American Association Of Physics Teachers Meeting
January 1999 Anaheim, CA

Poster DH9 Tues Jan 12

The Physics Of Using Field Line Animation In The Teaching Off Electromagnetism

supporting

Paper FC6 Wed Jan 13 at 4 pm

Using 3D Animation To Engender A Sense Of Wonder In The Student Of Electromagnetism

John W. Belcher, Stanislaw Olbert
and Mark Bessette

Department of Physics
Massachusetts Institute of Technology

Supported by grants from The Helena Foundation, the MIT Classes of 51/55, the MIT School of Science, and MIT Academic Computing

Introduction

Animation of field line motion is a significant help in developing intuition about electromagnetic fields, as the shape of field lines is a remarkable guide to their dynamical effects.

We are producing 3D animations which illuminate the physics of electromagnetic phenomena in a visually compelling way. We shoot video of actual demonstrations and animate those demonstrations, including field lines.

These are not cartoons--the field configurations and dynamics are calculated quantitatively. We use a commercial animation package (Kinetix 3D Studio MAX 2.5) with the field lines embedded in the animation as 3D objects (tubes).

Here, we discuss the concept of field line motion and how to define that motion in a physically meaningful (but not unique) way. We also give examples of experiments and animations in the areas of magnetoquasistatics and dipole radiation, and provide the supporting physics and mathematics.

The experiments and animations discussed here are available at

web.mit.edu/jbelcher/www/anim.html

and will be shown at this meeting in

Paper FC6--Wed Jan 13 at 4 pm

Field Line Motion In Magnetoquasistatics

Magnetic field lines are defined in the usual way--that is, $dy / dx = B_y / B_x$, etc. We make no attempt to have the density of field lines correspond to field strength (this is impossible in 2D projections of 3D fields in any case[1]). How do we define field line motion? Consider the following thought experiment. We have a solenoid carrying current provided by the emf of a battery. The axis of the solenoid is vertical. We place the entire apparatus on a cart, and move the cart horizontally at a constant velocity **V**. Our intuition is that the magnetic field lines associated with the currents in the solenoid should move with their source, i.e., with the cart.

How do we make this intuition quantitative? First, we realize that in the laboratory frame there will be a "motional" electric field given by $\mathbf{E} = -\mathbf{V} \times \mathbf{B}$. We then imagine placing a low energy test electric charge in the magnetic field of the solenoid, at its center. The charge will gyrate about the field and the center of gyration will move in the laboratory frame because it $\mathbf{E} \times \mathbf{B}$ drifts ($\mathbf{v} = \mathbf{E} \times \mathbf{B} / B^2$) in the $-\mathbf{V} \times \mathbf{B}$ electric field. This drift velocity is just **V**. That is, the test electric charge "hugs" the "moving" field line, *moving at the velocity our intuition expects.* In the more general case (e.g., two sources of field moving at different velocities), the motion we show in our computer visualizations has the same physical basis:

The motion of a given field line in our quasistatic animations is what we would observe in watching the motion of low energy test electric charges spread along that field line.

We also use this definition of the motion of file lines in situations which are not quasistatic, for example dipole radiation in the induction and radiation zones. In this case (but **not** in the quasistatic cases) *the calculated motion of the field lines is non-physical,* as their speed exceeds that of light in some regions. However, animations of the field line motion is still useful, as the direction of that motion indicates the direction of energy flow.

Calculating Field Line Motion In Magnetoquasistatics

To calculate field line motion consistent with our definition above, we need to insure that our velocity field is *flux preserving* [2,3,4]. For any general vector field $\mathbf{G}(x,y,z,t)$, the flux of that field through an open surface S bounded by a contour C which moves with velocity $\mathbf{v}(x,y,z,t)$ is given by

[1] Wolf, A., S.J. Van Hook, E.R. Weeks, "Electric field line diagrams don't work", Am. J. Phys., **64**, 714 (1996).
[2] Stern, David P., "The Motion of Magnetic Field Lines", Sp. Sci. Rev., **6**, 147-173 (1966).
[3] Vasyliunas, V. M., "Non uniqueness of magnetic field line motion:, J. Geophy. Res., **77**, 6271-6274 (1972).
[4] Rossi, B. and S. Olbert, *Introduction to the Physics of Space* (McGraw-Hill, London, 1970) p. 382.

