# A Thermodynamic Framework for Analyzing and Improving Manufacturing Processes

Matthew Branham, Timothy G. Gutowski, Alissa Jones, and Dusan P. Sekulic

Abstract—In this paper, we present the formulation of a framework for the quantitative thermodynamic analysis of manufacturing processes and systems. Since manufacturing typically involves the input of high-quality material/energy and/or dissipation of low-quality energy/waste to manipulate a material, an approach that combines both the first and the second laws of thermodynamics is appropriate. This formulation helps emphasize that the improvement of manufacturing processes and systems is more a question of conserving the quality of energy than merely conserving energy. We conclude with two examples of its application, the first a comparison of metal casting technologies and the second a contrast between high-throughput CNC machining and a slower process rate grinding operation.

Index Terms—Manufacturing, Thermodynamics, Exergy

### I. INTRODUCTION

ANUFACTURING accounts for approximately one-third of energy consumption in the United States [1], while metabolizing a large percentage of the raw materials entering the global economy. That manufacturing activities are often associated with degradation of the environment should not be surprising, particularly given the consumption of natural resources inherent in manufacturing. Fundamentally, the connection between manufacturing and the environment is unbreakable; the natural environment serves as both the source of raw materials as well as the sink for the wastes of industrial processes. The challenge is to mitigate the impact of manufacturing on the environment while still obtaining the desired products.

The framework presented in this paper was created with this challenge in mind. A generalized manufacturing system is defined as a sequence of energy conversion, material processing, and manufacturing steps, each of which can be described by individual thermodynamic balances. This graphical presentation is useful for both quantifying and

Matthew Branham, Department of Mechanical Engineering, Massachusetts Institute of Technology, Cambridge, MA 02139 USA (phone: 617-253-7530; fax: 617-253-1556; e-mail: mbranham@mit.edu).

Timothy G. Gutowski, Department of Mechanical Engineering, Massachusetts Institute of Technology, Cambridge, MA 02139 USA (e-mail: gutowski@mit.edu).

Alissa Jones, Department of Mechanical Engineering, Massachusetts Institute of Technology, Cambridge, MA 02139 USA (e-mail: alissa.jones@gmail.com)

Dusan P. Sekulic, Department of Mechanical Engineering, University of Kentucky, Lexington, KY 40506 USA (e-mail: sekulicd@engr.uky.edu).

visualizing energy and material flows in a manufacturing process. We then progress to show how this analysis can be used to evaluate the environmental burden of a process by comparing metal casting technologies, as well as in comparing material removal efficiencies in CNC machining and grinding. Before closing, we discuss how the model might be applied at the process level with the goal of process improvement.

# II. GENERAL MANUFACTURING MODEL

Manufacturing can be conceived as a complex system featuring a sequence of thermodynamic processes in the same manner as proposed by Gyftopoulos [2] for an energy conversion/materials processing system. Each stage in the process has energetic and material inputs, the material inputs of which are either taken from the environment directly as raw materials or indirectly as material processed in an earlier stage. The useful output from a given stage is then passed on to become the input for a succeeding stage, until a final product is produced which is sold to consumers. Each manufacturing stage inevitably generates entropy and a stream of waste materials. Waste material is, as a rule, discarded to the environment (although in some cases, it can be recycled). In addition, low-quality energy flows (e.g., waste heat) are dissipated (and in some cases recovered).

Figure 1 depicts this generalized model of a manufacturing system. The manufacturing subsystem ( $\Omega_{MF}$ ) is powered by a work rate fed from an energy conversion subsystem ( $\Omega_{ECMF}$ ). The input materials are shown to be manipulated upstream in the materials processing subsystem ( $\Omega_{MA}$ ), which also has its own energy conversion subsystem ( $\Omega_{ECMA}$ ). This network representation can be infinitely expanded to encompass ever more complex and detailed manufacturing systems and/or to include all elements of the life cycle of the product.

At every stage, the manufacturing system interacts with the environment (shown here at reference pressure  $p_0$  and reference temperature  $T_0$ ) either by dumping low-quality heat (accompanied by corresponding entropy flow) or by sourcing and sinking the materials used in the process. The characteristics of this environmental reference state are critical to the calculation of the thermodynamic interactions taking place in the process. By convention, the reference pressure is selected as 101.3 kPa and the reference temperature as 298.15K.

