Mixing Entropy and Product Recycling

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Abstract— In this paper, we explore the relationship between the mixture of materials used in a product and the extent of end-of-life materials recycling from retired products in the United States. This is done for 14 common products, which are either widely recycled or not recycled. The results demonstrate the utility of using a normalized mixing entropy measure, identical to Shannon Information, to resolve the products that are recycled and not recycled. The success of this measure is explained by outlining an analogy between recycling systems and communications theory.

Two key observations are required: 1) the same axioms which establish Shannon Information, "H", as a measure of the information content of a message, can also apply to a measure of mixing for materials, and 2) just as message codes can be represented as tree diagrams, so too can recycling systems. Using a well known communications theory result, Shannon's Noiseless Coding Theorem, this analogy shows that "H" for material mixtures represents a reasonable lower bound on the cost of separation.

Keywords-recycling; mixing; entropy; product design

I. INTRODUCTION

In this work we seek a compact representation of the potential to recycle the materials used in a product. It is generally recognized that the two most important components of the recycling problem are: 1) the value of the materials (consumer demand), and 2) the ease of isolation [1]. The first issue is relatively straight forward to address. The second issue can be addressed by focusing on the materials separation processes used to break down, identify, and sort the materials in a product. This paper will start by developing a measure for the "complexity" of a mixture of materials, then relate this measure to the materials separation process. Some candidate measures for the complexity of a materials mixture include: 1) Shannon Information, 'H'; 2) number of materials, 'M'; and 3) number of perfect binary separation steps, 'M-1'.

The first, Shannon information, will be reviewed in this paper. The others have been discussed and used by other researchers. For example, Ishii has suggested design guidelines for recycling that include identifying the number of materials used in a product [2, 3]. Similarly, one could use the number of perfect binary separation steps, M-1. This measure was used by Sodhi et al. [4]. There are two important advantages to using Shannon information. One is that it can be developed rigorously and there exists a large body of results already derived from it. The second is that it is easier to apply since the counting of materials is naturally modulated by the concentration dependence of H. Furthermore, it will be noted that log M is in fact contained in H. In particular, this paper will show that Shannon's measure of information and Shannon's Noiseless Coding Theorem can be reinterpreted to represent a measure of material mixing and a lower bound on the operational cost of separation, respectively.

II. OVERVIEW OF THE ANALOGY

Information theory was developed to understand the behavior of a communication system, as shown in Fig. 1 [5, 6, 7]. Of particular interest to the recycling problem are the parts of the system that encode information, send it in the form of a message along a channel, and decode it at the other end. It is important to realize that the information and the encoding are quite different entities. Information helps resolve uncertainty, while encoding symbolically represents this information and can be done in many different ways. For example, many encoding schemes are made up of words, which in turn are made up of an alphabet of symbols. If the scheme is the English language then the alphabet is a through z. If the scheme is a binary code then the alphabet has only two symbols, usually 0 and 1. Of particular interest is how efficient a code is for transmitting the information in a message. Alternative code words can be represented by tree diagrams that graphically represent the length of a word 'n_i'. The average word length in a message can then be measured as the sum of the individual word lengths times their individual probabilities of occurrence. This is given in (1).

$$\bar{n} = \sum_{i=1}^{M} p_i n_i \qquad (1)$$

Shannon's noiseless coding theorem then shows that H is a lower bound for the average word length in a message. This important result enables encoders to assess the efficiency of codes, as well as the capacity of the communication channel.
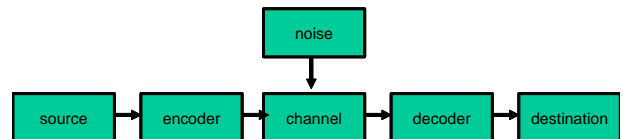

Figure 1. Communication system.

In what follows, it is shown that within certain reasonable constraints, these results can be used to interpret the behavior of product recycling systems. In summary: Shannon information, H, will be reinterpreted as a measure of mixing. Tree diagrams will be used to represent material recycling systems and the value $\overline{n}$ will be reinterpreted as a measure of the effort or cost to separate the materials. Finally, Shannon's Noiseless Coding Theorem will relate the measure of mixing, H, to the operational cost for separation.

III. MEASURE OF MATERIAL MIXING

A measure of materials mixing can be formulated in a manner identical to how Shannon developed a measure of information. One can start by hypothesizing a measure of material mixing, 'H', which is a function of the number of component materials in a mixture, 'M', and 'c_i', which is the probability of the occurrence of material i. Hence,

$$\sum_{i=1}^{M} c_i = 1$$

(2)

This measure of mixing should have the following properties; 1) H should be continuous in each concentration c_i, 2) If all c_i are equal, $c_i = 1/M$, then H should be a monotonically increasing function of M, and 3) If a mixture is composed of two different mixtures, the total measure of material mixing should be the weighted sum of the individual values. This third property is illustrated in Fig. 2.