$$\frac{d}{dt}\int_S \mathbf{G} \quad d\mathbf{S} = \int_S \frac{\partial \mathbf{G}}{\partial t} \quad d\mathbf{S} + \int_S (\quad \mathbf{G})\mathbf{v} \quad d\mathbf{S} - \oint_C (\mathbf{v} \times \mathbf{G}) \quad d\mathbf{l} \qquad (1)$$

If we apply this equation to $\mathbf{B}(x,y,z,t)$ and use $\quad \mathbf{B} = 0$ and $\frac{\partial \mathbf{B}}{\partial t} = - \quad \times \mathbf{E}$, we have

$$\frac{d}{dt}\int_S \mathbf{B} \quad d\mathbf{S} = -\oint_C (\mathbf{E} + \mathbf{v} \times \mathbf{B}) \quad d\mathbf{l} \qquad (2)$$

If we then define the motion of our contours so that the magnetic flux through the surfaces they bound is constant as a function of time, and consider circular contours and fields with azimuthal symmetry, then equation (2) guarantees that their motion satisfies $\mathbf{E} + \mathbf{v} \times \mathbf{B} = 0$, which is the same as $\mathbf{v} = \mathbf{E} \times \mathbf{B} / B^2$, assuming that $\mathbf{v}$ and $\mathbf{B}$ are perpendicular (justified by the fact that there is no meaning to the motion of a field line parallel to itself). This is just the drift velocity of low energy test electric monopoles that we refer to above. This definition of field line motion is not unique (see Vasyliunas[3]).

To show how this works in practice, consider an animation of the motion of the field lines of a magnet levitating above a disk with zero resistance (Figure 1). The magnet is constrained to move only on the axis of the disk, and the dipole moment of the magnet is also constrained to be parallel to that axis. The magnet will be repelled by eddy currents in the disk, and at some point there will be a balance between the downward force of gravity and the upward force of repulsion. We then consider small displacements about this equilibrium position, which will be periodic. The field lines themselves are given by Davis and Reitz[5], and have azimuthal symmetry. How do we trace the *motion* of a field line?

We do this by starting our integration very close to the magnet at a constant angle from the vertical axis, following a given field line out from that point. To animate a line, we use the same starting angle at every point in the oscillation. The field line traced out will be different when the magnet is at different distances from the disk. But the flux inside any open surface whose bounding contour is defined by the intersection of a horizontal plane and the field line when rotated azimuthally will have constant flux inside it, since $\quad \mathbf{B} = 0$. Therefore the motion of the field line so defined satisfies the prescription we have given above, and reflects the drift motion of low energy test electric charges spread along it.

Examples Of Animations: I. Faraday's Law

We have developed a series of Faraday's Law animations involving the fields of a permanent magnet, represented by a 3D dipole, and the fields of a nonmagnetic conducting ring carrying current induced by the motion of the magnet. We now discuss the physics and mathematics involved in calculating the current in the ring and the dynamics of the motion of the magnet, if that is appropriate. The animations are: (1) a magnet moved by an experimenter through a coil of wire; (2) a permanent magnet falling through a nonmagnetic conducting ring with resistance (including zero resistance).

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[5]Davis, L. C. and J. R. Reitz, "Solution to Potential Problems near a Conducting Semi-Infinite Sheet or Conducting Disk", Am. J. Phys. **39**, p. 1255 (1971).

Equation of Motion

We have a 3D dipole with dipole moment $\boldsymbol{\mu} = \mu\,\hat{\mathbf{z}}$. It moves on the axis of a circular loop of radius *a*, resistance *R*, inductance *L*, with inductive time constant *L/R*. It moves downward under the influence of gravity. We constrain the motion to be along the *z*-axis, and the magnetic dipole moment to be parallel to that axis. The equation of motion is