This set of subsystems can be completely described in

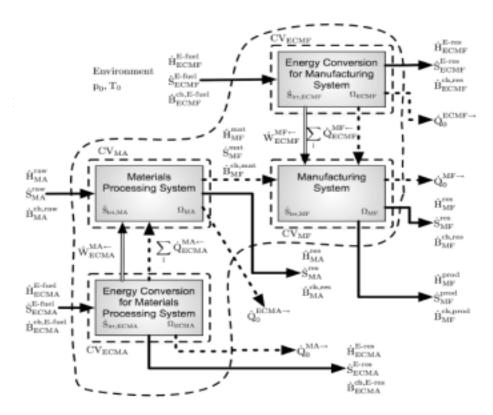


Figure 1: General Manufacturing Model

thermodynamic terms by formulating mass, energy, and entropy balances [3]. Beginning with the manufacturing system  $\Omega_{\text{MF}}$ , we have three basic equations:

## Mass Balance:

$$\frac{dm_{MF}}{dt} = \left(\sum_{i=1} \dot{N}_{i,in} * M_{i}\right)_{MF} - \left(\sum_{i=1} \dot{N}_{i,out} * M_{i}\right)_{MF} \tag{1}$$

where  $\dot{N}$  is the number of moles of the i<sup>th</sup> component entering or leaving the system and  $\dot{M}$  is the molar mass of that

or leaving the system and  $M_{\rm i}$  is the molar mass of that component.

# **Energy Balance:**

$$\frac{dE_{MF}}{dt} = \sum_{i} \dot{Q}_{ECMF}^{MF \leftarrow} - \dot{Q}_{0}^{MF \rightarrow} + \dot{W}_{ECMF}^{MF \leftarrow} + \dot{H}_{MF}^{mat} - \dot{H}_{MF}^{prod} - \dot{H}_{MF}^{res}$$
(2)

where  $\dot{H}$  gives the sum of the enthalpy rates of all materials, products, and residue bulk flows. Note that for reacting systems,  $(\dot{H}_i/\dot{N}_i) = h_i = h_{f,i}^o + \Delta h_i$ , where  $h_{f,i}^o$  represents the specific enthalpy of formation and  $\Delta h_i$  accounts for the change in the enthalpy from the reference temperature and pressure to the actual temperature and pressure of the  $i^{th}$  component.

# **Entropy Balance:**

$$\frac{dS_{MF}}{dt} = \sum_{i} \frac{\dot{Q}_{ECMF}^{MF \leftarrow}}{T_{i}} - \frac{\dot{Q}_{0}^{MF \rightarrow}}{T_{0}} + \dot{S}_{MF}^{mat} - \dot{S}_{MF}^{prod} - \dot{S}_{MF}^{res} + \dot{S}_{irr,MF}$$
(3)

where  $\dot{S}_i$  gives the sum of the entropy rates of all components. Note that in the entropy balance, the entropy of each bulk flow (e.g., reacting systems) can be defined in terms of specific absolute entropy  $s_i^o$  and specific entropy difference evaluated at the given temperature and pressure and at the reference values,  $\Delta s_i$ .

In steady state, the terms to the left of the equality sign are zero, so the next step is to solve the energy and entropy balance equations for  $\dot{Q}_0$  and then equate them. This yields an expression for the work requirement for the manufacturing process:

$$\dot{W}_{ECMF}^{MF\leftarrow} = ((\dot{H}_{MF}^{prod} + \dot{H}_{MF}^{res}) - \dot{H}_{MF}^{mat}) 
-T_{0} * ((\dot{S}_{MF}^{prod} + \dot{S}_{MF}^{res}) - \dot{S}_{MF}^{mat}) 
-\sum_{i>0} \left(1 - \frac{T_{0}}{T_{i}}\right) \dot{Q}_{ECMF}^{MF\leftarrow} + T_{0} \dot{S}_{irr,MF}$$
(4)

The quantity H-TS appears often in thermodynamic analysis and is referred to as the free energy. When related to the reference state  $(p_0, T_0)$  the free energy becomes equivalent to exergy (B). That is,  $B = (H-T_0S)_{P,T} - (H-T_0S)_{P_0,T_0}$ . Exergy represents the maximum amount of work that could be extracted from a system as it is reversibly brought to equilibrium with a well-defined environmental reference state (hence the requirement that the exergy reference be  $p_0$  and  $T_0$ ). It is very important to note that the material-related terms in (4) may be split into contributions that account for the so-

IEEE International Symposium on Electronics and the Environment May 19-20, 2008, San Francisco USA

called physical and chemical exergies (see below). For a further discussion of exergy and its meaning, see Bejan [3].

The physical exergy is that portion of the exergy which is a function of temperature and pressure only. Therefore, the quantity

$$\dot{B}_{i}^{ph} = \dot{N}_{i} [(h_{i} - h_{i,o}) - T(s_{i} - s_{i,o})]$$
 (4a)

is known as the *physical exergy* rate of the bulk flow system, defined as the availability between the system in its given state and the system in equilibrium with its surroundings  $(T_0, p_0)$ .