In 1948, Shannon showed that the only H satisfying these assumptions is of the form:

$$H = -K \sum_{i=1}^{M} c_i \log c_i$$

(3)

where K is a constant. (By convention we will use K = 1 and take logarithms to the base 2. Then H is given in bits). This same function can be derived from a statistical mechanics approach using Boltzmann's equation to obtain an extensive form of the entropy of mixing. Normalization by the amount of material, leads to an intensive measure of materials mixing identical to (3).

Given the following decompositions,
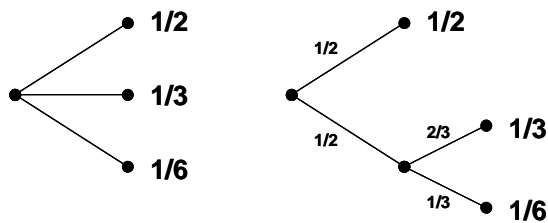
The following should be true,

$$H\left(\tfrac{1}{2}, \tfrac{1}{3}, \tfrac{1}{6}\right) = H\left(\tfrac{1}{2}, \tfrac{1}{2}\right) + \tfrac{1}{2} H\left(\tfrac{2}{3}, \tfrac{1}{3}\right)$$

Figure 2. Decomposition of a choice from three possibilities [6].
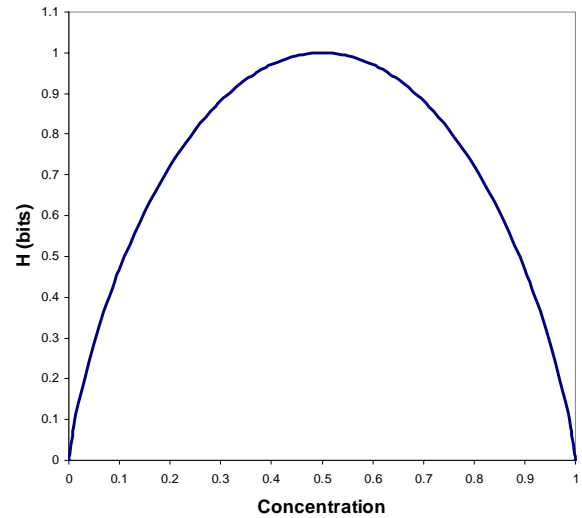

Figure 3. Material entropy for a binary mixture with concentrations c and 1-c.

Before proceeding, it is important to point out the constraints on the function H, and their implications for the recycling analogy. One constraint is related to the symmetry of (3). For example, Fig. 3 shows (3) for a binary mixture. The symmetry implies that all materials are equal. In other words, the measure H for a two component mixture of ratio 90:10 would be the same as for a mixture of 10:90. Hence, this measure would not be appropriate for measuring mixtures of pollutants with benign materials. Secondly, the lattice interpretation suggests the species are of equal sizes. Variations in size can be treated, for example as in the Flory-Huggins theory of polymer-solvent mixing, but (3) as it now stands, does not take this into account [8]. Finally, in as much as (3) represents a normalized entropy measure of mixing, any energy barriers for mixing or demixing would have to be treated separately.

Note that these three restrictions (1. equal value, 2. equal size, and 3. no energy barrier) are not generally met by the materials mixtures in the product. However, these conditions are approximately met in the materials recycling process after fairly routine steps (i.e. removal of hazardous materials and shredding and sizing). The requirement of equal value can be interpreted as having the goal to separate all materials.

IV. CODING AND RECYCLING

Just as codes can represent information, recycling schemes can represent materials. Some materials will have a long list of separation steps necessary for extraction, while others will have only one or a few. This is directly analogous to long and short word lengths (e.g. "Constantinople" versus "Oz").

Both code words and materials can then be represented by tree diagrams. In general, recycling systems can be drawn as tree diagrams starting at the trunk with products which have been collected or which are part of a mixed waste stream, and leading to the branch ends where separated materials and wastes come out. These diagrams only employ two elements:

links and nodes. The former denote conveyance or transportation and the latter denote various processes such as cleaning, shredding, magnetic separation etc. The trees essentially show mass flow through the system.

In what follows, it will be very useful to develop a vocabulary for our representation of recycling systems that is compatible with information theory. This will allow one to easily grasp the analogy, and perhaps more importantly to follow the mathematical proofs, which are readily available in the information theory literature [7].