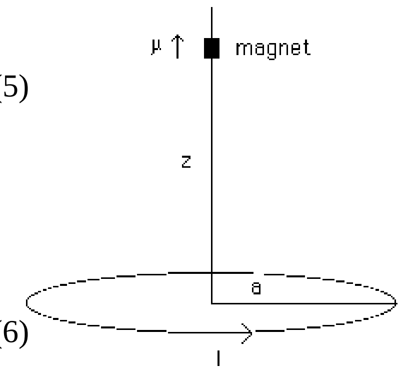
$$m\frac{d^2z}{dt^2} = -mg + \mu\frac{dB_z}{dz} \tag{3}$$

where B_Z is the field due the current *I* in the ring (taken to be positive in the direction show on the sketch). The expression for B_Z is

$$B_z = \frac{\mu_o I a^2}{2(a^2 + z^2)^{3/2}} \tag{4}$$

so that equation (3) is

$$m\frac{d^2z}{dt^2} = -mg - \frac{3\mu\mu_o I a^2}{2}\frac{z}{(a^2+z^2)^{5/2}} \tag{5}$$



An Equation for I from Faraday's Law

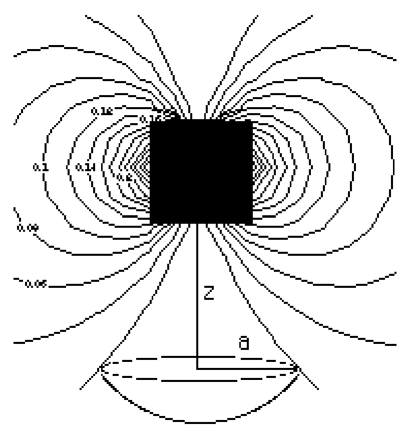
Faraday's Law is

$$\oint \mathbf{E}\cdot\mathbf{dl} = -\frac{d}{dt}\int\left[\mathbf{B}_{dipole} + \mathbf{B}_{ring}\right]\cdot\mathbf{dA} = -\frac{d}{dt}\int\mathbf{B}_{dipole}\cdot\mathbf{dA} - L\frac{dI}{dt} \tag{6}$$

where L is the inductance of the ring. If $\mathbf{E} = \rho\,\mathbf{J}$, where ρ is the resistivity of the ring and $\mathbf{J}$ is the current density, then $\oint\mathbf{E}\cdot\mathbf{dl} = \oint\rho\,\mathbf{J}\cdot\mathbf{dl} = I\oint\rho\,dl/A = IR$, with $R = \oint\rho\,dl/A$ where *R* is the resistance of the ring. So we have

$$IR = -L\frac{dI}{dt} - \frac{d}{dt}\int\mathbf{B}_{dipole}\cdot\mathbf{dA} \tag{7}$$

We now need to determine the magnetic flux through the ring due to the dipole field. To do this we calculate the flux through a spherical cap of radius $\sqrt{a^2+z^2}$ with an opening angle given θ given by $\sin\theta = a/\sqrt{a^2+z^2}$ (this is the same as the flux through the ring because $\mathbf{B} = 0$). The flux through a spherical cap only involves the radial component of the dipole field, given by



$$B_r = \frac{\mu_o}{2\pi}\frac{\mu\cos\theta}{r^3}$$

$$\int\mathbf{B}_{dipole}\cdot\mathbf{dA} = \int\frac{\mu_o}{2\pi}\frac{\mu\cos\theta}{r^3}2\pi r^2\sin\theta\,d\theta$$

$$\int\mathbf{B}_{dipole}\cdot\mathbf{dA} = -\frac{\mu_o\mu}{r}\int\cos\theta\,d\cos\theta = \frac{\mu_o\mu}{2}\frac{a^2}{\left(a^2+z^2\right)^{3/2}}$$

Using this expression for the flux in (7), and assuming that $\mu = M_O$ is constant in time, yields

$$IR = -L\frac{dI}{dt} + \frac{3\mu_o a^2 M_o}{2}\frac{z}{\left(a^2+z^2\right)^{5/2}}\frac{dz}{dt} \tag{8}$$

Equations (5) and (8) are our coupled ordinary differential equations which determine the dynamics of the situation when the magnet is falling toward the ring under the influence of gravity.

