Reducing Technical Uncertainty in Product and Process Development Through Parallel Design of Prototypes

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Abstract

When developing a new product or process developers may conduct prototyping experiments to test the technical feasibility of design alternatives. We model product and process prototyping as combinations of Bernoulli experiments with known rewards, costs and success probabilities. Experimental outcomes are observed and the design with the highest observed reward is chosen. The model balances the cost of building and testing the prototypes against improvements in expected profits. We present a prototype design methodology that yields the optimal combination of Bernoulli trials with varying parameters and show how the mode of experimentation determines the preferred *type* of product.

1. Introduction

1.1. Research Objectives

We specify the optimal prototype design, or combination of prototypes, for the Bernoulli case and demonstrate how the mode of experimentation determines the preferred *type* of prototype.

We then consider the *prototype design* problem, namely: given a menu of possible prototypes, each characterized by its own payoff, probability of success, and cost*, what* prototypes should be built and *how many* of each should be developed in parallel?

The remainder of the paper proceeds as follows. Section 3 develops optimal hybrid policies that include parallel *and* sequential experiments. Section 4 addresses the problem of prototype design. Section 5 concludes the paper with a discussion of the results and managerial implications.

2. Discrete Outcome Homogeneous Parallel Prototypes

In this section we consider the simplest model of parallel prototyping experiments, each with the same potential reward, cost and success probability, conducted in a single period.

2.1. Example: Semiconductor Production Equipment

Applied Materials is a leading supplier of chemical vapor deposition (CVD) equipment and other process equipment used by manufacturers of semiconductor devices. Applied's CVD division pioneered the design of modular equipment that deposits and etches chemicals onto semiconductor wafers. Chipmakers such as Intel buy process equipment based on reliability, cost effectiveness, and performance, particularly the equipment's ability to squeeze smaller circuits more densely onto a given wafer. In short, improving the process for semiconductor manufacturing is the key to profitability for firms such as Applied Materials. Time-to-market matters a lot in this industry, since chipmakers commit to a highly integrated manufacturing process early on, when designing their fabs. After a point, changing the process technology becomes technically difficult and very costly.

In 1996, the CVD Division faced a competitive threat that was also an opportunity [Kori 1996]. A competitor had developed a new wafer-heating technology using ceramic hot plates rather than aluminum ones. Ceramic could generate higher temperatures more consistently, allowing the tungsten metal layer to be applied to the silicon wafer in a more precise manner. If Applied could develop it own ceramic heater for its modular CVD equipment the rewards would be significant and relatively predictable. But technological feasibility was uncertain due to the difficulty of maintaining a vacuum within a mixed-material (ceramic and aluminum) wafer-heater chamber. Engineers conducted brainstorming sessions, conceived of four potential designs, and identified several suppliers who could build and test these designs. The CVD division manager had to decide which designs to build and test in order to address this opportunity. We revisit this example in section 4.2.

2.2. The Single-Period Bernoulli model[1]

To model the outcomes of prototyping in a single period horizon, we consider a set of homogeneous Bernoulli trials. Each of n prototyping experiments costs c and results either in success with probability p, or in failure with probability $1-p$. Success in at least one trial gives rise to gross profit R, while failure in all n trials results in gross

[1] Model notation is summarized in Appendix 1.

profit of zero. No benefits are derived from achieving more than one success. R and c are assumed positive and $0 < p < 1$.

This model applies when the technology risk dominates and potential rewards are predictable. That is, the firm knows how profitable the innovation will be, but is uncertain as to its feasibility.

Unless specifically mentioned, the number of parallel prototypes, n, is treated as a continuous decision variable, even though in application it would be integer-valued.

The objective function for the Bernoulli model is given by:

$$\textbf{(1)} \qquad E[p_n] = R \cdot \left(1 - (1-p)^n\right) - c \cdot n .$$

It is easy to verify that (1) is strictly concave in n and is maximized at:

$$\textbf{(2)} \qquad n^* = \frac{\ln\left[\dfrac{-c}{R \cdot \ln(1-p)}\right]}{\ln(1-p)} .$$

The necessary and sufficient condition for a positive number of prototypes is

$$\textbf{(3)} \qquad R > \frac{c}{-\ln(1-p)} .$$

Condition (3) requires that the potential reward exceed the probability-adjusted cost of prototyping. Assuming (3) is met, then profit improves as c declines, $\dfrac{\partial E[p_{n^*}]}{\partial c} = -\dfrac{\ln\left[\dfrac{-c}{R \cdot \ln(1-p)}\right]}{\ln(1-p)} = -n^* < 0$. Higher values of p increase $E[p_{n^*}]$ since $\dfrac{\partial E[p_n]}{\partial p} = n \cdot R \cdot (1-p)^n$, which is positive for all n, but the same cannot be said of n^*,

the optimal number of parallel prototypes. Rather, n^* achieves its maximum when $p = 1 - e^{\frac{-c \cdot e}{R}}$, an interior point of (0, 1). This result derives from

$$(4) \qquad \frac{\partial n^*}{\partial p} = \frac{1 + \ln\left(\frac{-c}{R \cdot \ln(1-p)}\right)}{(1-p) \cdot [\ln(1-p)]^2},$$

which is positive for[2] $1 - e^{\frac{-c}{R}} < p < 1 - e^{\frac{-c \cdot e}{R}}$, negative for $1 - e^{\frac{-c \cdot e}{R}} < p < 1$, and zero when $p = 1 - e^{\frac{-c \cdot e}{R}}$. The fact that n^* is maximized for an interior value of p suggests that the optimal number of prototypes is highest when outcome variability, adjusted for parallelism, is maximized. In the case of Bernoulli trials, extremely high or low success probabilities reduce the variability of outcomes and lead to fewer parallel prototypes being built.

