \alpha \right)^2 / \left(2\sigma_2 \right) \right] & p_s > 0 \\ 0 & \text{otherwise} \end{cases} \tag{4}$$

Then, the optimal production amount is given by

$$Q_i^\star = \frac{1}{a_i} \left[\exp\left(\alpha + \frac{1}{2}\sigma^2 \right) - b_i \right] \tag{5}$$

since $\mathcal{E} \langle p_s \rangle = \exp\left(\alpha + \frac{1}{2}\sigma^2 \right)$.

If the farmer, i, makes the investment corresponding to the optimal output given in Eq. (3), then his profit, $\pi_{\mathbf{i}} = p_s Q_i - C_i(Q_i)$ also has the log-normal distribution whose expected value and the variance are given by

$$\begin{aligned} \mathcal{E} \langle \pi_{\mathbf{i}} \rangle &= Q_i^\star \mathcal{E} \langle p_s \rangle - C_i(Q_i^\star) \\ &= \tfrac{1}{a_i^2} \Big[(a_i - 1) \exp\left(2\alpha + \sigma^2 \right) + 2b_i \exp\left(\alpha + \tfrac{1}{2}\sigma^2 \right) \\ &\quad + b_i^2 - a_i c_i \Big] \end{aligned} \tag{6}$$

and

$$\begin{aligned}\text{var}(\pi_{\mathbf{i}}) &= \left(Q_i^\star\right)^2 \text{var}(p_s) \\ &= \tfrac{\exp(\sigma^2)}{a_i^2}\left[\exp\left(\alpha+\tfrac{1}{2}\sigma^2\right)-b_i\right]^2\left[\exp(\sigma^2)-1\right]\end{aligned} \tag{7}$$

since $\text{var}(p_s) = \exp(\sigma^2)\left[\exp(\sigma^2) - 1\right]$.

Suppose the farmer now has an option that he can sell any amount of his output at the futures market in the spring for the delivery in the following fall. With the introduction of futures market, the profit-maximization cost function in Eq. (2) is modified to include the amount of crop sold at the futures markets as

$$\min_{Q_i, Q_f} \mathcal{E}\left\langle C_i(Q_i) - p_f Q_f - p_s\left(Q_i - Q_f\right)\right\rangle, \tag{8}$$

where p_f and Q_f represent the forward price and the amount of crop sold at the futures market respectively. The solution to Eq. (8) is given as

$$Q_i^\star = \frac{1}{a_i}\left[\exp\left(\alpha + \frac{1}{2}\sigma^2\right) - b_i\right] \tag{9}$$

$$Q_f^\star = \begin{cases} +\infty & p_f > \mathcal{E}\langle p_s\rangle \\ \frac{1}{a_i}\left[\exp\left(\alpha+\frac{1}{2}\sigma^2\right)-b_i\right] & p_f = \mathcal{E}\langle p_s\rangle \\ -\infty & \text{otherwise} \end{cases} \tag{10}$$

From Eq. (9) we note that the optimal output does not depend on the presence of futures market. Intuitively, this implies that the decision of supplier can be decoupled as production- and marketing-decisions. When making the production decisions, a supplier is only concerned with the probability distribution of spot prices and not of forward prices. On the other hand, the marketing decisions are determined based on the *relationship* between the forward prices and the expected value of spot prices as shown in Eq. (10). Suppose the forward prices are set above the expected value of the spot prices. In this case the optimal (marketing) decision rule prescribes for an infinite amount of sales at the futures market. Even though he does not produce an infinite amount of crops himself, the assumption regarding the perfect competition at the spot market made earlier in the section allows the supplier to supply the amount committed in the forward contracts with the purchase of an infinite amount of crop at the spot market on the delivery day, which approach yields the expected value of the supplier's profit to be an infinity. When the forward prices are set below the expected spot prices, the supplier can again obtain the expected value of an infinite profit by buying at the futures market and selling at the spot market. The above are the classic examples of a well-known arbitrage opportunity created by the price discrepancies between spot and futures markets. This suggests, in a mature market environment, the forward prices have to have the interpretation of being the expected value of spot prices.