Alternately, the terms  $h_{f,i}^o - T_0$  so are known as *chemical exergies* ( $b_i^{ch} = \dot{B}_i^{ch} / \dot{N}_i$ ) and represent the additional potential that can be extracted by bringing the chemical potentials ( $\mu_i$ ) of a given system to equilibrium with its surroundings in the ultimate dead state ( $T_0$ ,  $p_0$ ,  $\mu_0$ ). Note that chemical exergies are calculated by using enthalpy of formation and absolute entropies. In addition to requiring a reference temperature and pressure, the definition of chemical exergies also requires a reference state with a specified chemical composition. This reference is typically taken to be representative of the compounds in the earth's upper crust, atmosphere, and oceans. In this article, exergy values are calculated using the Szargut reference environment [4].

We simplify (4) with:

$$\dot{W}_{ECMF}^{MF\leftarrow} = ((\dot{B}_{MF}^{prod} + \dot{B}_{MF}^{res}) - \dot{B}_{MF}^{mat})^{ph} 
+ (\sum_{i=1}^{n} b_{0,i}^{ch} * \dot{N}_{i})_{MF}^{prod} + (\sum_{i=1}^{n} b_{0,i}^{ch} * \dot{N}_{i})_{MF}^{res} -$$
(5)

$$(\sum_{i=1}^{n} b_{0,i}^{ch} * \dot{N}_{i})_{MF}^{mat} - \sum_{i>0} \left(1 - \frac{T_{0}}{T_{i}}\right) \dot{Q}_{ECMF}^{MF \leftarrow} + T_{0} \dot{S}_{irr,MF}$$

Using the same analysis for the system  $\Omega_{\text{ECMF}}$  yields:

$$\dot{W}_{ECMF}^{MF\leftarrow} = (\dot{B}_{ECMF}^{E-fuel} - \dot{B}_{ECMF}^{E-res}) 
+ (\sum_{i=1}^{n} b_{0,i}^{ch} * \dot{N}_{i})_{ECMF}^{E-fuel} - (\sum_{i=1}^{n} b_{0,i}^{ch} * \dot{N}_{i})_{ECMF}^{E-res} 
- \sum_{i>0} \left(1 - \frac{T_{0}}{T_{.}}\right) \dot{Q}_{ECMF}^{MF\leftarrow} - T_{0} \dot{S}_{irr,MF}$$
(6)

Finally, the rate of exergy lost during these two processes (excluding waste) is given by:

$$D_{ECMF+MF} = T_o(\dot{S}_{irr,MF} + \dot{S}_{irr,ECMF}) \tag{7}$$

With these equations and the appropriate ones for the other subsystems, we have completely described the manufacturing system in terms of loss of potential. We save the discussion of the interesting aspects of these equations for the following sections.

# III. MODEL APPLICATION TO IRON CASTING

With regard to manufacturing process analysis, industrial ecology is principally concerned with the way a process interacts with the surrounding environment. Thus, the system boundaries are drawn such that inputs and outputs are at reference temperature and pressure  $(T_0, p_0)$ . That is to say, the

application of exergy analysis to industrial ecology focuses on the change in chemical exergy of the materials in the process while drawing the system boundaries sufficiently large to eliminate the change in physical exergy.

In order to demonstrate the industrial ecology application of this thermodynamic model, we draw on an example from the casting industry [5]. One common issue in the industry is the relative merit of the various melting technologies. industry mainstay has been the cupola furnace, which uses metallurgical coke as a heat source and runs continuously. The heel electric-induction melter presents a second option. As the name implies, it is electrically powered and runs constantly in order to maintain the metal as a liquid. A more modern variant is the batch electric-induction melter, which uses advanced control electronics to throttle back electric power consumption when possible. In general, the cupola furnace can accept a wider variety of (and less pure) inputs than the induction furnaces. From an energy and exergy perspective, the heel electric-induction furnace is the poorest technological choice since it uses electricity (the highest quality energy) and runs continuously. The question to answer is whether the cupola or the batch induction furnace is superior.

Figures 2 and 3 show the inputs to the two melter types required in the production of 1000kg of melt (instead of on a rate basis, as with the equations of section II). On the surface, it is difficult to tell which process is superior. An interesting comparison begins to evolve, however, in an exergy analysis as the boundaries of the analysis are progressively expanded.