It is straightforward to verify that reducing the cost per experiment increases the optimal number of parallel prototypes $\left(\text{i.e.} \frac{\partial n^*}{\partial c} < 0\right)$, as expected. Increasing the potential reward, R, also pushes n^* higher $\left(\text{i.e.} \frac{\partial n^*}{\partial R} > 0\right)$.

How is total R&D spending affected by lower experimentation costs? We have

[2] **Note that $p \leq 1 - e^{\frac{-c}{R}}$ violates condition (3) and, therefore, need not be considered here.**

$$(5) \qquad \frac{\partial (n^* \cdot c)}{\partial c} = \frac{1+\ln\left(\frac{-c}{R\cdot \ln(1-p)}\right)}{\ln(1-p)},$$

which is positive when $0 < \frac{-c}{R\cdot\ln(1-p)} < \frac{1}{e}$. So reductions in the unit prototyping cost decrease total spending when *c* is small. If prototyping costs are sufficiently high, i.e., if $\frac{1}{e} < \frac{-c}{R\cdot\ln(1-p)} < 1$, then reductions in the unit cost per experiment will lead to *higher* total spending. Total spending is concave in *c* $\left(\frac{\partial^2 n^* \cdot c}{\partial c^2} = \frac{1}{c\cdot\ln(1-p)} < 0\right)$ and achieves a maximum at $c = \frac{R\cdot(-\ln(1-p))}{e}$, consistent with equation (4)'s result that the maximum number of prototypes occurs at this same point.

For example, with $R = 100$ and $p = ½$, optimal total spending peaks when *c* = 25.5, i.e., when the cost of prototyping is approximately one quarter of the potential profit, and then falls on either side of that unit cost, as seen in Figure 1. $n^* \cdot c$

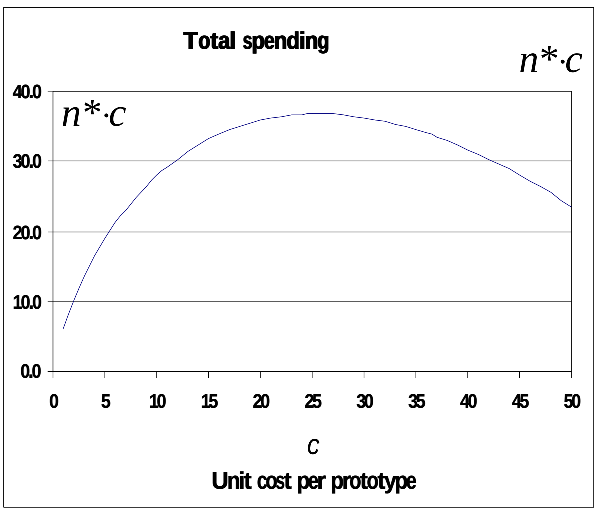


Figure 1

Managers may object to the notion of parallel prototyping on the grounds of limited resources. Our results quantify the value of an extra dollar of R&D when the budget,

M, constrains the team to building fewer than n^* prototypes. It is straightforward to show that shadow price of *M*, is $l = \frac{R}{c} \cdot (-\ln(1-p))(1-p)^{M/c} - 1$. Clearly, the shadow price of a marginal R&D dollar is higher when the potential reward is high and the cost per prototype is low. Firms that employ the one-shot mode of prototyping by building a *single* prototype, that is by setting $M = c$, could derive a return of $l = \frac{R}{c} \cdot (-\ln(1-p))(1-p) - 1$ on each additional dollar of R&D budget. For example, when $R = 10$, $p = 1/2$, and $c = 1$, and *M* is limited to 1, $l = 2.47$. An additional dollar of R&D budget yields a 247% return at the margin. Even when restricting the analysis to integer values of *n*, the return is 150% on building a second prototype. Figure 1 reveals that the opportunity cost of constraining the budget for R&D is exacerbated when high prototyping costs begin to decline. The one-shot firm facing high prototyping costs starts out with below optimal R&D funding (assuming $n^* \geq 2$), and experiences even higher opportunity costs as *c* declines and $n^* \cdot c$ increases.

3. Hybrid Parallel/Sequential Policies

In this section we relax the constraint that all prototypes must be built in a single period. We assume an infinite time horizon with discount factor b, $0 < b \leq 1$. Any number of i.i.d. prototyping experiments can be run in each period. We show that for sufficiently low b, the optimal policy can be characterized as a hybrid sequential/parallel policy in which n^* parallel prototypes are built in each period until a "good enough" result is achieved. If b is sufficiently close to 1, that is if delays to market are not very costly, then a pure sequential policy will be optimal. We can then compare the expected profits of the optimal pure parallel, pure sequential, and hybrid policies.