After making the assumption that forward prices and the expected value of spot prices are equal, we deduce that the optimal decision for a supplier is

$$\begin{aligned} Q_i^\star &= \tfrac{1}{a_i}\left[\exp\left(\alpha + \tfrac{1}{2}\sigma^2\right) - b_i\right] \\ &= Q_f^\star \end{aligned} \tag{11}$$

The expected value and the variance of the supplier's profit are given by

$$\begin{aligned} \mathcal{E}\left\langle \pi_{\mathbf{i}} \right\rangle &= \tfrac{1}{a_i^2}\left[(a_i - 1)\exp\left(2\alpha + \sigma^2\right) + 2b_i \exp\left(\alpha + \tfrac{1}{2}\sigma^2\right)\right. \\ &\quad \left. + b_i^2 - a_i c_i\right] \\ &= \pi_{\mathbf{i}} \end{aligned} \tag{12}$$

and

$$\mathrm{var}(\pi_{\mathbf{i}}) = 0 \tag{13}$$

i.e., the profit is a deterministic value as all of supplier's output is sold at the futures market before production. With the use of forward contracts, the suppliers are able to eliminate the risks associated with the volatility in spot prices.

In the electricity industry where there is no practical means for storage, the volatility in spot prices is quite significant. Thus, most market scenarios for the deregulation of electricity anticipate the development of a futures market, including basic call and put options [7], forward contracts [13], callable forwards [9], and bid-based power pools [10]. Interruptible contracts [15], also known as recallable contracts or non-firm contracts, may be treated as callable forwards.

The result given in Eqs. (9) and (10) is consistent with the earlier literature. In [5], [11] and [8] it is shown that when production is non-stochastic with only uncertainty in the spot price, the optimal production rule is to set marginal cost equal to the forward prices as given in Eq. (11). Since the optimal decision rule based on the given profit-maximization cost function results in a deterministic value for profit, the same decision rule is also optimal when the optimization problem is solved based on the mean-variance function. Indeed, the solution to

$$\min_{Q_i, Q_f} \mathcal{E} \langle \pi_{\mathbf{i}} \rangle + \lambda \text{var}(\pi_{\mathbf{i}}) \tag{14}$$

matches the expressions in Eqs. (9) and (10), assuming the forward price is the expected value of the spot price. In Eq. (14), the supplier's preference in risk aversion reflected in λ.

However, suppliers in many industries are faced with production uncertainty along with price uncertainty. When the output is stochastic, the above generalization is, in general, not valid. In [2] the optimal decision rule different from Eqs. (9) and (10) is derived for the mean-variance cost function when there are uncertainties in production and price; it is shown that the optimal decision rule considers the supplier's preference in risk aversion and that the decoupling assumption between the production- and the marketing-decision cannot be made in general.

In electricity markets, there is a significant uncertainty in production. Many markets allow the production decisions to be made by the market makers called the system operators rather than by individual suppliers. For example, in the Pennsylvania-New Jersey-Maryland (PJM) market, suppliers initially bid supply curves to the system operator. The system operator subsequently determines the optimal generation output by individual suppliers using the optimal power flow (OPF) program.[1] Finally, the suppliers generate the amount of electricity determined by the system operators at the spot prices. The reason that the electricity industry operates this way is due to the strict requirement that the instantaneous supply of electricity has to continuously balance the instantaneous demand.

The coupling between production- and marketing-decisions becomes stronger if we relax the assumption of the forward price being the expected value of the spot price. The deregulation process of electricity industry is still at its infant stage. Plus, in many electricity markets being operated within the U.S., trades are conducted on the hourly basis in the spot markets as loads vary hour by hour, while futures markets allow trades only on the monthly basis. This results in a considerable discrepancies between the spot prices and the forward prices. Thus, in formulating optimal decision rule for electricity supplier, it is preferable to use the mean-variance cost function formulation given in Eq. (14) than to use Eq. (8).

In the following section, we describe the so-called decentralized unit commitment formu-

[1]The Optimal power flow program determines the most economical mix of generation for given system load.

lation for modeling the decision process of electricity suppliers.

III. Supplier's Profits and Risks in Unit Commitment Problem

Determining unit commitment decisions is an essential problem to the suppliers in the electricity markets. The problem determines the optimal production level of each individual generator over a certain period of time, usually one week. The decisions are typically made at daily intervals for each of 24 hour decision horizon. An extensive study of this problem is performed in [1]. In the study, the problem is posed as a stochastic optimization as described below.

