Modeling and Design of Multi-stage Separation Systems

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Abstract—**Interest in recycling has surged in recent years due to shifting material costs, environmental concerns over material production and disposal, and laws in many countries designed to improve material recycling rates. In response, recycling systems are becoming more complex as increasing material recovery is required from products with complicated material mixtures such as WEEE (Waste Electric and Electronic Equipment). One common approach to increasing system separation performance is the use of multi-stage separation systems. The problem of estimating the performance and designing multi-stage separation processes has rarely been tackled from a system engineering perspective, resulting in poor integration and sub-optimal configuration of industrial multi-stage separation systems. This paper presents a systematic approach to modeling and analyzing multi-stage separation processes. Individual separation processes modeled as Bayesian binary separation steps are incorporated into network models through mass flow rate equations. The model can be used to evaluate the performance of these multi-stage separations under varying conditions, informing decisions about system configuration and process performance. Several basic examples demonstrate the utility of this model for design decisions. The industrial value is demonstrated through a real case study featuring PET plastic and aluminum flake separation in the beverage container recycling industry.**

Index Terms—**Recycling, Separation, Sustainability, Systems Engineering**

I. Introduction

Increasing interest in recycling is driven by a multitude of factors, including fluctuations and increases in material prices [1], environmental concerns over material production and disposal, and laws and directives from high consumption countries designed to improve material recycling rates such as the European Union WEEE (Waste Electric and Electronic Equipment) Directive (Directive 2002/96/EC) or California's Electronic Waste Recycling Act (SB 20). As yield requirements rise and products with more complicated material mixtures enter the waste stream, the performance of recycling systems must increase to reach these goals.

Improvements in recycling performance can be achieved through several means, including individual process improvement, utilizing increasing numbers of separation steps, and optimizing the operational parameters of a process or system. Of these options, research efforts have largely focused on the performance of individual separation processes. A wide range of experimental studies and physical process models have evaluated the performance of separation processes including recycling processes, decontamination processes, and mineral purification processes [2]. While investigating and improving the performance of individual separation processes can have a significant impact on recycling performance, understanding the effects requires evaluating these individual processes in the context of complete separation systems of multiple separation steps.

Most separation systems utilize multiple steps. In some cases, multiple steps performing the same operation are used to purify materials to meet output purity goals [3], while in some cases multiple steps are required to create a greater number of distinct output material streams [4]. The evaluation of the performance of these multi-stage systems is a particularly critical issue due to the high volatility of the price of the output material products, the evolving nature of input material composition and quantity, and increasing pressure to improve system efficiency. The high capital costs and long payback times associated with recycling systems call for efficient management over time. While complex multi-stage separation systems are prevalent, the problem of designing and estimating the performance of these systems has rarely been tackled from a system engineering perspective.

Multi-stage recycling system modelling has been proposed in several specialized forms. Linear circuit analysis has been proposed for systems of similar steps [5]. This technique predicts the recovery of a multi-stage process as compared to a single step. Analytical solutions to mass balance equations describing material flow have been presented for certain specific multi-stage separation configurations, specifically chained separation steps with internally recycling streams [6]. Other models have investigated overall plant profitability, conceptual separation process planning, for modelling the impact of product design on the separation efficiency [7]–[9]. More comprehensive separation modelling has been proposed for mineral processing systems, particularly mineral froth-flotation cells [10]. Models of this type are designed to optimize very specific separation systems that incorporate process-specific separation performance models and restrictive

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operational constraints. The solutions produced by models of this type are restricted to very specific problems, and thus are unable to provide insight into more general problems under varying material and market conditions.

In this paper we present a multi-stage separation system model that is capable of analyzing arbitrary networks of separation steps processing a binary material mixture. This general model can analyze separation systems from any field, including mineral processing and recycling, and include any type of separation process. The model presented here combines the Bayesian separation model with manufacturing system models through mass balance equations. Given a separation system configuration, the Bayesian separation efficiencies of each step, and the input mass flow rates to the system, our model generates and solves mass flow rate balance equations, yielding the mass flow rates at each separation step. The flow rates define the concentration and recovery of material at any point in the system, as well as the overall rate of mass processing in the system. In particular, these performance measures can be used to compare separation systems with different configurations or with different Bayesian separation efficiencies at various steps.

II. Multi-Stage Material Separation Model

The multi-stage binary material separation model combines several components to create a predictive model for separation system performance. Individual separation processes are modeled using the Bayesian separation model. These individual separation steps are combined into complete separation systems by manufacturing network flow models, based on mass balance equations.