The first step is to define the boundaries of the manufacturing system ( $\Omega_{MF}$  from Fig. 1), which in this case shall be the systems shown in Figs. 2 and 3. As mentioned earlier, the material inputs are taken at  $T_0$  and  $p_0$ , and electricity is taken as the energy input into the batch melter. Table 1 shows that a comparison only at the level of  $\Omega_{MF}$  gives the conclusion that the batch electric-induction melter is the superior technology, with an exergy loss of 2,230MJ compared to 3,250MJ for the cupola per tonne of material melted. If one were to cease the analysis at that, however, much important information would be left out.

The next step is to expand the analysis to the energy conversion system supplying the melters,  $\Omega_{\text{ECMF}}$ . For the batch melter,  $\Omega_{\text{ECMF}}$  represents the power plant supplying the melter with electricity. For this example, it is assumed that the electricity is generated using a 65% coal/35% natural gas mix. Including the primary exergy required to generate electricity now gives an exergy destruction total of 5,100MJ per 1,000kg of melt for the batch melter.

The coke fueling the cupola furnace is also a secondary energy source in that it is derived from coal. However, since the boundary of  $\Omega_{MF}$  has been drawn such that the coke is a material input instead of an energetic input, in order to include the exergy destroyed in the production of coke we expand the analysis of the cupola to include  $\Omega_{MA}$  and  $\Omega_{ECMA}$ , the materials production system (there are no inputs from  $\Omega_{ECMF}$  for the cupola furnace as defined here). Using data from Considine et. al. [6], the production of the 86kg of metallurgical coke required in the production of 1,000kg of melt results in the

# **Cupola Melting Analysis**

# Metallic Input Materials and Alloys 1026 kg (B=8684MJ) Limestone Flux & Limespar 42 kg (B=35.4MJ) Metallurgical Coke 86 kg (B=3946MJ) Natural Gas (Afterburners) 5 kg (B=832MJ) Oxygen Enrichment 33 kg (B=4MJ) Nitrogen, Oxygen, Water Vapor 1477 kg (B=135MJ) Nitrogen, Oxygen, Water Vapor 1477 kg (B=135MJ)

Figure 2: Inputs and outputs to a cupola for the production of 1000kg of iron melt (exergy content) [5]

