# Material Recycling at Product End-of-Life

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***Abstract*—This work focuses on developing a compact representation of the material recycling potential for products at end-of-life. This representation is based on two measures: the value of the materials used in a product and the mixture of materials used in a product. These measures are similar to those used in constructing the Sherwood plot, which relates metal prices to the concentration of metals in a given ore grade. While the Sherwood plot provides insight into the relative attractiveness of mining different ores, the work here provides insight into the relative attractiveness of recycling different products. This information can in turn be used to help guide both product design and recycling policy.**

***Keywords-recycling; materials; separation; mixing; product design***

## I. MATERIAL SEPARATION

The ability to isolate a single material from a mixture of materials is critical to many industries, from metal extraction to pharmaceutical production to material recycling. In each of these industries, it is the difficulty in separating a single target material from a mixture of materials that largely dictates the market price of the material being isolated. A plot demonstrating this relationship between the difficulty of separation, as represented by the concentration of the target material in the original material stream, and the market price of the target material, was first formulated by Thomas Sherwood in 1959 [1,2]. This simple relationship between material concentration and material price has been shown to hold true for a diverse set of materials, from virgin metals to biological materials to pollutants [2,3,4,5]. An example of the Sherwood plot appears in Fig. 1.

As shown in Fig. 1, the simple relationship between the difficulty of material separation and the market price of that material holds true across many different materials and over many orders of magnitude. The underlying explanation is that the market value for a given target material is primarily driven by the amount of material that must be processed in order to isolate the target material. For target materials that occur in high concentrations, relatively small amounts of material must be processed to isolate a given amount of target material. Thus, target materials occurring at high concentrations generally have lower market values. For target materials that occur in low concentrations, relatively large amounts of material must be processed to isolate a given amount of target material. Thus, target materials occurring at low concentrations generally have higher market values.

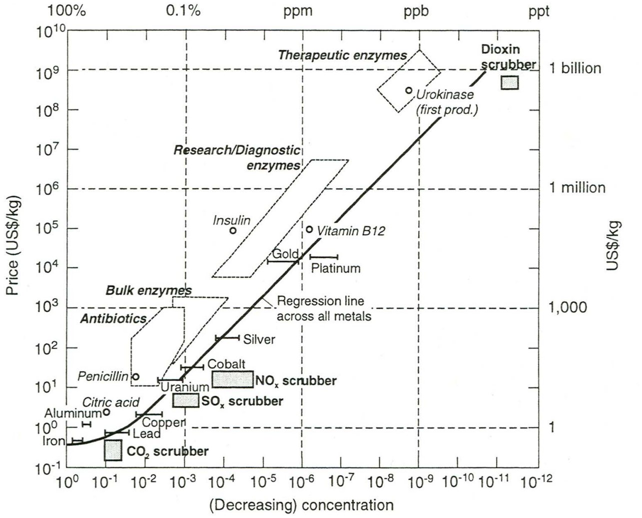


Figure 1. Sherwood plot showing the relationship between the concentration of a target material in a mixture of materials and the market value of the target material. Figure from Grübler [5].

The Sherwood plot can also be explained through simple economic models of revenues and costs. In the case of metal separation from ores, profitability requires that revenues from the sale of the target metal exceed the costs of extracting and isolating the metal. Thus, for profitable metal extraction,

$$k_v m_p c_v > k_c m_p \,, \tag{1}$$

where $k_v$ is the market value of the metal ($/kg of metal),
$m_p$ is the total mass of ore processed (kg of ore),
$c_v$ is the concentration of the metal in the ore (kg of metal per kg of ore), and
$k_c$ is the cost of processing the ore ($ per kg of ore).

Simplifying (1) yields

$$k_v > \frac{1}{c_v} k_c \,, \tag{2}$$

where the left-hand side of (2) is identical to the ordinate of the Sherwood plot, while the right-hand side of (2) represents the abscissa of the Sherwood plot, $1/c_v$, multiplied by the constant $k_c$, the cost of processing the ore per unit mass. The right-hand side of (2) accounts for the metal extraction costs that scale with the amount of ore processed. In the case of metals, this includes separation costs such as mining and milling costs, but does not include costs that do not scale with ore grade, for example smelting and refining costs [6]. However, as can be seen in Fig. 1, the economics of metal separation from ores, and in particular from low-grade ores, is typically dominated by the costs associated with the large material flows required to isolate a given amount of metal.