A. Bayesian Separation Model

The Bayesian material separation model describes the performance of individual separation processes through the use of two separation parameters [11]. These separation parameters, r and q, describe the recovery of the two components of a binary material mixture. Fig. 1 illustrates the basic concepts of the Bayesian model.

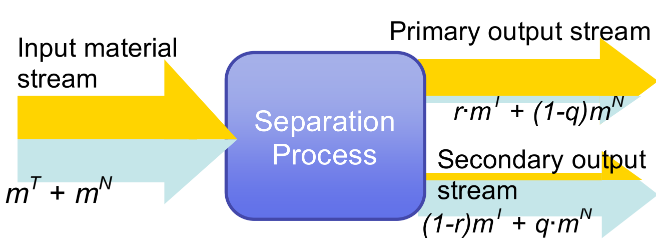

Fig. 1. Diagram of Bayesian material separation model features, including primary and secondary output streams, and separation parameters r and q.

In the Bayesian material separation model, a process separates a binary mixture of the target material, denoted T, and the non-target material, denoted N. The material flow rate of target material entering the process is denoted m^T, and the flow rate of the non-target material is denoted m^N. The process is assumed to separate the materials into two output streams, a primary output stream and a secondary output stream. The separation parameter r represents the recovery of the target material in the primary output stream, that is, the fraction of target material entering the separation process that is recovered in the primary output stream, while the separation parameter q represents the recovery of the non-target material in the secondary output stream, that is, the fraction of non-target material entering the process that is recovered in the secondary output stream. In a separation process, $r+q>1$. Thus the concentration of target material in the primary output stream is higher than in the input stream, while the concentration of non-target material in the secondary output stream is higher than in the input stream.

Many separation systems have been profiled using the parameters r and q. Fig. 2 shows these separation parameters for a wide variety of separation processes, primarily recycling processes. This figure shows the wide range of values that can be achieved by separation processes.

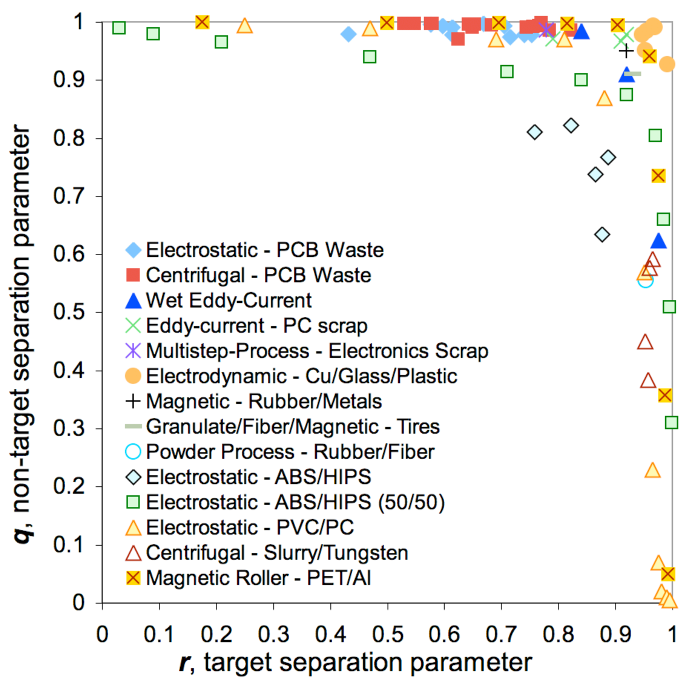

Fig. 2. Separation performance parameters of multiple processes in terms of r and q [2], [12].

B. Network Flow Model

In addition to a model for single step performance, a model of connections within a network is also required to build a complete system model. Given a system of binary separation steps, each separation step has two output streams, a primary and a secondary output stream. These outputs could connect to any other separation step, or to external outputs. A complete network model must include a list of directional edges that represent the connections of those primary and secondary output streams, as well as a list of edges connecting external inputs to the input of separation steps. Fig. 3 shows a sample network illustrating basic connections.