Case 1: Electric Induction Melting of		Case 2: Cupola Melting of 1000kg of	
1000kg of product ( $\Omega_{ extsf{MF}}$ only)		product (Ω <sub>MF</sub> only)	
<u>Material</u>	Exergy of	<u>Material</u>	Exergy of
	Inputs, B		<u>Inputs, B</u>
Scrap Metallics		Scrap Metallics	6,230 MJ
Cast Iron Remelt	2,530 MJ	Cast Iron Remelt	2,530 MJ
Electricity	1,720 MJ		2,740 MJ
Totals <b>B</b> inputs	10,480 MJ	Totals <b>B</b> inputs	11,500 MJ
<u>Material</u>	Exergy Useful	<u>Material</u>	Exergy Useful
	Products, B		<u>Products, B</u>
Gray Iron Melt	8,250 MJ	Gray Iron Melt	8,250 MJ
Total Exergy Loss	2,230 MJ	Total Exergy Loss	3,250 MJ
Bwaste+ Blost	2,230 1110	B <sub>waste</sub> + B <sub>lost</sub>	3,230 MO
Case 4: Electric Induction Melting of 1000kg of product including electricity		Case 3: Cupola Melting of 1000kg of	
1000kg of product inc	luding electricity	product including pro	duction of coke
1000kg of product inc production (Ω <sub>MF</sub>		product including pro ( $\Omega_{MF}$ , $\Omega_{ECMF}$ , $\Omega$	
production (Ω <sub>MF</sub>	and Ω <sub>ECMF</sub> )	$(Ω_{MF}, Ω_{ECMF}, Ω$	ма, Ωесма)
production (Ω <sub>MF</sub>	and Ω <sub>ECMF</sub> )  Exergy of Inputs, B	$(Ω_{MF}, Ω_{ECMF}, Ω$	<sub>MA</sub> , Ω <sub>ECMA</sub> ) <u>Exergy of</u>
production (Ω <sub>MF</sub> Material	and $\Omega_{ECMF}$ )  Exergy of Inputs, B  6,230 MJ	$(Ω_{MF}, Ω_{ECMF}, Ω_{Material})$	MA, Ω <sub>ECMA</sub> ) <u>Exerqy of</u> <u>Inputs, B</u>
production (Ω <sub>MF</sub> Material  Scrap Metallics	and Ω <sub>ECMF</sub> ) Exergy of Inputs, B 6,230 MJ 2,530 MJ	$(\Omega_{MF},\ \Omega_{ECMF},\Omega_{Material}$ Scrap Metallics	MA, Ω <sub>ECMA</sub> ) Exergy of Inputs, B 6,230 MJ
production (Ω <sub>MF</sub> Material  Scrap Metallics  Cast Iron Remelt	and Ω <sub>ECMF</sub> )  Exergy of Inputs, B  6,230 MJ  2,530 MJ  4,590 MJ	(Ω <sub>MF</sub> , Ω <sub>ECMF</sub> , Ω <u>Material</u> Scrap Metallics  Cast Iron Remelt	MA, Ω <sub>ECM</sub> A) Exergy of Inputs, B 6,230 MJ 2,530 MJ
production (Ω <sub>MF</sub> Material  Scrap Metallics  Cast Iron Remelt  Electricity Production	and Ω <sub>ECMF</sub> )  Exergy of Inputs, B  6,230 MJ  2,530 MJ  4,590 MJ	(Ω <sub>MF</sub> , Ω <sub>ECMF</sub> , Ω <u>Material</u> Scrap Metallics  Cast Iron Remelt  Coke Production	MA, Ω <sub>ECMA</sub> ) <u>Exerqy of</u> <u>Inputs, B</u> 6,230 MJ  2,530 MJ  3,490 MJ
production (Ω <sub>MF</sub> Material  Scrap Metallics  Cast Iron Remelt  Electricity Production	and Ω <sub>ECMF</sub> )  Exergy of Inputs, B  6,230 MJ  2,530 MJ  4,590 MJ	(Ω <sub>MF</sub> , Ω <sub>ECMF</sub> , Ω <u>Material</u> Scrap Metallics  Cast Iron Remelt  Coke Production	MA, Ω <sub>ECMA</sub> ) <u>Exerqy of</u> <u>Inputs, B</u> 6,230 MJ  2,530 MJ  3,490 MJ
production (Ω <sub>MF</sub> Material  Scrap Metallics  Cast Iron Remelt  Electricity Production  Totals B inpres	and Ω <sub>ECMF</sub> )  Exergy of Inputs. B  6,230 MJ  2,530 MJ  4,590 MJ  13,350 MJ	(Ω <sub>MF</sub> , Ω <sub>ECMF</sub> , Ω <u>Material</u> Scrap Metallics  Cast Iron Remelt  Coke Production <i>Totals B inpecs</i>	MA, Ω <sub>ECMA</sub> ) Exergy of Inputs, B 6,230 MJ 2,530 MJ 3,490 MJ 12,250 MJ
production (Ω <sub>MF</sub> Material  Scrap Metallics  Cast Iron Remelt  Electricity Production  Totals B inpres	and Ω <sub>ECMF</sub> )  Exergy of Inputs, B  6,230 MJ 2,530 MJ 4,590 MJ 13,350 MJ  Exergy Useful Products, B	(Ω <sub>MF</sub> , Ω <sub>ECMF</sub> , Ω <u>Material</u> Scrap Metallics  Cast Iron Remelt  Coke Production <i>Totals B inpecs</i>	MA, Ω <sub>ECMA</sub> )  Exergy of Inputs, B  6,230 MJ 2,530 MJ 3,490 MJ 12,250 MJ
production (Ω <sub>MF</sub> <u>Material</u> Scrap Metallics  Cast Iron Remelt  Electricity Production <u>Totals B inpres</u> <u>Material</u>	and Ω <sub>ECMF</sub> )  Exergy of Inputs, B  6,230 MJ 2,530 MJ 4,590 MJ 13,350 MJ  Exergy Useful Products, B	(Ω <sub>MF</sub> , Ω <sub>ECMF</sub> , Ω <u>Material</u> Scrap Metallics  Cast Iron Remelt  Coke Production  Totals <b>B</b> inpres <u>Material</u>	MA, $\Omega_{\text{ECMA}}$ )  Exergy of Inputs, B  6,230 MJ 2,530 MJ 3,490 MJ 12,250 MJ  Exergy Useful Products, B
production (Ω <sub>MF</sub> <u>Material</u> Scrap Metallics  Cast Iron Remelt  Electricity Production <u>Totals B inpres</u> <u>Material</u>	and Ω <sub>ECMF</sub> )  Exergy of Inputs, B  6,230 MJ 2,530 MJ 4,590 MJ 13,350 MJ  Exergy Useful Products, B	(Ω <sub>MF</sub> , Ω <sub>ECMF</sub> , Ω <u>Material</u> Scrap Metallics  Cast Iron Remelt  Coke Production  Totals <b>B</b> inpres <u>Material</u>	MA, $\Omega_{\text{ECMA}}$ )  Exergy of Inputs, B  6,230 MJ 2,530 MJ 3,490 MJ 12,250 MJ  Exergy Useful Products, B