Hence we will use trees to represent the procedure needed to separate a material. The material X_i, will have a probability of occurrence (concentration) c_i, and a unique set of operations needed to separate it from the feed stream. These operations will be taken from an 'alphabet' of possible operations which number 'D'. The separation system for a particular material will be comprised of n_i steps (number of nodes). This combination of steps (which could be called a word in information theory) will be unique for this material. It then follows that the complete system (a message) to separate a mixture of materials will be made up of the many individual material systems (words). Using this analogy, the same scheme which is used to represent the information in a message can be used to represent the steps necessary to separate a mixture of materials. The great advantage of this analogy is that several important results in information theory can be applied and interpreted for recycling systems.

V. OPERATING COST FUNCTION

Using tree diagrams we can then make the following simple observation; short diagrams are desirable and long ones are not. For recycling, short diagrams mean simple, less costly systems and lower operating costs. For information systems, short diagrams mean efficient representation of information (and hence greater channel capacity). This observation can be represented mathematically by defining the normalized number of separation steps, '$\bar{n}$', as

$$\bar{n} = \sum_{i=1}^{M} c_i n_i \tag{4}$$

where there are M materials with separation system steps n_1, n_2,…, n_M. In an earlier paper, a cost measure similar to (4) was used by Sodhi et al. to optimize a float-sink separation system for plastics [4].

VI. LOWER BOUND ON '$\bar{n}$'

We are now in a position to take advantage of Shannon's Noiseless Coding Theorem, which for our problem states that H(X) is a lower bound for '$\bar{n}$'. Mathematically, Shannon's theorem states

$$\bar{n} \geq \frac{H(X)}{\log D}. \tag{5}$$

TABLE I.   OPTIMUM CONCENTRATIONS FOR MATERIALS WITH GIVEN SEPARATION SCHEMES.

Material	Number of steps, n_i, in separation scheme	c_i (optimum)
X_1	1	1/2
X_2	2	1/4
X_3	3	1/8
X_4	3	1/8

The proof, which requires two intermediate results, is readily available in the information theory literature and therefore is not reproduced here [5, 6, 7]. The theorems say, in words, that of all the ways we can choose among the 'D' processes to separate X = X_1, X_2,…, X_M, H(X) is the most compact. Furthermore, equality in (5) is obtained when

$$c_i = D^{-n_i} \tag{6}$$

Again, in words, (6) says that one should use more of those materials which are easier to separate, and less of those that are difficult. For example consider a mixture of four materials, X_1, X_2, X_3 and X_4, with separator schemes of length, n_1, n_2, n_3, and n_4, that are taken from an alphabet of only two separation processes, D = 2. The materials, separation schemes and optimal concentrations in the product are shown in Table 1. Note that the theorem does not say that the optimum solution is necessarily feasible (this applies to both recycling and encoding).

VII. RECYCLING DATA AND DISCUSSION

These results suggest that H could be used to represent the difficulty of separating materials from their product or waste stream. To test this idea we plotted data for 14 products which are currently 'recycled' or 'not recycled' in the United States. The working definition for 'recycled' corresponds to a recycling rate of roughly 30% or better nationwide. The results of this analysis are given in Fig. 4.

The ordinate is an estimate of the total material value of a given product in the United States. It serves as an estimate of the potential market value of the materials in a product. This value is calculated by multiplying the material value of a single product by the number of such products retired annually. In this analysis, the material value of a single product is estimated by taking one tenth of the retail value of the product. While more detailed material values can be obtained by using market prices for each individual material, the approximation used here does a reasonable job capturing material value and greatly simplifies the analysis. Such an approximation also insulates the analysis from the sometimes volatile fluctuations of material markets.
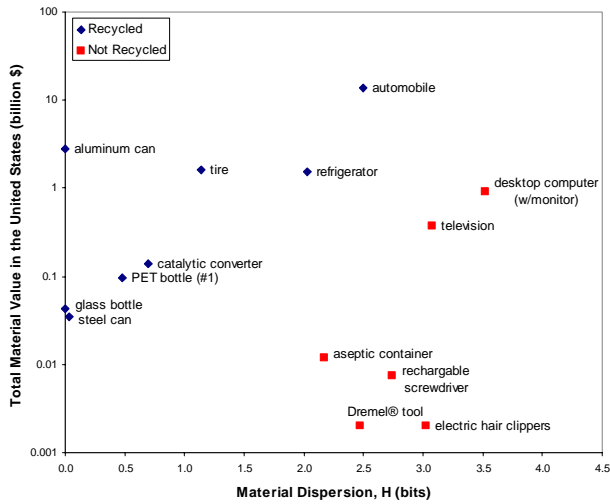
Figure 4. Total material value versus material entropy for 14 products (United States data).