3.1. Purely Sequential Experimentation

As a basis for comparison against pure parallel and hybrid parallel/sequential processes, we use the result in Weitzman [1979] for the optimal sequential search policy, referred to as *Pandora's rule* since it consists of opening boxes (building prototypes) with unpredictable contents (stochastic outcomes). Each possible prototype is parameterized by the cost of running it and the probability distribution of possible rewards. A reservation price, z, is assigned to each experiment and is the solution to the following equation:

(6) $$z = -c + z \cdot \int_{-\infty}^{z} f(x)dx + b \cdot \int_{z}^{\infty} x \cdot f(x)dx ,$$

where c is the cost of the prototype, $f(x)$ the density function of the profit distribution, and b the discount factor per period. Equation (6) reveals that the reservation price equals the net expected value of running sequential experiments until realizing the first outcome greater than or equal to the reservation price.

Weitzman proves that the optimal pure sequential search policy is to open the box with the highest reservation price, observe the stochastic outcome, and stop if the outcome is higher than the reservation prices for all remaining boxes. If the highest observed outcome does not exceed all remaining reservation prices, then the box with the next highest reservation price is opened. In the case of a finite number of boxes where the stopping rule is not met, the highest previously observed outcome is kept, i.e., there is recall. When replacement is permitted, for example when an unlimited number of the same box may be opened, Weitzman's optimal policy suggests opening just one type of box, the one with the highest reservation price, until exceeding that reservation price.

When delay is of no consequence, $b = 1$ and there is no economic advantage to building prototypes in parallel. The optimal policy is then purely sequential, with one prototype built at a time, and Weitzman's Pandora's rule characterizes the optimal policy.

We illustrate the optimal pure sequential policies using the Bernoulli and Uniform distributions. When $b = 1$, these distributions yield particularly simple results.

3.1.1. Bernoulli Sequential Policy

Applying (6) to the case of Bernoulli trials, the reservation price is $R - \frac{1}{p} \cdot c$, i.e., the reward *R* minus the cost of building $\frac{1}{p}$ prototypes, which is the expected number of Bernoulli trials until the first success. As long as $R > \frac{c}{p}$, the expected value of a pure sequential search is positive. In this case, single Bernoulli trials are run until the first success.

3.2. Optimal hybrid Policy

Next we consider a hybrid parallel/sequential policy in an infinite-horizon setting. The optimal hybrid policy balances time-to-market, cost of product development and product profitability. Higher parallelism in prototyping improves development speed and profitability, but at the price of higher development costs. Parallelism makes sense in situations where prototyping costs are low relative to the potential rewards, and for which speed-to-market has significant profit impact. When each prototype is relatively costly and time-to-market minimally impacts profitability, the sequential approach dominates. Given the conflicting costs and benefits of pure parallel and pure sequential modes, a hybrid policy of building parallel prototypes each period is worth analyzing.

To determine the optimal hybrid policy, we note that *n* i.i.d. parallel experiments can be viewed as a *single*, composite experiment with prototyping cost $n \cdot c$ and distribution function $F_n(x) = [F(x)]^n$ and density $f_n(x) = n \cdot f(x) \cdot [F(x)]^{n-1}$. Thus, the hybrid problem becomes a special case of the optimal sequential problem, where the choice of the number of parallel experiments within a period is recast as a choice from alternative experiments, parameterized by n. The composite experiment with parameter n (corresponding to n parallel experiments) has reservation price z_n which solves

(7) $$z_n = -n \cdot c + \int_{z_n}^{\infty} x \cdot n \cdot f(x) \cdot [F(x)]^{n-1} dx + b \cdot z_n \cdot [F(x)]^n .$$

By maximizing z_n over n, we obtain the optimal composite experiment (consisting of n^* parallel prototypes) with the highest net expected value. Since the experiment with the highest reservation price, z_{n^*}, should be run first, it follows that that the optimal hybrid policy in the infinite horizon problem is to run n^* experiments in parallel in each period until a result greater than z_{n^*} is observed. The expected value of the optimal hybrid policy is z_{n^*}, which can be compared with the expected value of pure sequential and pure parallel policies.

We next consider the optimal hybrid parallel/sequential policies for Bernoulli and Uniformly-distributed prototyping experiments. We calculate their expected values and compare them with the pure parallel and pure sequential cases.

3.2.1. Bernoulli Hybrid Policy

To find n^*, the number of prototypes that maximizes the reservation price, z_n, we note that:

(8) $$z_n = -n \cdot c + \left(1 - (1-p)^n\right) \cdot R + b \cdot (1-p)^n \cdot z_n$$, which yields

(9) $$z_n = \frac{R \cdot \left(1 - (1-p)^n\right) - n \cdot c}{1 - b \cdot (1-p)^n}$$

The first order condition for (9) is:

(10) $$(1-p)^n \cdot [-\ln(1-p)\cdot(R\cdot(1-b)+b\,nc)+b\,c]=c\,,$$

which can be solved numerically for n^*. **Figure 2 illustrates the optimal solution as a function of the discount factor** b **for** $R=10$, $p=\frac{1}{2}$, and $c=1$.