Table 1: Comparison betwen a cupola melter and a batch electric induction melter per 1000kg of melt produced. Clockwise from top left: Cases 1 and 2 consider only the final production step  $(\Omega_{MF})$  using secondary energy in the form of coke (cupola) or electricity (batch melter). Case 3 extends the analysis of the cupola to include the production of coke for fuel  $(\Omega_{MF}$  and  $\Omega_{MA})$ . Case 4 extends the analysis of the batch melter to include the exergy consumed in the production of electricity (assuming a 65% coal and 35% natural gas mix). [5]

# **Batch Induction Melting Analysis**

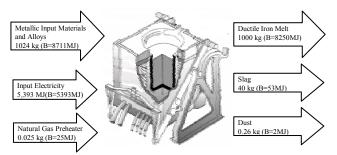


Figure 3: Inputs and outputs to a batch electric induction melter for the production of 1000kg of iron melt (exergy content) [5]

destruction of an additional 750MJ of exergy for a total of 4,000MJ for the cupola, still less than the 5,100MJ for the batch melter. Given our assumptions, the cupola furnace appears to be the better technology for the metal casting industry on an exergy basis.

There is a final dimension to this analysis that has not yet been addressed, and that has to do with the composition of the input materials. Both melters often use at least some virgin (pig) iron as an input into the melter. The amount of pig iron used can dramatically affect the environmental impact of casting. Let's assume for example that pig iron composes 5% of the metallic inputs, or about 50kg, for both the cupola and the batch melter. This input comes from the system  $\Omega_{MA}$  and includes the energy required for the production of those materials ( $\Omega_{ECMA}$ ). The production of 50kg of pig iron results in the destruction of 1,420MJ of exergy, increasing the exergy destroyed in the cupola case to 5,420MJ (a 36% increase) and in the batch melter to 6,520MJ (a 28% increase). It is evident that the relative proportion of pig iron used in each of the melters can significantly affect the performance of the two technologies.

#### IV. SUBTRACTIVE PROCESSES

Subtractive manufacturing processes – milling, grinding, etching, electric discharge machining, and the like – pose a problem for exergy analysis. Traditional ratios of efficiency in exergy analysis reward processes in which a large percentage of the input material is incorporated into the output product. By definition, however, the aim of a subtractive process is the removal of some material from the work piece in operation. Traditional measures of efficiency have difficulty treating the material that flows through a process but which is not operated For example, engraving a part number on a large aluminum work piece should not be considered more efficient than engraving the same figure on a smaller piece on the basis that a larger percentage of the input material is available in the output product. Sorin et. al. treat this dilemma by identifying two separate exergy streams: the so-called "transiting exergy" which flows through the process unaltered, and the "utilizable exergy" which is manipulated and active in the process [7].

It would be useful to have a metric for the evaluation of the efficiency (in exergy terms) of subtractive processes that is independent of the "transiting exergy" that enters a process. One approach to establishing an efficiency ratio would be to compare an actual manufacturing process with the theoretically best process possible. In exergy terms, this would be expressed as a ratio between the theoretical minimum work rate required to carry out a subtractive process and the actual utilizable exergy rate input into the process. We thus define the efficiency of removal  $(\eta_R)$  as:

$$\eta_{R} = \frac{\dot{W}_{\text{min}}}{\dot{B}_{inputs destroyed}}$$
 (8)

The task now is to define the minimum work required to remove material from a work piece. Ideally, the definition of minimum work should be process-independent. The task is complicated, however, by the reality that there are many different mechanisms for material removal (shearing, eroding, dissolving, etc.). Here we discuss three potential approaches to arriving at a minimum work value.

The first approach is to assume that the material is a crystalline solid. The minimum energy to rupture that crystal (fracture along a specified plane) can then be calculated, assuming that only the two adjacent layers of atoms that will be split to form new surfaces are acted upon in the rupture process. Following the example of Courtney [8], the work per unit area is given by:

$$\frac{W}{A} = \frac{2Ea_0}{\pi^2} \tag{9}$$

Rate of Exergy Inputs - CNC Machining				
Mass in	Moles per	Primary		
(kg/sec)	second	exergy (W)		
1.81E-04	1.00E-02	9.02		
1.60E-05		719.5		
		4800		
		5529		
(cm <sup>3</sup> /sec)				
Specific Exergy				
Consumption (J/cm <sup>3</sup> )		1223		
Minimum Work of Al				
(J/cm <sup>3</sup> )	55.4			
	n⊳	4.5%		
	Mass in (kg/sec) 1.81E-04 1.60E-05  IRafe sec) Exergy on (J/cm³)	Mass in (kg/sec) second 1.81E-04 1.00E-02 1.60E-05  IRafe sec) 4.52 Exergy on (J/cm³)		