In short, the Sherwood plot addresses the fundamental relationship between material concentration and material value as it relates to the separation of materials. It can be used to easily assess the relative attractiveness of separating different materials, from mining a metal to isolating a pollutant. The work presented here aims to develop a variant of the Sherwood plot that can be used to assess the relative attractiveness of recycling different products.

## II. MATERIAL RECYCLING FOR PRODUCTS

Adapting the Sherwood plot to address product recycling requires some modifications. Unlike scenarios such as metal separation and pollutant extraction, in which concentrations of the target materials are typically quite low, the concentration of target materials in end-of-life products can be quite high. While the concentration of target materials in end-of-life products clearly depends on which materials are targeted, including common metals and plastics as target materials results in concentrations of target materials that are well above 0.75 for many products. Thus, material concentration does not effectively differentiate among products. However, an analogous metric, material mixing, can serve to differentiate.

As mentioned earlier, the material concentration metric used in the Sherwood plot addresses the difficulty of material separation. For material recycling at product end-of-life, it is not the concentration of a target material, but rather the mixture of materials that determines the difficulty of material separation. Furthermore, while the materials plotted in Fig. 1 typically involve the separation of a single target material from unwanted materials, the separation of materials from end-of-life products typically involves the separation of multiple materials, including metals, plastics, and other materials.

In order to quantify material mixing in a product, it is perhaps easiest to consider how materials in a product are separated. Consider, for example, the case of two hypothetical products with material compositions as shown in Fig. 2. If these products were to be recycled at end-of-life for material recovery, hazardous materials and valuable components would first be manually removed. The product would then be shredded, and various mechanical separations would then take place in order to isolate individual materials. The diagram of separation steps for a given product resembles a branching tree, as seen in Fig. 2, with shredded materials from end-of-life products entering the main branch and separated material categories exiting the final branches. Each node in the tree represents a separation step. For any given tree, fewer nodes mean fewer separation steps, and thus a product with relatively less material mixing. More nodes mean more separation steps, and thus a product with greater material mixing. This can be seen in Fig. 2, where Product A, composed of an equal mixture of two materials, requires only one separation step to isolate a material. Product B, composed of an equal mixture of four materials, requires two separation steps to isolate a material. The number of separation steps can thus serve as a measure of material mixing.

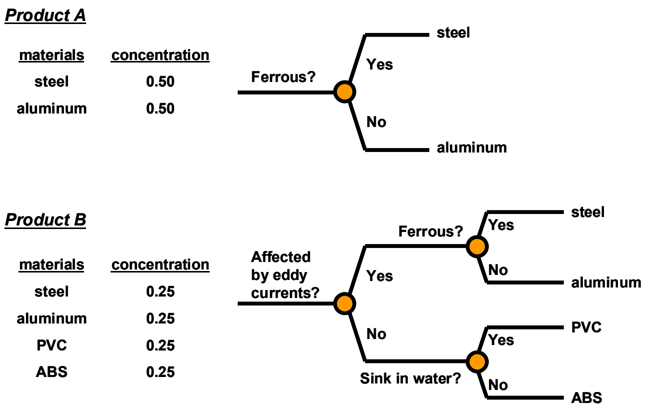


Figure 2. Two hypothetical products along with their respective branching trees for material separation. Note that Product B, with more materials, requires a longer branching tree with more separation steps.

Simple economic models of revenues and costs can again be developed, this time addressing the economics of material recycling for products at end-of-life. For the recycling of a single product to be economically profitable,

$$\sum_{i=1}^{M} m_i k_i > m_p \bar{n} k_n , \tag{3}$$

where $M$ is the number of materials in the product,
$m_i$ is the mass of material $i$ (kg),
$k_i$ is the market value of material $i$ ($ per kg)
$m_p$ is the mass of the product (kg),
$\bar{n}$ is the average number of separation steps, and
$k_n$ is the processing cost per mass-separation step ($ per mass-separation step).