Let us consider the case of a single generator. If the generator is turned on, a startup cost of S is incurred. Similarly, it requires a shutdown cost, T, when the generator is turned off. Each generator must also observe minimum up time and minimum down time constraints; a generator may not be on for fewer than t_{up} consecutive hours or or off for less than t_{dn} consecutive hours. Given these constraints, the profit of an electricity supplier at hour k, $\pi[k]$, is given by

$$\begin{aligned}\pi[k] &= u[k]\left(p_s[k]Q_G[k] - C_G(Q_G[k]) - I(x[k] < 0)S\right) \\ &\quad - (1 - u[k])\left(c_f + I(x[k] > 0)T\right) \qquad (15)\end{aligned}$$

where

u	: Decision on production, 1 if the unit is on, 0 otherwise
Q_G	: Production output
C_G	: Cost of production
I	: Conditional statement, 1 if true, 0 otherwise
x	: Unit status, to turn on, $x[k] < t_{dn}$, and to turn off, $x[k] > t_{up}$
c_f	: Fixed costs incurred when the generator is off

Here we assume that the suppliers submit to the market maker/system operator supply bids at the beginning of each hour k. The supply bid describes how much the supplier is willing to produce at various spot prices. The spot price, $p_s[k]$ is determined exogenously, and the system operator decides on the output by individual suppliers based on respective supply bids.

From an individual supplier perspective the optimal output is then computed by solving

$$\min_{u[k],Q_G[k](p_s[k])} \sum_{k=0}^{23} -\pi[k] = \min_{u[k],Q_G[k](p_s[k])} \sum_{k=0}^{23} (1-u[k]) \tag{16}$$
$$\times (c_f + I(x[k]>0)T) - u[k]\,(p_s[k]Q_G[k]$$
$$-C_G(Q_G[k]) - I(x[k]<0)S)$$

for the profit-maximization cost function.

The dynamic programming technique is well suited for solving problems of the form in Eq. (16) in which each stage of the problem is a one hour time interval. The unit status, $x[k]$ and the spot price, $p_s[k]$ are treated as the evolving states of this system in applying the dynamic programming technique. The state transition equation of x_k is given by [14]:

$$x[k+1] = \begin{cases} \max(1, x[k]+1) & : \ u_k = 1 \\ \min(-1, x[k]-1) & : \ u_k = 0 \end{cases} \tag{17}$$

The state transition equation of $p_s[k]$ is formulated as an appropriate stochastic process. For example,

$$p_s[k+1] = p_s[k] + \alpha \ln\left(\frac{\bar{p_s}}{p_s[k]}\right) + w[k] \tag{18}$$

where $\bar{p_s}$ and $w[k]$ are the perceived mean of spot prices and random white noise process respectively. Clearly Eq. (16) is a stochastic optimization problem with the stochastic state transition equation such as Eq. (18). Due to the nonlinear constraints such as minimum up time and minimum down time, no general closed form solution is derived. The details of numerical solution to Eq. (16) is presented in [1].

Suppose we extend the result of solving the unit commitment problem to include the presence of the futures market where electricity suppliers can sell any amount of their outputs through forward contracts. Then, the profit of a electricity supplier at hour k, $\pi[k]$, given in Eq. (15) becomes

$$\pi[k] = \sum_{i=0}^{k-1} p_{f,i}[k]Q_{f,i}[k] + u[k](p_s[k](Q_G[k] - \sum_{i=0}^{k-1} Q_{f,i}[k]) - C_G(Q_G[k]) - I(x[k]<0)S) \tag{19}$$
$$-(1-u[k])(c_f + p_s[k]\sum_{i=0}^{k-1} Q_{f,i}[k] + I(x[k]>0)T)$$

where

$p_{f,i}[k]$	:	Forward prices at stage i for delivery at stage k
$Q_{f,i}[k]$	:	Amount of electricity to be delivered at stage k at forward price, $p_{f,i}[k]$

In [1], the solution to Eq. (19) is derived as

$$Q_G[k] = \frac{\mathcal{E}\left\langle p_s[k] \right\rangle - b}{2a} \tag{20}$$

$$Q_{f,k-1}[k] = \frac{\mathrm{cov}(p_s^2[k], p_s[k]) - 2b}{4a\mathrm{var}(p_s[k])} \tag{21}$$

using the one-stage model for the quadratic generation cost function, i.e.,

$$C_G(Q_G[k]) = aQ_G^2[k] + bQ_G[k] + c \tag{22}$$

For the profit-maximization cost function, the solution in Eqs. (20) and (21) indicates the decoupling between the production- and the marketing-decisions, and the forward price is implicitly and *necessarily* assumed to be the expected value of spot prices. This suggests that in order to capture the inter-temporal effect of forward contracts, the problem needs to be formulated using the mean-variance cost function on at least the two-stage model.