Table 2: Efficiency of removal for CNC machining of aluminum. Assumes an exergy content for the proprietary coolant of 45,000kJ/kg, which is an average among a variety of long-chain hydrocarbons. For aluminum 7075, the minimum work of plastic deformation to failure is 55.4 J/cm<sup>3</sup>. Data from [12].

where A is the cross-sectional area of the fractured plane, E is the modulus of elasticity and  $a_0$  is the initial interatomic spacing of the crystal material. Using E=200GPa and  $a_0$  (the side length of a BCC crystal) = 0.287nm for steel, plugging into (9) gives a minimum specific work of  $11.6J/m^2$ . Alternatively, one could equate the minimum specific fracture work with the surface energy per unit area created (=2 $\gamma$ ). Using Murr's calculated value of  $\gamma$ =1715ergs/cm² [9], we arrive at a specific work of fracture of  $3.43J/m^2$ , which is of the same order of magnitude as that determined using the theoretical fracture strength method. It should be stressed that this analysis captures only the separation of two crystalline material layers from one another, which is very different from practical experience in which stress is divided among the multitude of layers composing a bulk material.

A second measure is to evaluate the work required to plastically deform to fracture the material to be removed, which yields a volume-based measure of minimum energy. Ideally, the energy required for elastic deformation is recoverable, so the minimum work is given by the area solely underneath the plastic region of the curve. Assuming a linearly increasing plastic region:

$$\frac{W_{\min}}{V} = \int_{\varepsilon} \sigma d\varepsilon = \sigma_{yield} (\varepsilon_{total} - \varepsilon_{yield}) 
+ 0.5(\sigma_{\max} - \sigma_{yield}) * (\varepsilon_{total} - \varepsilon_{yield})$$
(10)

where for AISI 1212 steel,  $\sigma_{yield} = 415 MPa$ ,  $\sigma_{max} = 540 MPa$ ,  $\varepsilon_{total} = 0.10$ , and  $\varepsilon_{yield} = \sigma_{yield}/E = 415 MPa/200,000 MPa = 0.002$ . Plugging into the above equation yields  $\mathbf{W}_{min} = \mathbf{46.8 MJ/m}^3 = \mathbf{46.8 J/cm}^3$ .

A final approach would be to seek a more practical value of "minimum" work, since the second approach yields impossibly small energies (1-100 J/cm³) for material removal. One way to do this would be to assume a straightforward cutting process

Rate of Exergy Inputs - Grinding						
	Mass in	Moles per	Primary			
Inputs	(kg/sec)	second	exergy (W)			
H2O	1.81E-04	1.00E-02	9.02			
Coolant Concentrate	1.60E-05		719.5			
Electricity			975			
- · ·			4=0.4			
Total			1704			
Material Per	Material Removal Rate					
(cm <sup>3</sup> /sec)		0.0166				
Specific Exergy Consumption (J/cm³)		· · · · · · · · · · · · · · · · · · ·	102624			
Minimum Work of Fe T						
Removal	(J/cm <sup>3</sup> )	46.8				
		$\eta_{R}$	0.046%			

Table 3: Efficiency of removal for grinding of steel. Data from [13].

Bridgeport Torque TC3 Automated Milling Machine		Okamoto ACC-8-20DX Surface Grinder	
	11:1 Mix of H2O and a severely hydrotreated		
Cutting fluid	petroleum oil		
Cutting fluid			
make-up rate	0.1875gal/hr		
Al Removal	Energy Rate		<b>Energy Rate</b>
Rate (cm <sup>3</sup> /sec)	(W)	Rate (cm <sup>3</sup> /sec)	(W)
4.52	4810	0.0166	975
9.03	5960		
12.04	6990		

Table 4: Characteristics of a Bridgeport CNC mill and a surface grinder [12,13].

is used (shearing). The energy determined in this way will be reflective of the greater energies required to generate the large shear strains seen in metal cutting, and although it will not be a "minimum theoretical" value for work it might be viewed as a more realistic one. As an example of how this might work, manufacturing texts suggest the shear energy per unit volume is approximately  $(2-4)\tau$ , and since the Brinell hardness is  $\sim 2\pi\tau$ , we can say that  $u\sim (0.5-1)H_B$ . Using  $u=0.5H_B$  and plugging in hardness values for steel, we get energies in the range of 3.0-5.5kJ/cm³, more than two orders of magnitude larger than that given by (10) [10].

