Bayesian Material Separation Model with Applications to Recycling

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Abstract

In this paper we present a probabilistic model of material separation processes. The model, based on Bayes' Theorem can estimate the performance of any binary separation process. Given input material data and the probabilistic characteristics of the separation process, the model estimates the masses and concentrations of the output streams. Applications of this model to separation processes, including those used in material recycling, are demonstrated.

Keywords

Material Separation, Material Purification, Recycling, Bayes' Theorem.

INTRODUCTION

In this paper, we present a mathematical model for a process that concentrates a target material from a mixture. The model can be used for analyzing the performance of recycling processes and material recycling systems as well as other separation and purification processes which are mentioned later. The work here addresses some of the same issues as in an earlier work by Murphy et al on plastics recycling (2001) [1]. Our approach is based on Bayes' Theorem, and requires an estimate of two probabilities; the probability of correctly identifying the target material in the mixed input stream, and the probability of correctly identifying the non-target material. From these, one can then obtain the probabilities for a false negative and a false positive. The model applies to any binary separation process, i.e. any separation process that has only two output streams; the stream with the higher concentration of the target material, and the stream with the lower concentration of target material. By using Bayes' Theorem, the four probabilities, and conservation of mass, one can then derive a complete set of equations to rigorously describe a material separation process. This model is then developed to show its practical utility, and compared with data in several different areas.

DEVELOPMENT OF THE MODEL

Consider a mixture of a target material, A, with mass m_A, and everything else, called "not A" or A^c, with mass m_A^c. The concentration[1] of the target material A is just the probability of A, and is represented as c. The concentration of A^c is just $1 - c$.

$$P(A) = c \tag{1}$$

$$P(A^c) = 1 - c \tag{2}$$

Now consider a test, call it B, which says it has identified A and the compliment B^c which says it has identified A^c. We define the following conditional probabilities:

$$P(B \,/\, A) = r \tag{3}$$

$$P(B^c \,/\, A) = 1 - r \tag{4}$$

$$P(B^c \,/\, A^c) = q \tag{5}$$

$$P(B \,/\, A^c) = 1 - q \ . \tag{6}$$

Equation (3) gives the probability that the test, when given material A, correctly identifies A. Equation (4) gives the probability that the test, when given material A, incorrectly identifies the material as A^c—this is the false negative. Equation (5) is the probability that the test, when given material A^c, correctly identifies A^c. Equation (6) is the probability that the test, when given material A^c, incorrectly identifies the material as A—this is the false positive. We assume that in order to be a viable separation process, $0.5 < r \leq 1.0$ and $0.5 < q \leq 1.0$.

From Bayes' theorem [2], we can write the probability of A, given the test result B as

$$P(A \,/\, B) = \frac{P(B \,/\, A)P(A)}{P(B \,/\, A)P(A) + P(B \,/\, A^c)P(A^c)} \ . \tag{7}$$

But the probability of A given B is just the concentration of a new stream call it c_{j+1}. This is given in equation (8).

[1] $c = m_A/(m_A + m_A^c)$, $1\text{-}\, c = m_A^c/(m_A + m_A^c)$

$$c_{j+1} = \frac{rc_j}{rc_j + (1-q)(1-c_j)} \quad . \tag{8}$$

In this notation, c_j is the concentration of A in the incoming stream. Hence the tests B and B^c constitute a material separation step, which attempts to find the target material A, and reject the other material A^c to the other stream. This is portrayed in Figure 2.

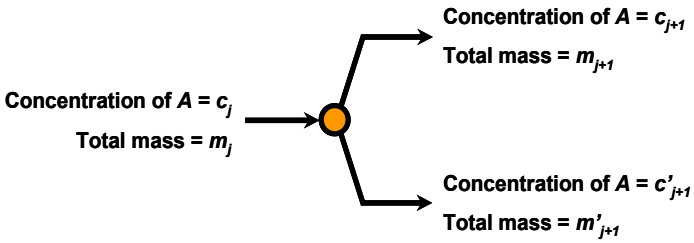

Figure 1. A binary material separation step that takes an incoming stream of mass m_j and target material concentration of c_j and outputs two streams: a concentrated stream of mass m_{j+1} and target material concentration of c_{j+1} and a dilute stream of mass m'_{j+1} and target material concentration of c'_{j+1}.