To determine the effect of time-to-market delays on the optimal policy, we might ask how low b **would have to be in order for** n^* **to change from one experiment per period to two or more per period. Using (9) with** n **= 1 and** n **= 2, the breakeven** b **is:**

(11) $$b_{hybrid}=\frac{R(p-p^2)-c}{R(p-p^2)-c+cp^2}$$

If the importance of time-to-market is such that $b \le b_{hybrid}$**, then two or more prototypes will be built per period. If** $b > b_{hybrid}$**, that is if time-to-market delays are not so costly, then a pure sequential policy will be optimal. As** R **increases, ceteris paribus, so does** b_{hybrid}**, meaning that parallelism is favored. This makes intuitive sense, since the marginal benefit of receiving** R **immediately, as opposed to receiving it after a sequential search, increases in** R**. Higher values of** c **have the opposite effect, favoring sequential search.**

We might ask how low b **would need to be to make the pure parallel policy more attractive than the pure sequential policy. Using (9) with** n **= 1, the pure sequential policy has an expected net profit of** $\frac{R\cdot p-c}{1-b\cdot(1-p)}$**. Combining (1) and (2) yields a maximum expected profit of** $E[p_{n^*}]=R\cdot\left(1-(1-p)^{\frac{\ln\left[\frac{-c}{R\cdot\ln(1-p)}\right]}{\ln(1-p)}}\right)-c\cdot\frac{\ln\left[\frac{-c}{R\cdot\ln(1-p)}\right]}{\ln(1-p)}$ **for the pure**

sequential case. Therefore, after a few calculations, the cutoff b at which parallel and sequential policies yield the same expected value becomes:

(12) $$b_{switch} = \frac{1}{1-p} \cdot \frac{E[p_{n^*}] - E[p]}{E[p_{n^*}]}$$, where $E[p] = R \cdot p - c$.

When $b < b_{switch}$ the pure parallel policy outperforms the pure sequential one. According to (12), b_{switch} depends on the incremental expected profit from n^* parallel prototypes compared with that from a single one. The greater the incremental benefit, the higher b_{switch} becomes, and the likelier that $b < b_{switch}$ so that parallelism will be favored. The incremental benefit of parallelism, $E[p_{n^*}] - E[p]$, is driven by inequality (3). To the extent that the potential reward, R, far exceeds $\frac{c}{-\ln(1-p)}$, parallelism will increase, i.e., n^* will climb in equation (2) and so too will the incremental benefit from parallel prototyping.

Comparing the expected net profit from the three policies: pure parallel using equation (1), pure sequential using equation (9) with $n = 1$, and hybrid parallel/sequential using equation (9) with the optimal n^* for each level of b, we can see how the hybrid policy dominates the performance of either pure policy. We note that for higher b's, where time-to-market is less important, the optimal hybrid policy converges to the pure sequential policy. For lower b's, where time-to-market is less important, the optimal hybrid policy converges to the pure parallel, single-period policy.

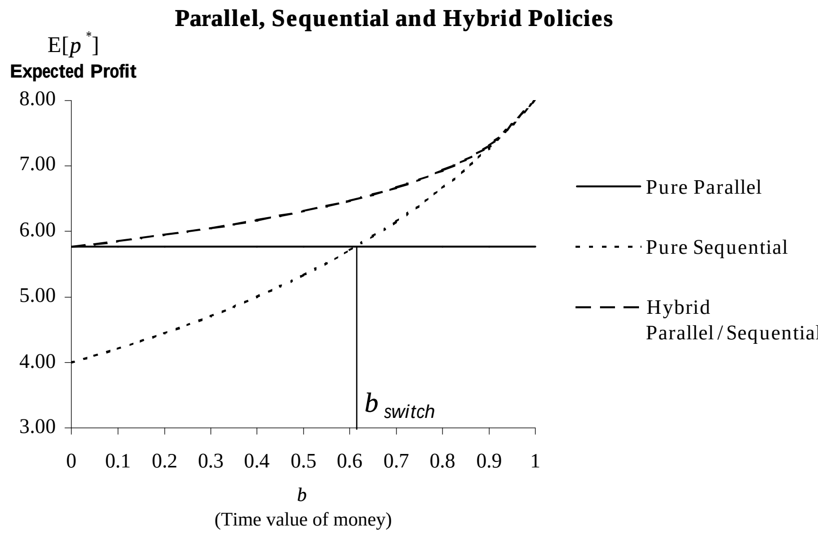


Figure 2

Figure 2 shows the performance of the three policies for the case of $R = 10, p = \frac{1}{2}, c = 1$ **as** b **varies between 0 and 1. Applying (12) to this example,** $b_{switch} = 0.62$**, the point at which the two pure policy graphs intersect. Improvements in either of the financial parameters, i.e., higher** *R* **or lower** *c***, increase the value of** b_{switch}**, thus making it more likely that expected profits from parallel prototyping will exceed those from sequential prototyping.**

4. Design of Prototypes

So far, we have considered the problem of choosing a policy for prototypes whose parameters were exogenous. Specifically, we considered the choice of sequential versus parallel prototyping, and within parallel prototyping - the choice of the number of prototypes to build. The problem of choosing the prototype itself is at least equally important. In fact, as we illustrate below, this problem interacts in meaningful ways with the nature of experimentation: *the choice of prototypes depends on the mode of experimentation*.