The left-hand side of (3) represents the revenues from the sale of separated materials extracted from a single product at end-of-life. The right-hand side of (3) represents the cost of material separation, and is dependent on the mass of the product, the average number of separation steps, and the processing cost per mass-separation step. The units of $k_n$, cost per mass-separation step, simply account for the fact that processing cost is dependent on both how much mass flows through a single separation step, as well as on how many separation steps that mass flows through.

Looking more closely at the processing cost per mass-separation step, it is clear that this cost is not independent of the quantity processed. Instead, as more mass is processed, the cost per mass-separation step, $k_n$, decreases. These economies of scale are due to the fact that separation equipment requires large up-front capital expenditures [7]. Amortizing these high capital costs over more mass can greatly reduce the processing cost per mass-separation step. To incorporate economies of scale in a very simple form, the following relationship is used:

$$k_n = \frac{k_s}{N_p m_p}\,, \tag{4}$$

where $k_n$ is the processing cost per mass-separation step ($ per mass-separation step),
$k_s$ is the processing cost per separation step ($ per separation step),
$N_p$ is the number of products disposed of, and
$m_p$ is the mass of a single product (kg).

Substituting (4) into (3) and simplifying yields

$$N_p \sum_{i=1}^{M} m_i k_i > \bar{n} k_s\,. \tag{5}$$

The left-hand side of (5) represents the total potential revenue from the sale of separated materials extracted from all end-of-life products of a given type. The right-hand side of (5) represents the cost of material separation, and is dependent on the average number of separation steps and the processing cost per separation step. The average number of separation steps, $\bar{n}$, can be calculated using

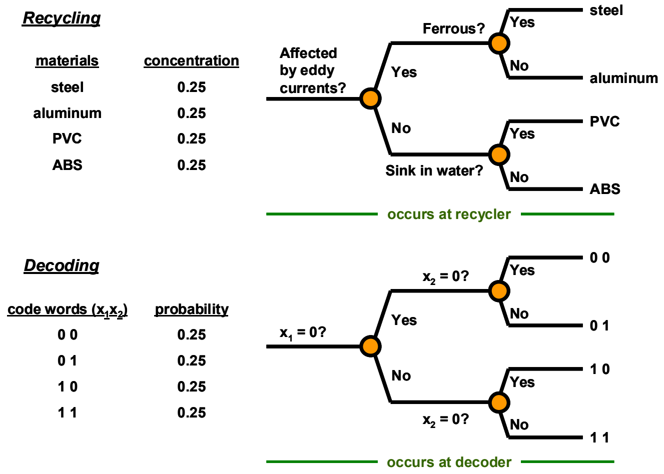
$$\bar{n} = \sum_{i=1}^{M} c_i n_i\,, \tag{6}$$

where $M$ is the number of materials in the product,
$c_i$ is the concentration of material $i$, and
$n_i$ is the number of separation steps necessary to isolate material $i$.

However, while $\bar{n}$ could be calculated for each product, this would require intimate knowledge of the separation processes used in recycling systems. Instead, given a set of reasonable constraints, a result from information theory can provide a simple result that can be used in place of $\bar{n}$.

## III. INFORMATION THEORY

Information theory was initially developed by Claude Shannon in the 1940's, to understand the behavior of a communication system [8,9,10]. While a communication system is focused on encoding a message, sending that message, then decoding the message, a product production system is focused on manufacturing a product, using that product, then recycling the product. In these two systems, strong parallels exist between decoding a message and recycling a product. Fig. 3 shows branching trees that are used both to decode messages in communication and to separate materials in recycling. In communication, the branching trees represent the procedure necessary to decode messages using a series of "yes" or "no" questions. In recycling, the branching trees represent the procedure necessary to separate materials using a series of binary separation processes. Thus, in both cases, short branching trees, with fewer decision nodes or separation steps, are desirable. For recycling, short branching trees represent lower material separation costs, while in communication, short branching trees represent more efficient messages, and thus greater channel capacity.



**Recycling**

| materials | concentration |
|---|---|
| steel | 0.25 |
| aluminum | 0.25 |
| PVC | 0.25 |
| ABS | 0.25 |

**Decoding**

| code words $(x_1x_2)$ | probability |
|---|---|
| 0 0 | 0.25 |
| 0 1 | 0.25 |
| 1 0 | 0.25 |
| 1 1 | 0.25 |

Figure 3. A comparison of branching trees for message decoding in communication and for material separation in recycling.