The merits of these different techniques we leave open for debate, and encourage the suggestion of new approaches to dealing with removal processes from an exergy standpoint. To close, however, we illustrate the use of the second approach to evaluating minimum work by a comparison between CNC machining and grinding.

In studies of manufacturing systems, it has been found that the efficiency of a manufacturing process is a strong function of the rate at which the process occurs [11]. That is, a process such as milling which operates at a moderate material processing rate is virtually always more efficient (on an energy per unit mass basis) than a process such as grinding which operates at a slower process rate.

Tables 2 and 3 present a comparison between these two processes using real-world data (machine characteristics given in Table 4). As anticipated, the efficiency of removal as defined in (10) is much smaller for grinding of steel (0.046%) than for CNC machining of aluminum (4.5%). A spot check convinces us that this difference is not due to the material difference. For example, we estimate the CNC machining of steel to have an efficiency on the order 2-4%.

Although both of the preceding examples were at the "industrial ecology" level of analysis, we would like to point out that the model presented in this paper is also capable of in depth thermodynamic analysis. That is, by allowing both temperature and pressure changes one can look for opportunities on a component by component level to improve efficiency and recover waste heat. These analyses will be quite similar to those already successfully employed in energy systems analysis [14-15]. Finally, our construction of a large multicomponent system as shown in Figure 1, and our subsequent analysis are equivalent to the so-called method of cumulative exergy analysis (see for example Hau and Bakshi [16]).

#### ACKNOWLEDGMENT

We wish to express our sincere appreciation to the National Science Foundation who supported this work in full through their Design, Manufacturing, and Innovation Grant #0323426.

# REFERENCES

- [1] Energy Information Administration. *Energy Consumption by Sector*, 1946-2006 <a href="http://www.eia.doe.gov/emeu/aer/txt/ptb0201a.html">http://www.eia.doe.gov/emeu/aer/txt/ptb0201a.html</a>>
- [2] Gyftopoulos, Elias P. and Gian Paolo Beretta. *Thermodynamics: Foundations and Applications*. Mineola, NY: Dover Publications, 2005.
- [3] Bejan, Adrian. Advanced Engineering Thermodynamics. New York: John Wiley & Sons, 1988.
- [4] Szargut, J., Morris, D.R., & Steward, F.R., 1988, Exergy Analysis of Thermal Chemical and Metallurgical Processes, Hemisphere Publishing Corporation and Springer-Verlag, New York, NY, USA.
- [5] Jones, Alissa. "The Industrial Ecology of the Iron Casting Industry," Master of Science in Engineering Thesis, Massachusetts Institute of Technology, Cambridge, MA. May 2007.
- [6] Considine, T., CJ Jablonowski, and DMM Considine, The Environment and New Technology Adoption in the US Steel Industry. 2001, The Pennsylvania State University.
- [7] Sorin, M., Lambert, J., and Paris, J., 1998, "Exergy Flow Analysis in Chemical Reactors." *Trans IChemE*, 76: 389-395.
- [8] Courtney, Thomas H. *Mechanical Behavior of Materials*. New York: McGraw-Hill, 1990.
- [9] Murr, Lawrence E. *Interfacial Phenomena in Metals and Alloys*. London: Addison-Wesley, 1975.
- [10] Degarmo, Paul E., Black, J.T., and Kohser, Ronald A. *Materials and Processes in Manufacturing*. Upper Saddle River, NJ, Prentice Hall: 1997.
- [11] Gutowski, T., Dahmus, J., Thiriez, A., Branham, M., and Jones, A., 2007, A Thermodynamic Characterization of Manufacturing Processes, IEEE International Symposium on Electronics and the Environment, Orlando, Florida, USA, May 7-10.
- [12] Kordonowy, David. "A Power Assessment of Machining Tools," Bachelor of Science in Mechanical Engineering Thesis, Massachusetts Institute of Technology, Cambridge, MA. May 2002.
- [13] Baniszewski, Beth. "An Environmental Impact Analysis of Grinding," Bachelor of Science in Mechanical Engineering Thesis, Massachusetts Institute of Technology, Cambridge, MA. June 2005.
- [14] Dincer, I., and Rosen, M. Exergy: Energy, Environment, and Sustainable Development. Boston: Elsevier, 2007.
- [15] Bejan, A., Tsatsaronis, G., Moran, M. *Thermal Design & Optimization*. New York: John Wiley & Sons, 1996.
- [16] Hau, J., and Bakshi, B. Expanding Exergy Analysis to Account for Ecosystem Products and Services. Environmental Science and Technology, 2004. 38 (13): p. 3768-3777.