We consider pure parallel experiments that are not necessarily identical (but are stochastically independent). Combinations of experiments with different parameters may be run simultaneously. Is it optimal to build prototypes with high rewards and low success probabilities, low rewards and high success probabilities, or a combination of both?

The prototype design problem is formulated as follows. Given a palette of K possible types of prototypes, each with a different reward distribution and cost per experiment, which combination of the K possibilities, and how many of each, should be run to maximize expected net profit? The parameters vary from one experiment type to another as a result of differences in technical feasibility, production cost, and customer utility of each type of design concept. Design of prototypes is distinct from traditional design of experiments in that the former seeks to maximize the expected value of the highest observed result, while the latter seeks to maximize learning about the underlying response surface [Box 1978].

We develop a policy that maximizes expected profit by selecting the proper types of prototypes and the optimal number of each to run in parallel. In addition, we show how the prototyping mode affects the types of experiments chosen.

For this section, we employ Bernoulli experiments with a single-period restriction; the techniques of section 3 show that the extension to the case of hybrid schedules is straightforward. In what follows, we first address the case of $K = 2$ candidate prototypes and then extend our results to the case of $K > 2$.

4.1. $K = 2$ Types of Prototypes

Given the choice of two experiment types of the Bernoulli variety, we consider four possible cases: (1) build n_1* of the Type-1 prototype $(n_1* > 0, n_2* = 0)$, (2) build n_2* of the Type-2 prototype $(n_1* = 0, n_2* > 0)$, (3) build $n^*_{1_{(1,2)}}$ Type-1's and $n^*_{2_{(1,2)}}$ Type-2's

$(n^*_{1_{(1,2)}}, n^*_{1_{(1,2)}} > 0)$, or (4) build neither prototype. We assume, without loss of generality, that $R_1 \geq R_2$.

The expected net profit from building n_1 prototypes of Type-1 and n_2 of Type-2 is given by

$$E(p_{n_1,n_2}) = R_1\left(1-(1-p_1)^{n_1}\right) + R_2(1-p_1)^{n_1}\left(1-(1-p_2)^{n_2}\right) - n_1 \cdot c_1 - n_2 \cdot c_2 \tag{13}$$

which is jointly concave in n_1 and n_2.

The solutions to the first order conditions for (13) are:

$$n^*_{1_{(1,2)}} = \frac{\ln\left[\dfrac{c_2 \ln(1-p_1) - c_1 \ln(1-p_2)}{(R_1 - R_2)\ln(1-p_1)\ln(1-p_2)}\right]}{\ln(1-p_1)} \tag{14}$$

, and

$$n^*_{2_{(1,2)}} = \frac{\ln\left[\dfrac{c_2 (R_2 - R_1)\ln(1-p_1)}{R_2\left[c_2 \ln(1-p_1) - c_1 \ln(1-p_2)\right]}\right]}{\ln(1-p_2)} \tag{15}$$

.

Expressions (14) and (15) lend themselves to a simple interpretation. Using the fact that $\ln x < 0$ for $0 < x < 1$, we notice that in order for both $n^*_{1_{(1,2)}}$ and $n^*_{2_{(1,2)}}$ to be positive, it must be the case that:

$$0 < \frac{c_2 \ln(1-p_1) - c_1 \ln(1-p_2)}{(R_1 - R_2)\ln(1-p_1)\ln(1-p_2)} < 1 \text{ and } 0 < \frac{c_2 (R_2 - R_1)\ln(1-p_1)}{R_2\left[c_2 \ln(1-p_1) - c_1 \ln(1-p_2)\right]} < 1.$$

These two conditions simplify to:

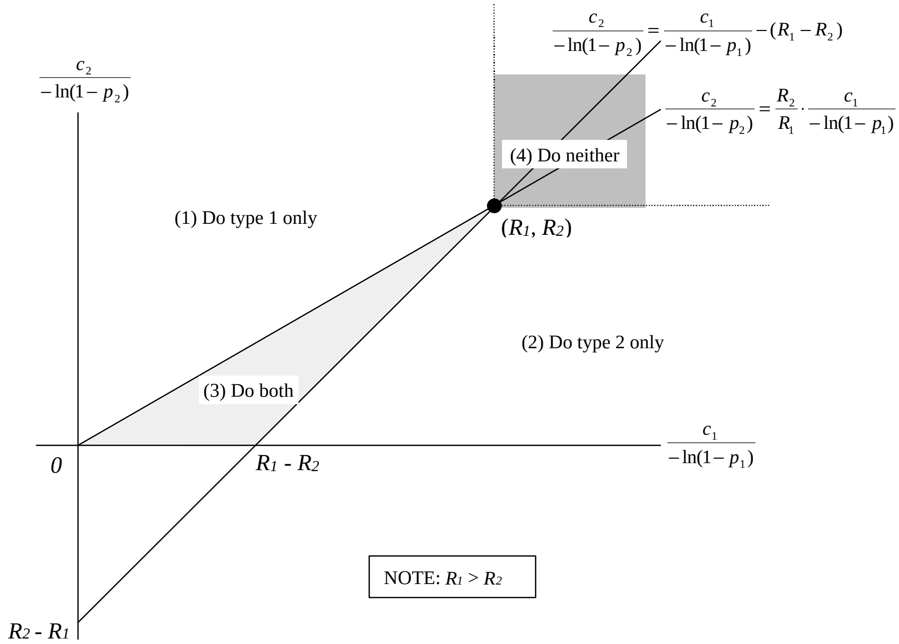
$$\frac{c_2}{-\ln(1-p_2)} < \frac{R_2}{R_1} \cdot \frac{c_1}{-\ln(1-p_1)} \tag{16}$$

and

$$\textbf{(17)} \qquad \frac{c_2}{-\ln(1-p_2)} > \frac{c_1}{-\ln(1-p_1)} - (R_1 - R_2).$$

If (16) and (17) are both met, then it is optimal to develop a combination of $n^*_{1_{(1,2)}}$ Type-1 and $n^*_{2_{(1,2)}}$ Type-2 prototypes. If (16) isn't met, but (17) is, then only Type-1 experiments should be run. The optimal number to run then, is $n_1^* = \frac{\ln\left[\frac{-c_1}{R_1 \cdot \ln(1-p_1)}\right]}{\ln(1-p_1)}$ from equation (2). On the other hand, if (17) is not met, but (16) is, then only Type-2 prototypes, in quantity $n_2^* = \frac{\ln\left[\frac{-c_2}{R_2 \cdot \ln(1-p_2)}\right]}{\ln(1-p_2)}$ should be built and tested. If neither (16) nor (17) is met, then neither experiment should be run. This analysis is summarized in Figure 3 below. Each of the four possible cases is represented by a region of the graph, with mixed prototype combinations being optimal when the parameters fall into the shaded triangle.

Figure 3



We also note that inequality (3) applies individually to each experiment type and is represented by the dotted lines in region (4) of Figure 3. Thus, when the p_i's and c_i's are such that (3) is violated for both sets of parameters, neither prototype is built.

4.2. Example: Applied Materials

Returning to the Applied Materials problem discussed in section 0, consider two possible engineering solutions to the problem of incorporating a ceramic wafer heating system into the modular CVD equipment[3]. Solution A provides a reward, R_A, of \$100M if successful, a probability of success, p_A, of 0.3 for each prototype built, and a cost per prototype, c_A, of \$4M. Solutions B 's parameters are \$75M, 0.4, and \$2M, respectively. Time-to-market is crucial, so all solutions must be built and tested in a single design cycle. The parameter values in this example correspond to the "Do both" region as seen in Figure 4.

[3] The parameter values given are representative of the actual data.

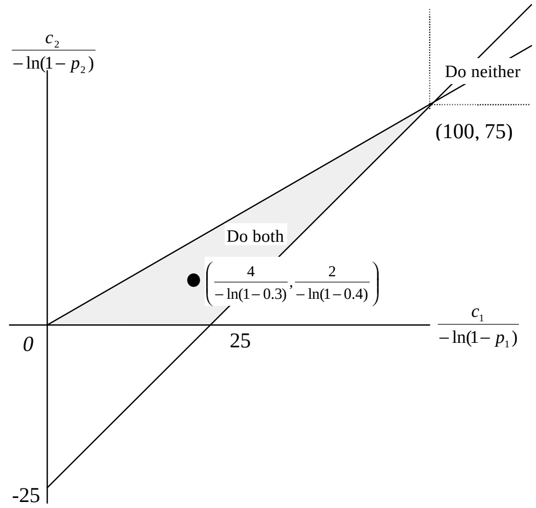


Figure 4

The optimal policy, using (14) and (15), is for Applied Materials to build Type-*A* prototypes at $n_A^* = 3$ suppliers and Type-*B* prototypes at $n_B^* = 4$ suppliers. It is worth noting that high risk, high reward prototypes and low risk, low reward prototypes were commissioned by Applied Materials in this instance. Both kinds of prototypes offer advantages to the firm. Type-*A* prototypes entail greater technical risk and higher development and testing costs, but offer significantly greater rewards since customers would buy more units of this particular design. Type-*B* prototypes, on the other hand, are easier to engineer and less costly to develop, but offer reduced customer utility, resulting in a smaller profit for the firm.