Given this analogy with communication systems, results from information theory can be applied to recycling systems, given certain constraints. Using Shannon's Noiseless Coding Theorem, a lower bound on $\bar{n}$, represented by $H$, can be calculated using

$$H = -K \sum_{i=1}^{M} c_i \log c_i\,, \tag{7}$$

where $K$ is a constant,
$M$ is the number of materials in the product, and
$c_i$ is the concentration of material $i$.

Setting $K = 1$, and taking logarithms to the base two, yields $H$ in bits. This value of $H$ represents a lower bound on the number of binary separations necessary to isolate a material. Thus, it serves as a measure of material mixing. The higher the $H$ value, the more separation steps necessary and the greater the material mixing; the lower the $H$ value, the fewer separation steps necessary, and the lower the material mixing.

In applying this result from communication systems to recycling systems, there are certain requirements that must be met. The materials that are separated must: 1) be of equal interest, 2) be of equal size, and 3) have no energy barriers to mixing and separating. While these conditions are not generally met by materials in a product, these conditions can be met through fairly routine steps in the product recycling process. Manual removal of hazardous materials and valuable components, along with the goal of separating all materials (meaning each material is a target material), fulfills the first requirement of equal material interest. The second requirement of equal size is met through shredding and sizing operations. The third requirement simply means that materials can be separated through mechanical means. Other separation methods, such as chemical separations, may occur later in the material recovery process, but are generally not completed at the material separation facility.

Additional information about information theory and its application to material recycling is provided in [11].

There do exist other approaches, besides information theory, that may result in alternative formulations for $\bar{n}$. For example, $M$, the number of materials in the product, could provide perhaps one of the easiest means of quantifying material mixing; more materials would mean greater mixing while fewer materials would mean less mixing. In fact, log $M$ is contained in $H$, and simply represents the case in which all materials occur in equal concentrations. The advantage of using $H$ is that the counting of materials is naturally modulated by the concentration of each material.

## IV. PRODUCT DATA

Using the result from information theory, (5) can be rewritten as

$$N_p \sum_{i=1}^{M} m_i k_i > H k_b \,, \tag{8}$$

where $k_b$ is the processing cost per bit ($ per bit).

In (8), $H$, a measure of material mixing, is now used instead of $\bar{n}$. For products in which the inequality in (8) is true, material recycling at product end-of-life would be economically profitable. For products in which the inequality in (8) is not met, material recycling at product end-of-life would not be economically profitable.

To test the effectiveness of (8) in determining the recycling potential of products, 17 common products are analyzed. For each product, the total potential revenue from material recycling is calculated using the left-hand side of (8), while material mixing is calculated using $H$. These values for each product are then plotted in Fig. 4. The products in Fig. 4 are classified as being either 'recycled' or 'not recycled' in the US, where the definition for 'recycled' corresponds to a recycling rate of 25% or better nationwide. Fig. 4 appears to be able to differentiate between products that are currently recycled in the US and those that are not.

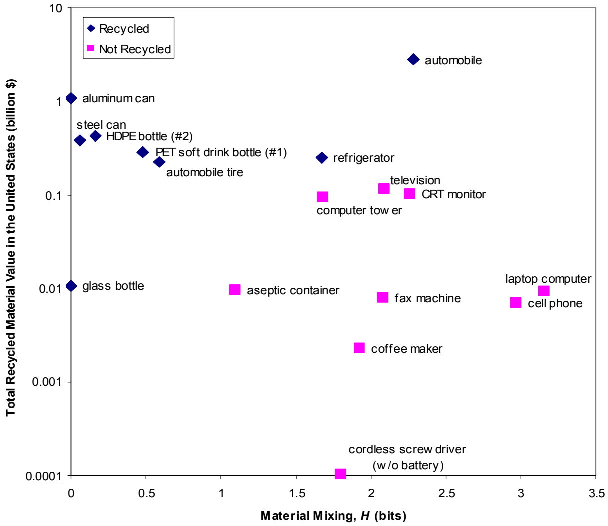


Figure 4. Total recycled material value in the US ($N_p \Sigma m_i k_i$) versus material mixing, $H$, for 17 products in the US.