Dimensionless Form of the Equation When The Motion Is Specified

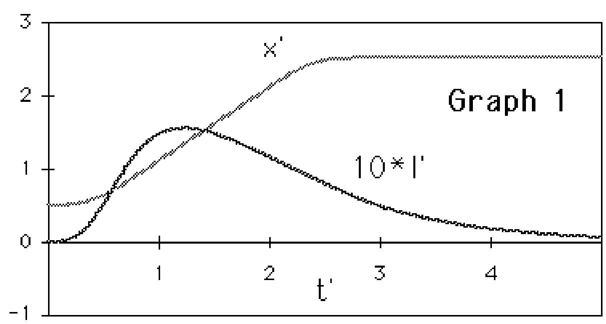
Before we consider the general case, in which the dynamics of the magnet are coupled to the current in the ring, let us first consider the simpler case where the motion of the magnet is specified. In the animations involving moving the magnet in and out of the coil, the magnet is at rest, then moves at constant speed, and is then at rest again. The current in the ring is determined by solving (8) given this as input. We assume that the magnet is moved at constant speed $\mathbf{v}_O$ along the x-axis in the time interval $0 < t < t_m$, and is at rest otherwise The position of the magnet at time t is $x(t)$. We measure all distances in terms of the distance a, and all times in terms of the time $\mathbf{a} / \mathbf{v}_o$. We define the dimensionless quantities

$$x' = \frac{x}{a} \qquad t' = \frac{t}{\mathbf{a}/\mathbf{v}_o} \qquad \lambda = \frac{L}{\mu_o a} \qquad I' = \frac{I}{I_o}, \quad where\ I_o = \frac{\mu_o M_o}{a\ L} \tag{9}$$

In terms of these variables, our equation (8) is

$$\boxed{\frac{dI'}{dt'} = -KI' + \frac{dG(x'(t'))}{dt'} \quad where \quad K = \frac{R}{L}\frac{a}{\mathbf{v}_o} \quad and \quad G(x') = -\frac{1}{2}\frac{1}{(1+x'^2)^{3/2}}} \tag{10}$$

Given the position of the magnet as a function of time, and the value of the dimensionless parameter K, we can solve equation (10) for the dimensionless current in the coil as a function of time. Once we have chosen our dimensionless parameter *K, and* the initial conditions, and solved our differential equation for $I'(t')$, how much freedom do we have in choosing the absolute value of the current? It can be shown that the overall shape of the magnetic field topology is determined once we make the one remaining choice of the dimensionless constant λ (eq. (9)), which up to this point we have not chosen (we have only picked values of K and the position of the magnet as a function of time to solve our dimensionless equation).



Once that choice is made, we have no additional freedom to affect the field topology. In all of our animations, we have chosen λ to be 2.

As an example of a numerical solution to equation (10), we show a solution in Graph 1 for the case where $K = 1$, and we have moved the magnet from $x' = 0.5$ to $x' = 2.5$. Figure 2, which is one frame of an animation of this process, shows the field configuration at an instant of time before the magnet comes to rest. The field lines have a hard time "getting through" the coil, since the sense of the current in the coil is such as to try to keep the number of field lines threading the coil from decreasing. Thus the field lines get "hung up" in the coil as they try to move through it. The intuitive sense that one gets in watching this animation is that the agent moving the magnet must do work to pull the field lines "through" the coil. This is a difficult point to get across in any other way. Figure 3 shows a similar case, except that we have taken $K = 0.3$ in this case (lower coil resistance, cf. equation (10)). Whereas Figure 2 shows many field lines but only in a single plane, Figure 3 shows one field line repeated many times in azimuth, to give a feel for its 3D character.