4.3. $K > 2$ types of prototypes

Applying the analysis of section 4.1 to the case where $K > 2$, the optimal design of prototypes for any set of Bernoulli experiments can be specified. We begin by assuming that the experiment types have been indexed such that $R_1 > R_2 > \ldots > R_K$. If $R_i = R_j$ for some $i \neq j$, the tie is broken by keeping only the experiment type with the lower value of $\frac{-c_i}{\ln(1-p_i)}$, as is clear from section 4.1. The objective function for *K* experiment types becomes:

(18) $$E\left(p_{n_1,\ldots,n_K}\right)=\sum_{i=1}^{K} R_i \cdot \prod_{j=1}^{i-1}\left(1-p_j\right)^{n_j} \cdot \left(1-\left(1-p_i\right)^{n_i}\right)-n_i \cdot c_i \text{.}$$

Consider, for example, the case of $K=3$. Solving the first order conditions yields:

(19) $$n^*_{1_{(1,2,3)}}=\frac{\ln\left[\dfrac{c_2\ln(1-p_1)-c_1\ln(1-p_2)}{(R_1-R_2)\ln(1-p_1)\ln(1-p_2)}\right]}{\ln(1-p_1)} \text{,}$$

(20) $$n^*_{2_{(1,2,3)}}=\frac{\ln\left[\dfrac{\ln(1-p_1)(R_1-R_2)\left(c_3\ln(1-p_2)-c_2\ln(1-p_3)\right)}{\ln(1-p_3)(R_2-R_3)\left(c_2\ln(1-p_1)-c_1\ln(1-p_2)\right)}\right]}{\ln(1-p_2)} \text{, and}$$

(21) $$n^*_{3_{(1,2,3)}}=\frac{\ln\left[\dfrac{c_3(R_3-R_2)\ln(1-p_2)}{R_3\left[c_3\ln(1-p_2)-c_2\ln(1-p_3)\right]}\right]}{\ln(1-p_3)}$$

Several facts are notable about (19) - (21). First, (19) corresponds exactly to (14), and (21) corresponds to (15) for the case where only Type-2 and Type-3 experiments are available. Second, each solution depends only upon its immediate neighbors, those with rewards just larger $\left(R_{i-1}\right)$ or just smaller $\left(R_{i+1}\right)$ than its own reward. It can be verified that this last fact holds for *all* $K>2$ and that $n^*_{i_{(1,\ldots,i,\ldots,K)}}=n^*_{i_{(i-1,i,i+1)}}$ for all K. Finally, it can also be verified that a necessary, but not sufficient condition for a positive number of experiments of Type-i (i.e., for $n^*_{i_{(i-1,i,i+1)}}>0$), is that each pair of experiments, $(i-1,i)$ and $(i,i+1)$ fulfill both conditions (16) and (17).

Given these results, the following algorithm defines the profit-maximizing design of prototypes for the case of Bernoulli trials:

1. **Verify that every prototype meets condition (3). This eliminates the "Do Neither" region of Figure 3.**

2. **Sort experiments by *R*, such that $R_1 > R_2 > ... > R_K$.**

3. **When $R_i = R_j$, eliminate the experiment with the higher $\frac{c}{-\ln(1-p)}$.**

4. **Check consecutive pairs, $(R_1, R_2), (R_2, R_3), ..., (R_{K-2}, R_{K-1}), (R_{K-1}, R_K)$, to see where they fall in Figure 3 .**

5. **If a prototype pair lies outside the "Do Both" region of Figure 3, keep the winner and eliminate the loser (Note: Eliminating a prototype reduces *K* by 1). If all pairs fall into the "Do Both" region, proceed to step 6, otherwise go back to step 4..**

6. **Check whether all "internal" prototypes, i.e., $R_2, ..., R_{K-1}$, result in a positive** $n^*_{i_{(i-1,i,i+1)}} = \frac{\ln\left[\frac{\ln(1-p_{i-1})(R_{i-1}-R_i)\left(c_{i+1}\ln(1-p_i)-c_i\ln(1-p_{i+1})\right)}{\ln(1-p_{i+1})(R_i-R_{i+1})\left(c_i\ln(1-p_{i-1})-c_{i-1}\ln(1-p_i)\right)}\right]}{\ln(1-p_i)}$.

7. **Eliminate prototypes for which $n^*_{i_{(i-1,i,i+1)}} \le 0$, in which case go back to step 4, otherwise continue.**

8. **Now that all "internal" prototypes have $n^*_{i_{(i-1,i,i+1)}} > 0$, the optimal solution is given by**

(22) $$n^*_{1_{(1,2)}} = \frac{\ln\left[\frac{c_2\ln(1-p_1)-c_1\ln(1-p_2)}{(R_1-R_2)\ln(1-p_1)\ln(1-p_2)}\right]}{\ln(1-p_1)},$$

(23) $$n^*_{i_{(i-1,i,i+1)}} = \frac{\ln\left[\frac{\ln(1-p_{i-1})(R_{i-1}-R_i)\left(c_{i+1}\ln(1-p_i)-c_i\ln(1-p_{i+1})\right)}{\ln(1-p_{i+1})(R_i-R_{i+1})\left(c_i\ln(1-p_{i-1})-c_{i-1}\ln(1-p_i)\right)}\right]}{\ln(1-p_i)}, \text{ and}$$