In Fig. 4, products that fall in the upper left-hand quadrant are recycled in the US, and represent products for which (8) is true. Products that fall in the lower right-hand quadrant of Fig. 4 are not recycled in the US, and represent products for which (8) is not true. Note that Fig. 4, much like the Sherwood plot in Fig. 1, plots material value versus a measure of material dispersion.

The values used to create Fig. 4 are provided in Table 1. The abscissas are calculated using only the bill of materials for a product. Looking at (7), it is clear that a bill of materials with part composition and mass data is sufficient to calculate $H$. The ordinates are calculated using the bill of materials for a product, price data for recycled materials, and market data for the number of such products retired annually in the US. Looking at the left-hand side of (8), the bill of materials provides $m_i$, price data provides $k_i$, and market data for retired products provides $N_p$. It is important to note that the method of calculating total recycled material value used here assumes that the materials in a product can be efficiently extracted, such that all material of a given type is recovered. In reality, separation processes have losses due to inefficiencies in the system. Thus, the recycled material value calculation completed here represents an upper bound on this value.

In the calculation of both the abscissa and the ordinate, it is important that the material counting scheme remains consistent. The 19 materials (18 materials and one 'other') considered, shown in Table 2, were chosen because of their high incidences of use in products today. Each material that is counted means that in a recycling system, materials of that type are separated from other materials. Thus, in a branching tree diagram, as shown in Figs. 2 and 3, each material listed in Table 2 would constitute a separate terminal branch on the tree.

TABLE I. PRODUCT DATA USED IN FIG. 4.

| Product | recycling rate | $N_p$ (million) | $\Sigma m_i k_i$ ($)[a] | H (bits) |
|---|---|---|---|---|
| automobile | 94% | 8.1 | $ 344.883 | 2.283 |
| refrigerator | 90% | 8.0 | $ 31.874 | 1.674 |
| automobile tire | 66% | 270.0 | $ 0.840 | 0.593 |
| steel can | 60% | 31,434.7 | $ 0.012 | 0.060 |
| aluminum can | 50% | 99,800.0 | $ 0.011 | 0.001 |
| glass bottle | 31% | 3,637.5 | $ 0.003 | 0.003 |
| PET soft drink bottle (#1) | 34% | 27,847.2 | $ 0.010 | 0.476 |
| HDPE bottle (#2) | 26% | 25,536.7 | $ 0.017 | 0.163 |
| computer tower | 11% | 35.4 | $ 2.644 | 1.679 |
| CRT monitor | 11% | 35.4 | $ 2.831 | 2.261 |
| laptop computer | 11% | 3.9 | $ 2.400 | 3.160 |
| television | 11% | 21.0 | $ 5.455 | 2.089 |
| aseptic container | 6% | 2,000.0 | $ 0.005 | 1.099 |
| cell phone | 1% | 100.0 | $ 0.069 | 2.970 |
| coffee maker | 0% | 4.0 | $ 0.566 | 1.928 |
| cordless screwdriver (w/o battery) | 0% | 1.0 | $ 0.102 | 1.795 |
| fax machine | 0% | 1.5 | $ 5.366 | 2.081 |

a. Recycled material values from [12].

TABLE II. MATERIAL COUNTING SCHEME USED TO GENERATE DATA IN TABLE 1 AND FIG. 4.

| Metals | Plastics | Non-Metal, Non-Plastic |
|---|---|---|
| aluminum | acrylonitrile/butadiene/styrene (ABS) | glass |
| copper | polycarbonate (PC) | paper |
| iron | polyethylene (PE) | rubber |
| lead | polyethylene terephthalate (PETE) | other |
| nickel | polypropylene (PP) | |
| steel | polystyrene (PS) | |
| tin | polyvinyl chloride (PVC) | |
| zinc | | |

While the material list shown in Table 2 is critical to the results presented in Table 1 and Fig. 4, other material lists, either more detailed or less detailed, could have instead been used, with only minor changes to the relative results. Fig. 5 illustrates this concept by calculating material mixing values for four different products using four different material counting schemes. The material counting schemes used, which include a low-level material decomposition (four materials), a mid-level material decomposition (ten materials), a high-level material decomposition (19 materials), and an ultra high-level material decomposition (36 materials), are presented in Table 3. From Fig. 5, it appears that once approximately ten materials are counted, the relative results, in terms of material mixing, remain unchanged. Also, at higher levels of material decomposition, it appears that material mixing values may in fact converge.