The concentration c_{j+1} is the concentration of A in the high concentration stream. The other concentration c'_{j+1} is the concentration of A in the low concentration stream. By a similar procedure this too can be determined from r, q and c_j. This is given in equation (9).

$$c'_{j+1} = \frac{(1-r)c_j}{(1-r)c_j + q(1-c_j)} \quad . \tag{9}$$

Note that for r>0.5, q>0.5 and c_j<1, $c_{j+1} > c_j > c'_{j+1}$,

Now that we know the concentrations c_{j+1} and c'_{j+1} we can solve for the masses of the output streams m_{j+1} and m'_{j+1}. With the mass of the input stream m_j we get the two following mass balance equations for the material A and the material A^c

$$\begin{aligned} c_j m_j &= c_{j+1} m_{j+1} + c'_{j+1} m'_{j+1} \\ (1-c_j) m_j &= (1-c_{j+1}) m_{j+1} + (1-c'_{j+1}) m'_{j+1} \end{aligned} \tag{10}$$

These equations can be written in the form

$$\boldsymbol{y} = \boldsymbol{C}\boldsymbol{x} \tag{11}$$

and can be solved for the masses $x^T = [m_{j+1}, m'_{j+1}]$ as a set of linear equations. The determinant of C is

$$\boldsymbol{det\, C} = c_{j+1}(1-c'_{j+1}) - c'_{j+1}(1-c_{j+1}) \tag{12}$$

Notice that we do not have a solution under the condition

$$c_{j+1} = c'_{j+1} \tag{13}$$

Solving equations (10) yields the mass of the stream with the higher concentration of A:

$$m_{j+1} = m_j \frac{c_j - c'_{j+1}}{c_{j+1} - c'_{j+1}} \tag{14}$$

and for the lower:

$$m'_{j+1} = m_j \frac{c_{j+1} - c_j}{c_{j+1} - c'_{j+1}} \tag{15}$$

We are also interested in the amount of the target material captured in the concentrated output stream. This mass ratio (the mass of A in the concentrated stream divided by the mass of A in the input) can be written as,

$$\frac{c_{j+1} m_{j+1}}{c_j m_j} = \frac{c_{j+1}}{c_j}\left[\frac{c_j - c'_{j+1}}{c_{j+1} - c'_{j+1}}\right] = r \tag{16}$$

And the mass ratio of target material in the dilute stream is,

$$\frac{c'_{j+1} m'_{j+1}}{c_j m_j} = 1 - r \tag{17}$$

By a similar procedure one can obtain the masses for A^c. The ratio of the mass of A^c in the concentrated stream (of A) divided by the mass of A^c in the input is,

$$\frac{(1-c_{j+1}) m_{j+1}}{(1-c_j) m_j} = 1 - q \tag{18}$$

and for the dilute stream,

$$\frac{(1-c'_{j+1})m'_{j+1}}{(1-c_j)m_j} = q \qquad (19)$$

APPLICATION OF THE MODEL

The model developed here can be used to describe materials separation processes such as those employed in materials recycling as well as those used in other separation applications. Here we will describe the behavior of this model and compare it with results published in the literature. First, it is useful to illustrate the range of results one can obtain from this model. For this purpose, it is convenient to combine the parameters r and q used in eq. 8 into a single new parameter β, where

$$\frac{1-q}{r} = 1-\beta \qquad (20)$$

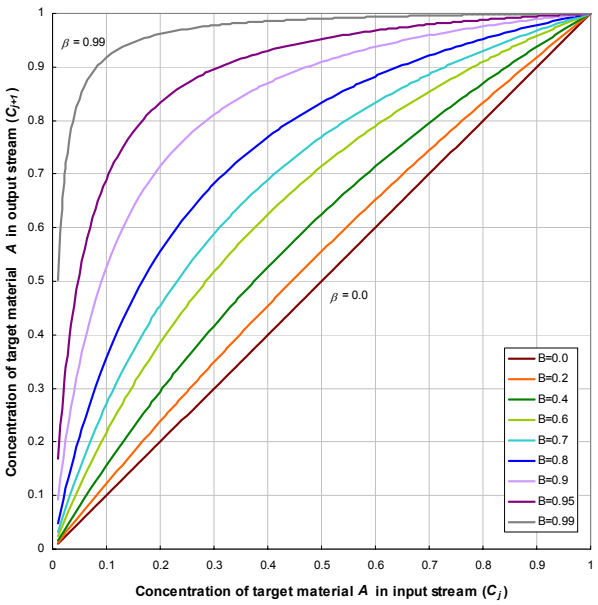

Figure 2. The performance of a separation process for various values of β.

Given the limits on r and q, β will be confined to the range of $0 < \beta \le 1$, where $\beta = 0$ would have no concentrating effect, (this would be the case $r = 0.5$, $q = 0.5$), and $\beta = 1$ would turn the output into pure target material. This would be the case $q = 1.0$ and $r > 0.5$. Recall from eq. (16) that the value of r will determine exactly how much of the pure target material will be captured. The range of output concentrations c_{j+1} from eq. (8) for different values of β as a function of the input concentration c_j is shown in Figure 2.