Given a set S, of n separation steps, we also need to define the set I_{ext}, of external inputs to the system, and O_{ext}, the set of external outputs of the system. With these sets defined, a complete description of the flow within a multi-stage separation system must include a set of primary connections, E_p, and a set of secondary connections, E_s, which are

$$E_p = \{(i,j) \mid \forall i \in S, where \quad j \in S \cup O_{ext}\} \tag{1}$$

$$E_s = \{(i,k) \mid \forall i \in S, where \quad k \in S \cup O_{ext}\} \tag{2}$$

and a set of external input connections, E_{ext},

$$E_{ext} = \{(l,i) \mid \forall l \in I_{ext}, where \quad i \in S\} \tag{3}$$

The total set of connections E, that represents the union of E_p, E_s, and E_{ext}, describes a complete system topology for the set of separation steps, S.

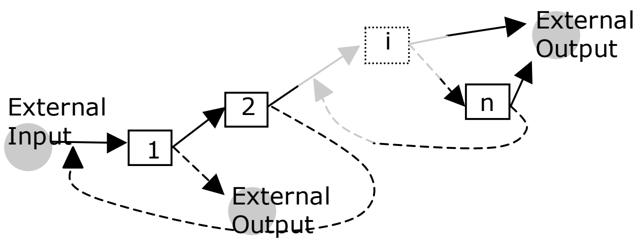

Fig. 3. System of multiple separation steps, with external input and output streams. Primary output streams are denoted by solid lines, secondary output streams are denoted by dashed lines.

C. Mass Balance Equations

With a model of the topology in hand, we can create mass balance equations describing the material flow in a multi-stage material separation system. Assuming steady flow conditions, mass balance equations can be used to determine the flow rates at individual separation processes and for system outputs. For a given separation processing step i, a member of S, the mass balances for the target and non-target material flow rates, $m^T{}_i$ and $m^N{}_i$, are

$$m_i^T = \sum_{(x,i)\in E_p} r_x m_x^T + \sum_{(y,i)\in E_s} (1-r_y) m_y^T + \sum_{(z,i)\in E_{ext}} m_z^T \tag{4}$$

$$m_i^N = \sum_{(x,i)\in E_p} (1-q_x) m_x^N + \sum_{(y,i)\in E_s} q_y m_y^N + \sum_{(z,i)\in E_{ext}} m_z^N \tag{5}$$

In each of these equations, the first summation term represents material directed to process i along connections from primary outputs, the second summation term represents material directed to process i along connections from secondary outputs, while the third summation term represents material directed to process i from external connections. We can create similar equations describing the mass flow rates to the system outputs, the elements of O_{ext}. For system output, i, an element of O_{ext}, these equations are identical to those presented as (4), (5), with the assumption that the final summation term of each of these equations, representing the contribution of external inputs to the mass flow rate, will be empty.

Completing these equations for each separation step in S and each system output in O_{ext} creates two systems of equations, one for the target material and one for the non-target material. For example, given the system shown in Fig. 4 (b), the system of equations describing the target mass flow rates for separation steps 1 and 2, and system outputs N_{out} and T_{out} are then formulated as

$$\begin{aligned} m_1^T &= (1-r_2) m_2^T + m_{in}^T \\ m_2^T &= r_1 m_1^T \\ m_{T_{out}}^T &= r_2 m_2^T \\ m_{N_{out}}^T &= (1-r_1) m_1^T \end{aligned} \tag{6}$$

A similar set of equations can be derived for the non-target material flow in the system. Assuming that the material flow rates of the elements of I_{ext}, the system inputs, are known, the two systems of mass balance equations along with these inputs provide a complete system of equations describing the flow of the two materials. These systems of equations are similar in form to the mass balance equations used in [9] and [13] to describe automotive recycling systems, but in a much-simplified form.

Under certain conditions, these equations are solvable using basic linear algebra techniques. Some of the important conditions include a restriction against self-feeding processes, a requirement that each processing step that is a member of N must have an input, and a requirement that the system of separation steps is connected. Self-feeding processes are processes that have one or both of their outputs directed to their own input. These self-feeding processes violate the steady flow system requirements as material flow builds up in self-feeding loops. We will also require that each separation step and each system output have at least one input. This ensures that each separation process and system output has a non-zero flow rate. In addition, we require that any system we analyze be connected, that is, that the separation system described represents one single system, rather than multiple separation systems running in parallel.