In selecting an appropriate level of material decomposition for a material counting scheme, it is important to consider actual recycling processes and actual markets for secondary materials. In general, the level of material decomposition should correspond to both the separation capabilities of the recycling system and the marketability of the recycled material streams.

## V. DESIGN AND RECYCLING TRENDS

The results shown in Fig. 4 suggest that there is an apparent recycling boundary between those products which society recycles and those that it does not. This boundary is shown in Fig. 6. From Figs. 4 and 6, as well as from the Sherwood plot, it is clear that society pursues materials with high value and low dispersion, and ignores materials with low value and high dispersion. In the case of products, both material value and material dispersion are specified in design. Thus, the recycling potential for products is a function of design, and can be varied through design activities such as material selection.

It is important to note that just as the recycling potential for products can change depending on material choices, the recycling boundary can also change, depending on recycling technology. The apparent recycling boundary in Fig. 6 is set at a level of recycling around 25%. As recycling technologies improve, this boundary could move towards the lower right-hand quadrant. Indeed, some of the products just below the apparent recycling boundary, including computer towers and CRT monitors, are increasingly discussed as potential candidates for wider-scale recycling. However, while recycling technologies may improve, design trends seem to be pushing products towards lower material value and greater material mixing. Designers are constantly motivated to reduce material costs in products, either by using less material or by using less expensive materials. At the same time, materials are being used in new and different applications, presenting designers with increasingly wider selections of potential materials.

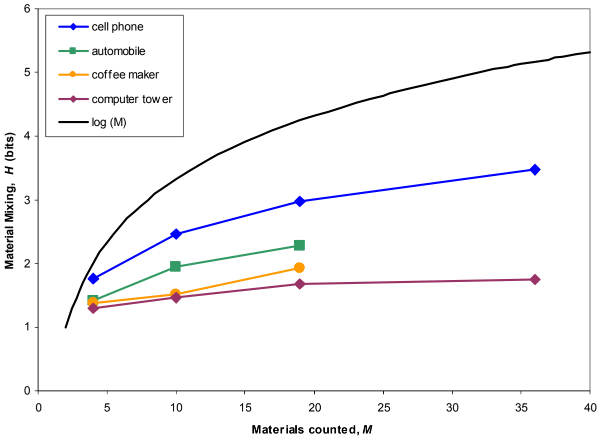


Figure 5. Material mixing, $H$, versus materials counted, $M$, for four different material counting schemes. The upper line, log $M$, represents the upper limit on material mixing values.

TABLE III. FOUR DIFFERENT MATERIAL COUNTING SCHEMES USED TO GENERATE FIG. 5.

| Low-level | Mid-level | High-level | Ultra High-level |
|---|---|---|---|
| ferrous metals | aluminum | aluminum | aluminum |
| non-ferrous metals | copper | copper | antimony |
| plastics | iron | iron | arsenic |
| other | lead | lead | barium |
| | nickel | nickel | beryllium |
| | steel | steel | cadmium |
| | tin | tin | chromium |
| | zinc | zinc | cobalt |
| | plastics | acrylonitrile/butadiene/styrene (ABS) | copper |
| | other | polycarbonate (PC) | gold |
| | | polyethylene (PE) | iron |
| | | polyethylene terephthalate (PETE) | lead |
| | | polypropylene (PP) | mercury |
| | | polystyrene (PS) | nickel |
| | | polyvinyl chloride (PVC) | palladium |
| | | paper | silver |
| | | glass | steel |
| | | rubber | tin |
| | | other | zinc |
| | | | acrylonitrile/butadiene/styrene (ABS) |
| | | | epoxy |
| | | | nylon |
| | | | phenolic resin |
| | | | polycarbonate (PC) |
| | | | polyethylene (PE) |
| | | | polyethylene terephthalate (PETE) |
| | | | polymethyl methacrylate (PMMA) |
| | | | polyoxymethylene (POM) |
| | | | polypropylene (PP) |
| | | | polystyrene (PS) |
| | | | polyvinyl chloride (PVC) |
| | | | tetrabromobisphenol A (TBBPA) |
| | | | paper |
| | | | glass |
| | | | rubber |
| | | | other |