It is important to note that products that are not separated from the mixed waste stream, such as aseptic containers and televisions, are penalized an additional bit. This increase in H reflects the separation step necessary to remove such products from the mixed waste stream. For products that are already separated from the mixed waste stream, such as aluminum cans and automobiles, this initial separation step is already completed. This accounting method helps to highlight the importance of policy with regards to product recycling. Some products examined in this analysis, if separated from the mixed waste stream, can become viable candidates for material recycling. For example, both the television and computer may cross the apparent recycling boundary, shown in Fig. 6, if a program to collect such products was implemented. In fact, some states, including Massachusetts and California, are beginning to implement programs for such products. Other products, such as the smaller electrical products, would still show little promise in terms of recycling, even if a collection program was implemented.

The results suggest that there is an apparent boundary between those products which society recycles and those that it does not. This boundary is shown in Fig. 6. This diagram is very similar to the Sherwood diagram used to relate ore grades and material values for mining [9, 10]. In both cases, society processes those materials with high value and low dispersion, and ignores those with low value and high dispersion. Since both material value and material dispersion are specified in design, Fig. 6 shows very clearly how the recyclability of a product is a function of design. Furthermore, it is worth pointing out that design trends constantly move products towards the lower right hand corner of the figure. Designers are motivated to constantly replace current materials with less expensive materials and less of them, and to increase functionality, often by using a greater variety of materials in a product. Hence, very significant resources are expended each day that move products toward lower 'recyclability'. In contrast, only modest resources are being expended to improve recycling technology to keep up with product design trends.

The abscissa is the measure of material mixing, H, calculated from a bill of materials for each product. In making this calculation, it is important that we count materials in a consistent manner. Fig. 5 shows the 17 different material types that were considered in this analysis. While Fig. 5 shows a branching tree that represents a possible material separation process, the actual separation nodes on the tree are not critical to the calculation of H. Instead, it is the different material types that are important, not the exact separation tree. While the number of material types can range considerably, depending on both the level of product decomposition and the level of material differentiation, as long as the material counting schemes for each product are consistent, useful conclusions can be drawn.
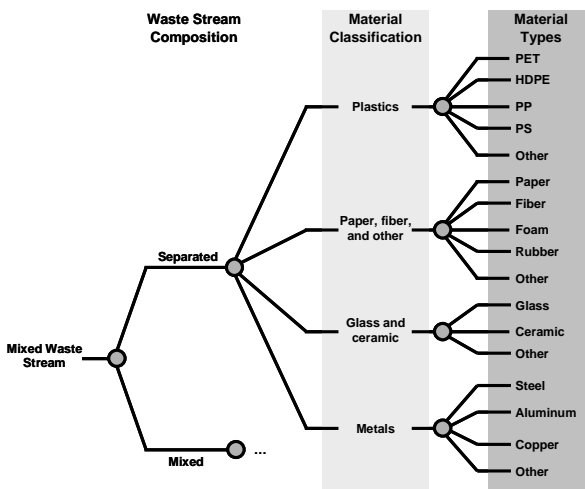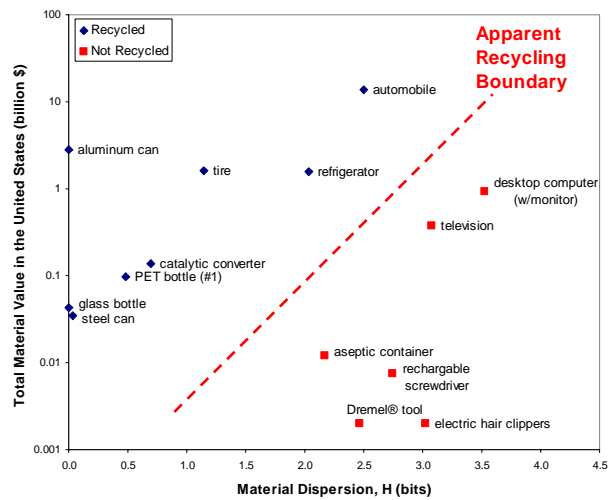

Figure 5. Material counting scheme used to calculate material entropy in Fig. 4.

Figure 6. Apparent recycling boundary for products in the United States.

VIII. Summary

This paper shows that some of the concepts developed in information theory can be applied to the recycling of materials. In particular, a measure of material entropy can be developed in a method completely analogous to Shannon's measure of information. Secondly, Shannon's Noiseless Coding Theorem can be applied, within certain constraints, to show that the material entropy measure represents a lower bound on the operational costs for material separation. The result is that material entropy and material value can be used to determine the recyclability of products.

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