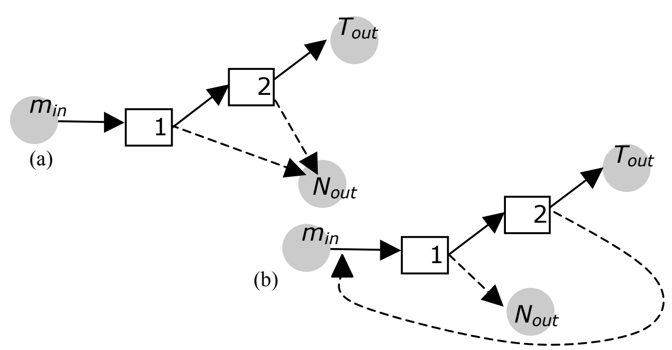

Fig. 4. (a) Separation system configuration with two purifying steps. (b) Separation system configuration with two purifying steps and an internally recycling stream directing the secondary output of the second step to the input of the first step.

III. Application to Simple Systems

The multi-stage material separation model presented here can analyze many types of material separation system. This model has some limitations; for example, it does not consider the possibility of stream splitting for the primary and secondary outputs of separation processes, separation processes that split materials into three or more separation steps, or incorporate material transformation steps, that is, steps that alter the incoming material physically and have one output, such as shredding or melting steps. Even with these limitations, this multi-stage separation model can be used to simulate many realistic separation systems. For example, typical single stream municipal waste recycling facilities consist of binary separation steps [14], [15].

While it's possible to model complex systems with a variety of separation processes, all of which can have different process efficiencies, and arbitrary connections between steps, the analysis of simple systems can be just as valuable. Many specific recycling systems, particularly purification systems, consist of only a few separation steps. Additionally, the analysis of simple systems can provide valuable insight into more complicated systems.

Here we consider the performance of simple multi-stage separation systems that mimic the behavior of many basic separation systems. We look specifically at the behavior of systems that feed one input material mixture into a chain of steps, with the goal of that system being the separation of the mixture into two separate output streams. That is, we consider systems where the set of external inputs, I_{ext}, consists of one input stream containing both target and non-target material, which will be denoted $m^T{}_{in}$ and $m^N{}_{in}$. The set of external outputs, O_{ext}, consists of two output material streams. In a typical separation system, the goal is to gather all the target material into one output stream, denoted T_{out}, and to gather all the non-target material into separate output stream, N_{out}. In mineral processing terminology, T_{out} is analogous to a system's concentrate, while N_{out} is analogous to its tailings.

Evaluating the performance of a binary separation system requires defining performance metrics. Here we use the basic performance metrics common in the mineral processing industry, recovery and grade of the target material output stream. We define the recovery, R, and grade, G, as

$$R = \frac{m^T_{Tout}}{m^T_{in}} \quad (7)$$

$$G = \frac{m^T_{Tout}}{m^T_{Tout} + m^N_{Tout}} \quad (8)$$

Recovery essentially measures the fraction of target material entering the system that is recovered in the target output stream T_{out}. Grade reflects the concentration of the target material in T_{out}.

A. Internally Recycling Streams

The most straightforward application of the multi-stage separation stream model presented in this paper is the basic evaluation of well-defined separation systems operating under fixed operating conditions. By evaluating systems with different configurations, we can compare the relative performance of the systems.

Here we consider an example that investigates the effects of internally recycling streams within separation systems. Fig. 4 shows two separation systems of two steps, one with an internally recycling stream, one without. By evaluating the performance of these systems under different separation parameters we can evaluate the effect of an internally recycling stream.

To evaluate these simple systems, we need to define the separation performance parameters for each step, and the input material concentrations. In all cases, we will consider a 50/50 mixture of target and non-target material. Consider an initial situation where r_1=r_2=0.9 and q_1=q_2=0.95. Under these conditions, for the system without an internally recycling stream as shown in Fig. 4 (a), we find R=0.8100 and G=0.9969. Evaluating the system with an internally recycling stream, as shown in Fig. 4 (b), under the same conditions, we find R=0.8901 and G=0.9971. The internally recycling stream leads to increases in both recovery and grade under these conditions. (There may be trade-offs in the form of additional capital or material processing costs.)

If we consider a situation where the values of r and q are reversed so that r_1=r_2=0.95 and q_1=q_2=0.9, the results are different. Under these conditions, the system without an internally recycling stream as shown in Fig. 4 (a) achieves a performance of R=0.9025 and G=0.9890. The system with the internally recycling stream as shown in Fig. 4 (b), operating under the same conditions, has a resulting performance of R=0.9475 and G=0.9885. Here, the performances of the two systems present a trade-off. The recovery of the system is improved by the use of the internally recycling stream, but there is a reduction in the output grade.

In cases where values of q are greater than the values of r, the effect of the recycling stream is a clear improvement over the system without a recycling stream. When the situation is reversed, there is an improvement in recovery, but a reduction in grade. In this case, the relative importance of grade and recovery would determine the value of adding the internally recycling stream to the configuration.