Dimensionless Form of the Falling Magnet Equations

We now put our more general coupled equations (5) and (8) into dimensionless form. We measure all distances in terms of the distance a, and all times in terms of the time $\sqrt{a/g}$ and define

$$z' = \frac{z}{a} \qquad t' = \frac{t}{\sqrt{a/g}} \qquad \alpha = \frac{R}{L}\sqrt{\frac{a}{g}} \qquad \beta = \frac{(\mu_o M_o)^2}{Lmga^3} \qquad I' = \frac{I}{I_o}, \quad where\ I_o = \frac{mga^2}{\mu_o M_o} \tag{11}$$

The time $\sqrt{a/g}$ is roughly the time it would take the magnet to fall under the influence of gravity through a distance a starting from rest. The current I_O is roughly the current in the ring that is required to produce a force sufficient to offset gravity when the magnet is a distance a above the ring. In terms of these variables, our equations (5) and (8) are

$$\frac{d^2 z'}{dt'^2} = -1 - F(z')I' \qquad\qquad \frac{dI'}{dt'} = -\alpha\, I' + \beta\, F(z')\frac{dz'}{dt'} \tag{12}$$

with

$$F(z') = \frac{3}{2}\frac{z'}{(1+z'^2)^{5/2}} \tag{13}$$

If we define the speed $v' = dz'/dt'$, then we can write three coupled first-order ordinary differential equations for the triplet (z', v', I'), using (12) and (13).

$$\boxed{\frac{dz'}{dt'} = v' \qquad \frac{dv'}{dt'} = -1 - F(z')I' \qquad \frac{dI'}{dt'} = -\alpha\, I' + \beta\, F(z')v'} \tag{14}$$

Given initial conditions for (z', v', I') and values of the parameters (α, β) (cf. eq. (11)) we use (14) to calculate the derivatives. For a given time step $\Delta t'$ we can then calculate new values of (z', v', I'). As before, the overall shape of the magnetic field topology is totally determined once we make the one remaining choice of the dimensionless constant λ, which up to this point we have not chosen (we have only to pick values of α and β to solve our dimensionless equation). Once that choice is made, we have no additional freedom to affect the field topology. In all of our animations, we have chosen λ to be 2.

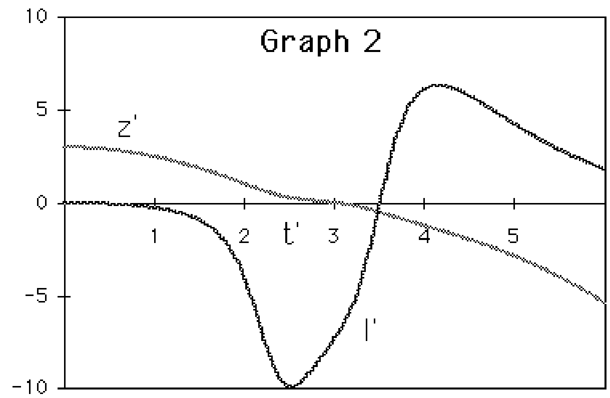
Conservation of Energy

If we multiply (5) by $v = \frac{dz}{dt}$ and (8) by I, after some algebra, we find that

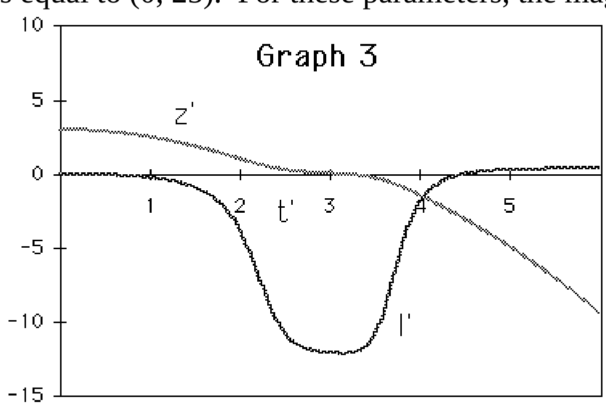
$$\frac{d}{dt}\left[\tfrac{1}{2}mv^2 + mgz + \tfrac{1}{2}LI^2\right] = -I^2 R \tag{15}$$

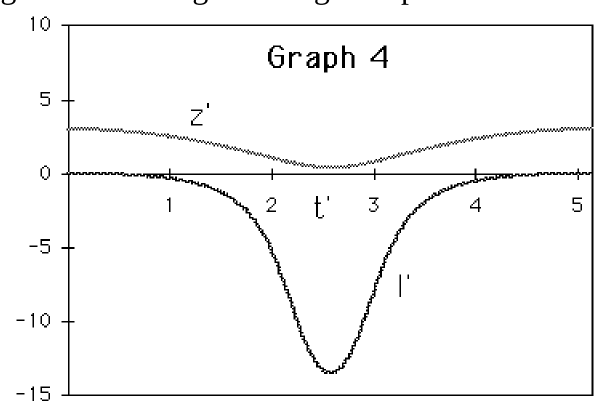
which expresses conservation of energy for the falling magnet plus the magnetic field of the ring. If $R = 0$, there is no dissipation of energy. For some values of the parameters, the magnet will levitate above the ring, as shown in one of our animations. For other values, the magnet can fall through the ring, as shown in another animation. But in no case can the total magnet flux through the ring change.