(24) $$n^*_{K_{(K-1,K)}} = \frac{\ln\left[\frac{c_K(R_K-R_{K-1})\ln(1-p_{K-1})}{R_K\left[c_K\ln(1-p_{K-1})-c_{K-1}\ln(1-p_K)\right]}\right]}{\ln(1-p_K)}$$

4.4. Impact of Prototyping Modes

Parallelism has an important impact on the *products* that the firm chooses to launch, not just on the number of prototypes built. Specifically, whereas we have shown that parallel prototyping may test a combination of products, the optimal sequential policy requires that only the best product, *ranked by reservation price*, be built. Assuming unlimited availability of each type of prototype, the optimal sequential policy is to determine the product with the highest reservation price, $z_i = R_i - \frac{c_i}{p_i}$, and repeatedly build versions of it until the first success. A combination of products would never be utilized under a such a sequential policy. In some sense, parallel policies foster product heterogeneity, while sequential policies lead to product homogeneity.

The following example highlights the relationship between the mode of experimentation and the choice of products. Consider three Bernoulli experiments, each of which improves upon one parameter relative to a base case as presented in Table 1.

Table 1: Example of Prototyping Modes

			Four types of Products			
			Base Case	Low Risk	High Reward	Easy-to-Prototype
Parameters	Probability of success	p	0.5	0.6	0.5	0.5
	Potential Reward	R	100	100	110	100
	Cost per prototype	c	5	5	5	1
Prototyping Modes	One-shot $E[p]$	$R \cdot p - c$	45	55	50	49
	Sequential $E[p]$ when $b = 1$	$R - \frac{c}{p}$	90	92	100	98
	Sequential $E[p]$ when $b = 0.9$	$\frac{R \cdot p - c}{1 - b \cdot (1 - p)}$	82	86	91	89
	Parallel $E[p_{n*}]$	$E[p_{n*}]$	74	79	83	92
	Parallel combination $E[p_{n_2*, n_3*}]$	$E[p_{n_2*, n_3*}]$	-	-	93	

The products in the example have been selected so that each improves upon a single parameter of the base case. The Low risk product has a higher probability of success, High Reward a higher potential payoff, and Easy-to-Prototype a lower cost per prototype.

The example shows that the preferred product differs for each prototyping mode. The one-shot firm prefers the Low Risk product, due to its higher success probability. The sequential experimenter prefers the High Reward product, because the firm is able to wait for its higher potential payoff. And the parallel experimenter prefers the Easy-to-Prototype product due to its low cost per prototype. In short, firms constrained to operate operating under particular development modes choose to build different kinds of products.

Further, if we allow parallel prototyping of *multiple* **types of products, expected profit increases and a combination of High Reward and Easy-to-Prototype products are built. Finally, the importance of time-to-market determine the globally optimal policy. When time-to-market has no importance (i.e.,** $b = 1$**), a purely sequential policy of building High Reward products generates the highest expected profit. But when time-to-market has value (i.e.,** $b = 0.9$**), profit is maximized with a parallel policy.**

To summarize, we have shown that the firm's choice of prototyping mode determines which types of products to build. When faced with a large array of potential ideas, the design team narrows its choices to an optimal combination of heterogeneous, parallel prototypes. As in section 3.2, the optimal combination becomes a composite experiment in the infinite horizon problem (so that policies can be optimized with hybrid parallel and sequential combinations of prototypes). For Bernoulli trials, we have presented an algorithm to identify the optimal design of prototypes, and established conditions under which heterogeneous combinations are optimal.

5. Discussion

We have shown how the *mode* **of prototyping determines the** *design* **of products. While sequential experimentation focuses exclusively on products with the highest reservation price, parallelism promotes heterogeneity. While firms relying upon the one-shot mode favor designs with more certain rewards, those employing the sequential mode prefer designs with high absolute rewards. Firms implementing parallel prototyping, on the other hand, favor designs that can be built and tested at low cost. In summary, the choice of prototyping mode profoundly impacts the types of products that the firm develops and the profitability derived from that endeavor.**

Appendix 1: Model parameters, variables and notation

n	**Number of prototypes to be built and tested; a decision variable.**
n^*	**Optimal number of prototypes to build without the abandonment option**
$n^*_{i_{(1,\dots,K)}}$	**Optimal number of Type-*i* given that types** $(1,\dots,i-1,i+1,\dots,K)$ **are built**
R	**The reward if a Bernoulli trial succeeds (0 if it fails)**
p	**The probability of success of a single Bernoulli trial.**
c	**Cost to build and test each prototype**
M	**Budget constraint on total R&D spending,** $n \cdot c \le M$
l	**Lagrange multiplier for the budget constraint**
K	**The number of available types of** (R_i, p_i, c_i) **experiments,** $i \in \{1,2,\dots,K\}$
b	**Discount factor per period,** $0 < b \le 1$
p_n	**Random variable for the maximum net profit available after *n* draws,**
z_n	**Reservation price used to optimally order sequential experiments;** **Solves** $z_n = -n \cdot c + \int_{z_n}^{\infty} x \cdot n \cdot f(x) \cdot [F(x)]^{n-1} dx + b \cdot z_n \cdot [F(x)]^n$

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