Notice that as the concentration of the input (c_j) approaches 1, the increase in concentration difference $(c_{j+1} - c_j)$ decreases dramatically for the higher range of β values. This is illustrated as the change in slope for the curves shown in Figure 2. In other words, it becomes increasingly difficult to obtain higher purity values.

To illustrate the effect of β, we analyze two systems with the same input concentration of target material, $c_j = 0.10$, the same total input mass, $m_j = 100$ kg, and the same desired output concentration of target material, $c_{target} \ge 0.99$, but with different values of β (0.9 and 0.6). The desired output target concentration is achieved by repeated separations of the concentrated output flow. The results are illustrated as tree diagrams in Figures 3 and 4.

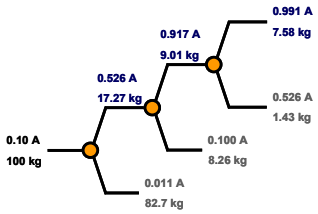

Figure 3. A material separation scheme with $r = q = 0.909$ and $\beta = 0.9$. The output target is reached in three steps.

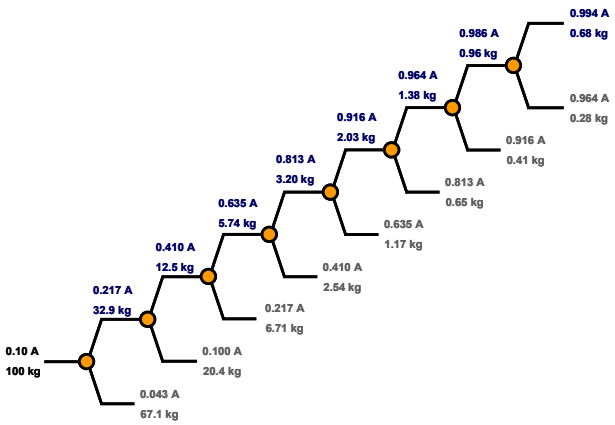

Figure 4. A material separation scheme with $r = q = 0.714$ and $\beta = 0.6$. The output target is reached in eight steps.

Figures 3 and 4 show not only the concentrating effects of separation processes, but also the amount of target material captured. In Figure 3, the target material is concentrated from 0.10 to 0.991 in three steps. Of the 10 kg of target material A that enter, 7.51 kg (0.991 x 7.58 kg) are captured. In Figure 4 the target material is concentrated from 0.10 to 0.994 in 8 steps. Of the 10 kg of target material A that enter, only 0.67 kg (0.994 x 0.68 kg) are captured. Note that as the materials proceed through the separation steps, the waste streams become highly concentrated in A, and could potentially re-enter the system in an earlier step. These issues of re-entrant flow are addressed by Albino in a separate study [3].

Albino's work also provides a way to calculate the concentrations and masses at any output for repeated separation steps as illustrated in Figures 3 and 4. For example after n separation steps, the concentration of the target stream is

$$c_{j+n} = \frac{r^n c_j}{r^n c_j + (1-q)^n (1-c_j)} \quad . \qquad (21)$$

Similarly, the total mass of material in that stream can also be calculated as [4]

$$m_{j+n} = m_j (c_j r^n + (1-c_j)(1-q)^n) \qquad (22)$$

These equations give the same values as shown on the top branches in Figures 3 and 4. For clarity, the results for the top branch in Figure 4 are plotted in Figure 5. This graph shows a phenomenon known as the "concentration dilemma", where the amount of material recovered decreases as final output concentration increases [5]. The effect is particularly prevalent when concentrating very dilute mixtures of the target material. One way to improve this process is to recapture the "waste" streams with concentrations higher than the original input. For realistic processes, this happens after the second separation step [3].

Note that equation 21 can be solved for the number of required separation steps n^*, given an input concentration c_j and a target concentration c_t.

$$n^* = \frac{log(\frac{1-c_t}{c_t}) - log(\frac{1-c_j}{c_j})}{log(1-\beta)} \qquad (23)$$

This result can be used to make an estimation of how separation costs would scale for different situations. For example, for very dilute solutions, it can be shown that costs would scale as

$$\text{cost} \sim n^* \sim \log(\frac{1}{c_J}). \qquad (24)$$

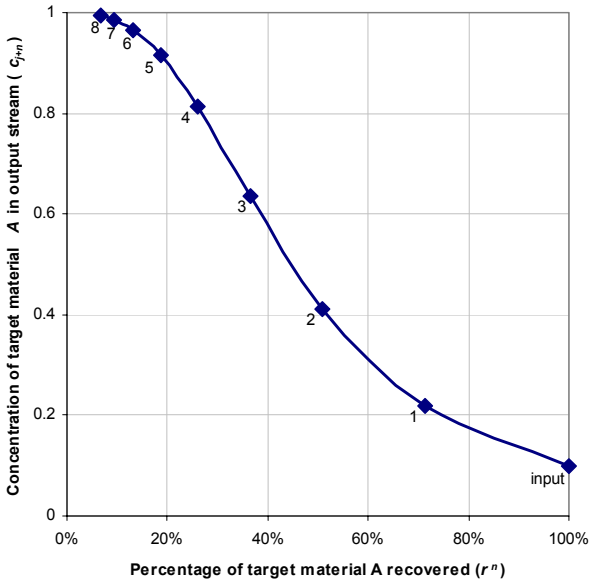

Figure 5. A plot of output concentration, c_{j+n}, and percent of mass recovered as a function of the number of separation steps already completed n. Values for n are shown next to the points on the plot.