IV. Mean-variance Hedging Strategy

This section, we present the two-stage model for unit commitment problem using the mean-variance cost function.

Assuming the generator is on, we can simplify Eq. (19) of the supplier's profit at the stages k and $k+1$, as

$$\pi[k] = p_s[k]Q_G[k] + p_{f,k+1}Q_{f,k+1}[k] - C_G(Q_G[k]) \tag{23}$$

$$\pi[k+1] = p_s[k+1](Q_G[k+1] - Q_{f,k+1}[k]) - C_G(Q_G[k+1]) \tag{24}$$

At the stage k, the supplier makes the marketing decision to sell the amount, $Q_{f,k+1}[k]$, of electricity for the delivery at the stage $k+1$ through the forward contract and the production decisions for the stages k and $k+1$, by solving a stochastic optimization problem given the mean-variance cost function:

$$\min_{Q_G[k], Q_G[k+1], Q_{f,k+1}[k]} \lambda \mathrm{var}(\pi[k] + \pi[k+1]) - \mathcal{E}\left\langle \pi[k] + \pi[k+1] \right\rangle \tag{25}$$

Combining Eqs. (25), (23) and (24) the two-stage forward contract problem then may be solved by using the dynamic programming technique:

$$J_{k+1}(p_s[k+1]) = \lambda \mathrm{var}(\pi[k+1]) - \pi[k+1] \tag{26}$$

$$J_k = \min_{Q_G[k], Q_G[k+1], Q_{f,k+1}[k]} \mathcal{E}\left\langle \lambda \mathrm{var}(\pi[k]) - \pi[k] + J_{k+1} \right\rangle \tag{27}$$

The solution to Eqs. (26) and (27) is given by

$$Q_G[k] = \frac{\mathcal{E}\left\langle p_s[k]\right\rangle - b}{2a + 2\lambda \mathrm{var}(p_s[k])} \tag{28}$$

$$Q_G[k+1] = \frac{\mathcal{E}\left\langle p_s[k+1]\right\rangle - b + 2\lambda \mathrm{var}(p_s[k+1]) Q_{f,k+1}[k]}{2a + 2\lambda \mathrm{var}(p_s[k+1])} \tag{29}$$

$$Q_{f,k+1}[k] = \frac{p_{f,k+1}[k] - \mathcal{E}\left\langle p_s[k+1]\right\rangle}{2\lambda \mathrm{var}(p_s[k+1])} + \frac{\mathrm{cov}(p_s^2[k+1], p_s[k+1]) - 2b}{4a \mathrm{var}(p_s[k+1])} \tag{30}$$

It is interesting to analyze the solution at the limiting cases. If $\lambda = 0$, then Eqs. (28), (29) and (30) are reduced to Eqs. (20) and (21). This is due to supplier's indifference towards risk, thus solving the optimization problem solely based on the expected profit as in Eq. (30). If $\lambda = \infty$, then Eq. (28) reduces to zero, and Eq. (29) yields $Q_{f,k+1}[k]$. Without prior commitment through forward contracts in the stage k, the supplier opt for no generation in order to minimize the variance of his profit. In the stage $k+1$ the supplier produces just enough to fulfill the maturing forward contracts without regarding the spot prices. This allows the supplier to eliminate any variance in his profit.

The approach in Eqs. (26) and (27) can be generalized to the optimization problem over several stage without assuming generator status:

$$J_N(\mathbf{x}_N) = g_N(\mathbf{x}_N) \tag{31}$$

$$J_k(\mathbf{x}_k) = \min_{\mathbf{u}_k \in U_k(\mathbf{x}_k)} \mathcal{E}_{\mathbf{w}_k}\left\langle g_k(\mathbf{x}_k, \mathbf{u}_k, \mathbf{w}_k) + J_{k+1}(\mathbf{u}_k)\right\rangle \tag{32}$$

where the cost functions g_k are:

$$g_N(x_N) = \mathcal{E}\left\langle p_s[N](x_N - Q_G[N]) + C_G(Q_G[N])\right\rangle \tag{33}$$

$$
\begin{aligned}
g_k(x_k) &= p_{f,N}[k](x_k - u_k) \\
&\quad + \lambda \text{var}\left(\sum_{i=k}^{N} \pi[i]\right) - \lambda \text{var}\left(\sum_{i=k+1}^{N} \pi[i]\right)
\end{aligned}
\tag{34}
$$