B. Process Improvements in Simple Systems

Another possible application of this model is to evaluate the effects of changing process parameters in a fixed separation system configuration. This type of problem might arise in real systems, where the option to replace an old process with a newer process may arise. Identifying the process in a separation system with the greatest impact may suggest the most effective way to prioritize improvements to the system.

In order to characterize the effect of process parameter improvement, we can use the metric $\partial R / \partial r_i$. This partial derivative metric measures the change in the system's recovery of target material when the target separation parameter of the ith processing step, r_i, varies, while holding other system parameters fixed. Conceptually, this represents the system recovery "pay-off" from changing the separation parameter of a particular step under otherwise fixed operating conditions.

As an example, we can investigate the effects of improving processing values in the simple three-step separation process shown in Fig. 5. This separation system features three purifying separation processes, with two internally recycling streams.

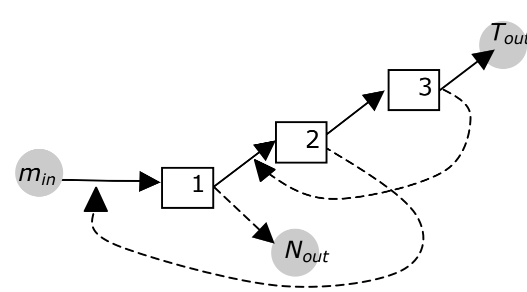

Fig. 5. Separation system with three steps and internally recycling streams.

Consider a situation where this system consists of three identical separation processes, with operational parameters $r_1=r_2=r_3=0.9$ and $q_1=q_2=q_3=0.95$. The overall recovery of this system is R=0.8890. While this may be acceptable performance for this system, over 10% of the target material processed by this system is diverted to waste. Given the opportunity to improve one processing step in this system, which step has the biggest impact on final recovery? Using the model presented in this paper, we can evaluate $\partial R/\partial r_i$ to find

$$\begin{aligned}\partial R/\partial r_1 &= 1.0962\\ \partial R/\partial r_2 &= 0.1205\\ \partial R/\partial r_3 &= 0.0120\end{aligned} \tag{9}$$

These values of $\partial R/\partial r_i$ imply that improving the target separation parameter of process *1* would have the greatest impact on the overall recovery of the system, while improvements to process *3* would have the least effect. In managing a real separation system operating under these conditions, improvements to process *1* would be the highest priority.

C. Selecting Separation Parameters

Many separation processes have the capability to operate at different separation parameter values by adjusting the splitting of output materials. Physically, the material output stream of most processes consists of continuous distributions of the target and non-target materials. These continuous distributions are divided by physical mechanisms to create to distinct output streams. The point of division determines the separation parameters of the process, as described in [2]. Fig. 6 shows a typical output distribution, with multiple potential separation points. Each possible division yields a different set of separation parameters.

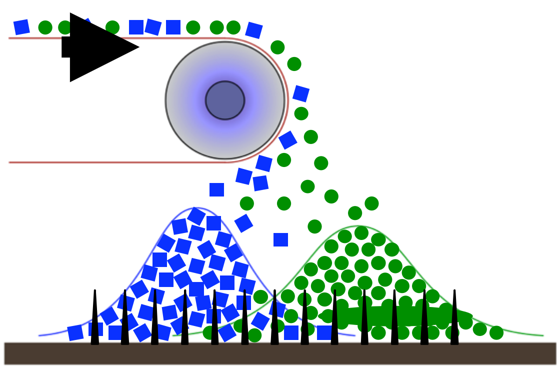

Fig. 6. A typical distribution of materials in the output of a separation process, with representations of possible division points.

Selecting the appropriate separation parameters for a given system configuration is a complex problem, but using the model presented in this paper we can evaluate the effects of set of separation parameters systematically. Here we investigate a real separation system with variable separation parameters.

Rare-earth magnetic rollers are commonly used to separate mixtures of polyethylene terephthalate plastic (PET) and aluminum flakes in the beverage container industry. PET from plastic bottles is commonly recycled for use in new bottles, but PET beverage container collections are often contaminated with aluminum cans. Any aluminum contamination that remains during the melting and forming process can clog forms and dies as well as create imperfections in the finished products. Highly effective removal of aluminum is critical; a variety of processes, including eddy-current separation, optical sorting, metal detection, and magnetic rollers, are used during different phases of secondary processing.