This result gives a scaling very similar to that of the "Sherwood Plot" which shows that *log (price)* scales with $log(1/c_j)$ for many different types of materials separated from dilute mixtures. See, for example, King [6] and Grübler [7].

In addition, there are still other results in the literature for which the model presented here appears to give similar behavior. These include the bio-magnification effect [8], and the "Shannon" result for the separation of a binary mixture where both materials are targeted [9].

Finally, we show the different character of the two probabilities r and q with two additional figures. In Figure 6 we have fixed r at a high value (r = 0.99) and allow q to vary. The parameter q represents the ability of the process to reject the non-target material. It clearly has a very strong

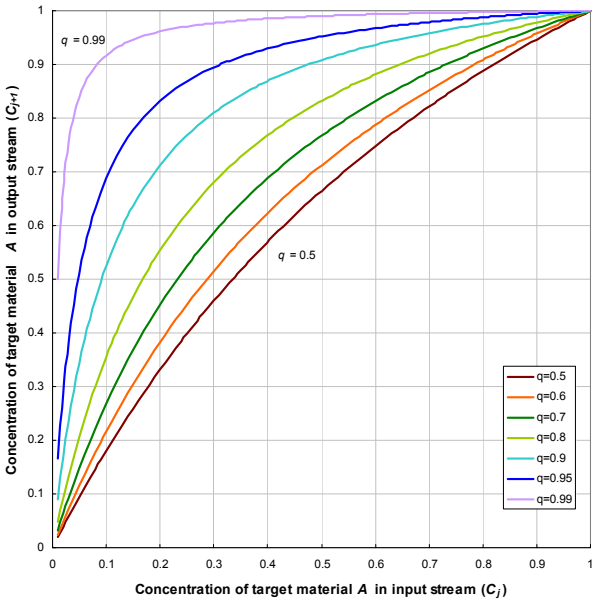

Figure 6. The performance of a separation process for various values of *q* while holding *r* = 0.99.

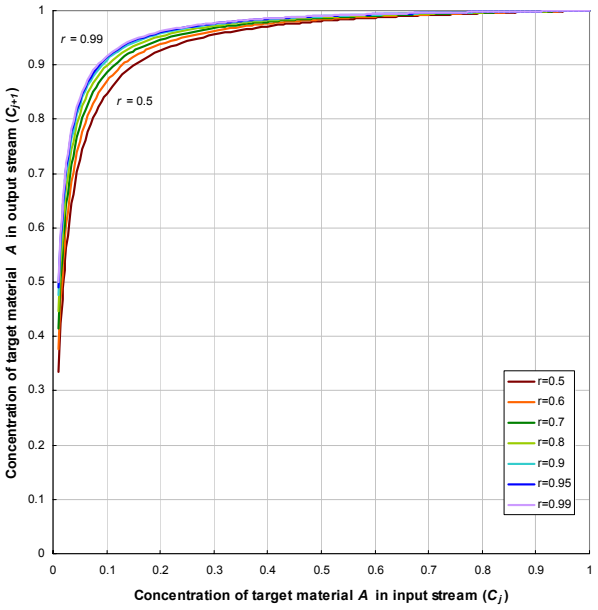

Figure 7. The performance of a separation process for various values of *r* while holding *q* = 0.99.

effect on purity of the concentrated output stream. The parameter r on the other hand controls the yield. In Figure 7 q is held constant at a high value (q = 0.99) and r is varied. This again underscores the importance of parameter q.

A comparison with data published in the literature for the physical separation of shredded plastics and circuit boards in different settings gives a wide range of values for r, q, and β. These are listed below in Table 1 [10, 11, 12].

Table 1. Values for *r*, *q*, and β for different material separation scenarios.

Separation Method	Materials Separated	*r*		*q*		β	
		min	max	min	max	min	max
electrostatic	ABS, HIPS	0.414	0.798	0.608	0.946	0.509	0.870
electrostatic	metals, non-metals	0.576	0.753	0.971	0.998	0.959	0.997
centrifugal	metals, non-metals	0.530	0.823	0.970	0.998	0.952	0.998

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