This problem is theoretically solvable; however, it requires a large amount of price information, including the mean and variance of all future forward prices up to the delivery date and the expected variance of the spot price at future times, which means along with the state transition equation defined for the spot prices as given in the Eq. (18), the state transition equation for forward prices has to be determined as well. A simple example will be defining the spot and the forward prices as correlated mean-reverting stochastic processed of the form

$$
p_s[k+1] = p_s[k] + \alpha \ln\left(\frac{p_{f,k}[\bar{k} - 1]}{p_s[k]}\right) + w[k] \tag{35}
$$

$$
p_{f,i+1}[k] = p_{f,i}[k] + \beta \ln\left(\frac{\bar{p_f}}{p_{f,i}[k]}\right) + w'[k] \tag{36}
$$

where $\bar{p_f}$ is the perceived mean of forward price as a stochastic process, and $w'[k]$ is a random white noise process uncorrelated with $w[k]$. The cost function can also be amended to include both fixed and variable transaction costs. This is computationally very complex. A complete study of this problem is left for future research.

V. Conclusion

This paper describes the opportunities for risk hedging available to competitive electric power suppliers through the use of forward contracts. The forwards contracts have a payoff which increase as the spot prices drop and decrease as the spot prices rise. Futures markets for electricity are in their infancy. Nevertheless, suppliers can use the forward contracts to reduce the risk of future profit according to his preference at the expense of the expected profit. A complete solution is derived for the two-stage model. A more general optimization problem involving spot markets and futures markets can be defined, which calculates the optimal forward contract position over time as spot and forward prices for electricity evolve; however, this problem is very complicated to solve.

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References

[1] Allen, E., Stochastic Unit Commitment in a Deregulated Electric Utility Industry, Ph.D. thesis, Massachusetts Institute of Technology, 1998.

[2] Anderson, R., Danthine, J.-P., "The Determinants of the Volatility of Futures Prices", Working Paper No. 33, Center for the Study of Futures Markets, Columbia Business School, New York, NY, June 1982.

[3] Bertsekas, D., Dynamic Programming and Optimal Control: Volume I., Athena Scientific, Belmont, MA, 1995.

[4] Citizens Power, The US Power Market: Restructuring and Risk Management, Risk Publications, London, England, 1997.

[5] Danthine, J.-P. "Information, futures Prices, and Stabilizing Speculation", *Journal of Economic Theory* 17, pp. 79-98, February 1978.

[6] Dixit, A., Pindyck, R., Investment under Uncertainty, Princeton University Press, Princeton, NJ, 1993.

[7] Electric Power Research Institute, "Option Pricing for Project Evaluation: an Introduction", TR-104755, Research Project 1920-05, *Final Report*, 1995.

[8] Feder, G., Just, R., Schmitz, A., "Futures Markets and the Theory of the Firm under Price Uncertainty", *Quarterly Journal of Economics* 94, pp. 317-328, March 1980.

[9] Gedra, T., Varaiya, P., "Markets and Pricing for Interruptible Electric Power", *IEEE Transactions on Power Systems 8*, pp. 122-128, 1993.

[10] Hogan, W. W., "Contract Networks for Electric Power Transmission", *Journal of Regulatory Economics* 4, pp. 211-242, 1992.

[11] Holthausen, D., "Hedging and the Competitive Firm under Price Uncertainty", *American Economic Review* 69, pp. 989-995, December 1979.

[12] Kawai, M., "Spot and Futures Prices of Nonstorable Commodities under Rational Expectations", *Quarterly Journal of Economics* XCVIII, pp. 235-254, 1983.

[13] Kaye, R., Outhred, H., Bannister, C., "Forward Contracts for the Operation of an Electricity Industry under Spot Pricing", *IEEE Transactions on Power Systems* 5, pp. 46-52, 1990.

[14] Shaw, J., "A Direct Method for Security-constrained Unit Commitment," *IEEE Transactions on Power Systems,* Vol. 10, No. 3, pp. 1329-1342, August 1995.

[15] Tan, C.-W., Varaiya, P., "Interruptible Electric Power Service Contracts", *Journal of Economic Dynamics and Control* 17, pp. 495-517, 1993.