Rare-earth magnetic rollers are typically the first aluminum removal step used after the containers are shredded. At this point, the level of aluminum contamination is typically between 500 and 2000ppm by weight. After the roller processing, a metal detector system removes any final aluminum contamination. However, each aluminum particle diversion removes a significant volume of plastic. The magnetic roller processing must reduce the aluminum contamination entering the metal detector to roughly 25ppm by weight. Fig. 7 shows a rare-earth magnetic roller process separating PET from aluminum.

Adjusting the splitting mechanism of the roller separation process yields a range of possibilities for the material separation parameters. Fig. 8 graphs the operational r, q pairs for the separation of PET plastic and aluminum created by varying the division point of the output stream. Taking the higher input contamination level, 2000pm, meeting the goal of reducing the final concentration of aluminum contamination to 25ppm by weight or less with a single step while maximizing the output recovery requires operating the rare-earth magnetic roller process at a the operational pair r=0.6954, q=0.9990. Under these conditions the total recovery of PET plastic is roughly 70%. The level of recovery can be greatly improved with the introduction of a second processing step.

Fig. 7. Rare-earth magnetic roller process.

Consider instead the two-step system configuration shown in Fig. 4 (b), which consists of two purifying steps, with an internally recycling stream. Given the goal of maximizing recovery while maintaining an output contamination of 25ppm by weight or less, the model presented in this paper can inform the selection of separation parameters for this separation system. By investigating the possible operating points for each process, we find that the system configuration that yields the highest recovery of PET plastic while meeting the purity requirements. The optimal separation parameters for the system, yielding a total PET recovery of R=0.9974, are r_1=0.90, q_1=0.99, r_2=0.97, q_2=0.74. In Fig. 8, these two operational points are labeled as 1 and 2. This two-step system achieves a recovery of nearly 100%, an absolute improvement of roughly 30% over a single-step magnetic roller system.

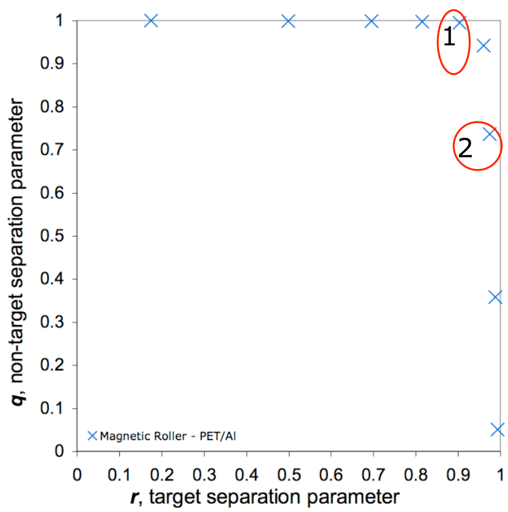

Fig. 8. Separation parameters for rare-earth magnetic roller processing of PET plastic (***r***) and aluminum flake (***q***) for different decision points [12].

With two variable separation steps, each with 9 discrete operational points, there are roughly 80 different system configurations to consider. Complexity increases with additional separation steps or continuous operating parameters to the point that a systematic model is necessary to evaluate all the possibilities.

IV. Conclusion

Multi-stage separation systems are a critical part of many industries, including recycling and mineral processing. The model presented in this paper combines the Bayesian separation model for process performance with network flow models through mass flow balance equations. The utility of this model is demonstrated through several simple examples, including the identification of optimal operating parameters for a two-step rare-earth magnetic roller system processing PET plastic and aluminum flake. By predicting the performance of separation systems, the model can inform the selection of separation processes, the configuration of those processes, and the operational parameters of those processes.

Future development of this model will focus on incorporating more features of real separation systems. Many separation systems process multiple materials. The Bayesian material separation model describes the separation of binary material mixtures; an extension of the Bayesian model covering multiple material processing would accommodate the construction of systems of mass flow rate equations for multiple materials within the framework of this model. Another possible extension for this work is the definition of cost and energy use metrics for evaluating the performance of the system in economic and energy terms. These metrics would require an understanding of the dollar and embodied energy value of capital equipment in the systems, including both separation processes and connecting elements such as conveyors, as well as operating costs and energy use. These values are expected to be affected by mass flow rates within the system as well as material types, machine sizing, and many other possible factors. Other future research possibilities include the applicability of this model to reconfigurable separation systems and modeling features that affect system production rates such as machine failures, system storage, and process capacity.

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