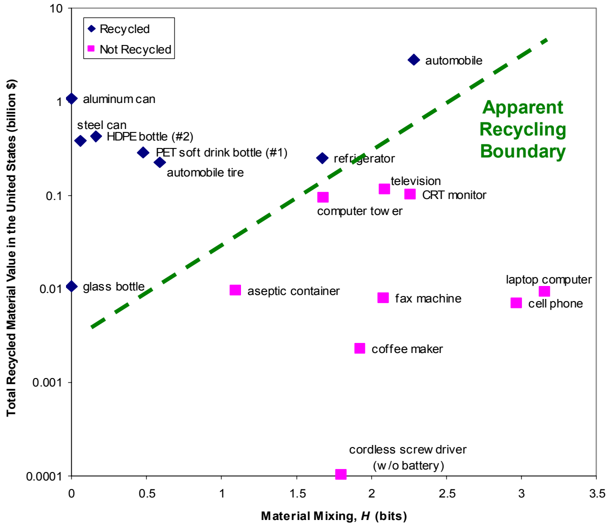


Figure 6. Apparent recycling boundary for products in the US.

These trends in product design can be seen in Fig. 7. Historical data for refrigerators and automobiles show general trends towards greater material mixing. It is interesting to note that in the case of automobiles, SUVs seem to move against this general trend. This is due in part to the fact that SUVs have higher percentages of certain materials, namely steel and aluminum, than do typical automobiles. This results in lower material mixing values. Also, SUVs have considerably more material (1500 kg for a 2000 automobile versus 1970 kg for a 2000s SUV), and thus a greater material value. Fig. 7 also shows differences in electronic products, comparing desktop computers to laptop computers. As consumer electronics continue to get smaller, there seems to be little chance that an effect similar to the SUV effect seen in automobiles, will occur.

In short, while recycling technologies are moving the apparent recycling boundary towards the lower right-hand quadrant in Fig. 6, design engineers are also moving products towards the lower right-hand quadrant. These competing trends are alarming, particularly when one compares the significant resources spent on design to the more modest resources spent on recycling.

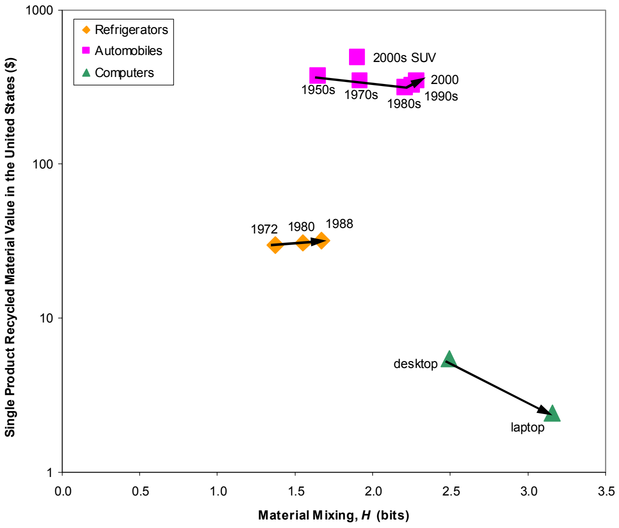


Figure 7. Design trends in refrigerators, automobiles, and computers. Note that the recycled material value is calculated for a single product, not for the entire market of products, as is the case in Figs. 4 and 6.

## VI. SUMMARY

This paper presents a means by which the material recycling potential of products can be evaluated. The work presented here captures and quantifies the two critical aspects involved in material recycling at product end-of-life, namely the value of the materials used in a product and the mixture of materials used in a product. These concepts can be applied to better understand and guide design, engineering, and policy decisions related to recycling.

## ACKNOWLEDGMENT

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