We show some examples of numerical solutions to equations (14). The first of these (shown in Graph 2 to the right) is for (α, β) values equal to (1, 35), with the magnet starting from rest at $z' = 3$, and the current in the ring initially zero. As the magnet falls, the current in the ring builds up in a sense so as to keep the flux through the ring from increasing. After the magnet falls through the ring, the current reverses direction so as to keep the flux through the ring from decreasing. When the magnet is above the ring, the sense of the current is to slow the magnet, as is also the case when the magnet is below the ring. The current is not zero just as the magnet crosses through the ring because of the inductance of the ring. Figure 4 shows many field lines in a plane after the magnet has fallen below the ring. The shape of the field lines implies that the field is transmitting an upward force on the magnet (tension parallel to the field). Figure 5 is a frame at the same time as Figure 4, except instead of showing many field lines in a single plane we show one field line repeated many times in azimuth.



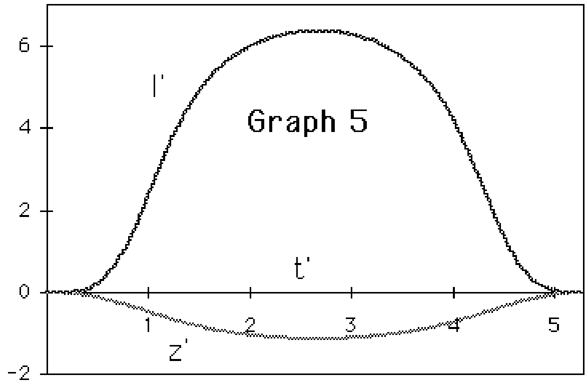
In Graph 3 below, we show a solution for the case where the resistance of the ring is zero, i.e. (α, β) values equal to (0, 25). For these parameters, the magnet falls through the ring. Graph 4





again shows a solution for the case where the resistance of the ring is zero, (α, β) values equal to (0, 35), but in this case the magnet levitates above the ring with a period of 5.14 dimensionless time units.

We can also find solutions where the motion of the magnet is periodic and the magnet is suspended below a ring with zero resistance. Graph 5 on the next page shows a solution for (α, β) values equal to (0, 18), where now we start the magnet out at $z' = 0$ with zero current in the ring. The motion of the magnet is periodic with a period of 5.28 dimensionless time units. Figure 6 shows one frame of an



animation of this motion when the magnet is at its lowest point, suspended below the ring by the tension in the field lines.

Examples Of Animations: II. Dipole Radiation

If $\mathbf{m}(t)$ is the time dependent magnetic dipole moment, the equation for the magnetic field for magnetic dipole radiation (including the quasistatic, induction, and radiations terms) is

$$\mathbf{B}(\mathbf{r},t) = \frac{\mu_o}{4\pi}\left\{ \underbrace{\frac{1}{r^3}\left[3\hat{\mathbf{n}}(\mathbf{m}\cdot\hat{\mathbf{n}})-\mathbf{m}\right]}_{\textbf{quasi-static}} + \underbrace{\frac{1}{cr^2}\left[3\hat{\mathbf{n}}(\dot{\mathbf{m}}\cdot\hat{\mathbf{n}})-\dot{\mathbf{m}}\right]}_{\textbf{induction}} + \underbrace{\frac{1}{rc^2}(\ddot{\mathbf{m}}\times\hat{\mathbf{n}})\times\hat{\mathbf{n}}}_{\textbf{radiation}} \right\} \tag{16}$$

where the expression on the right is evaluated at the retarded time $t' = t - r/c$. We have animated this field using the approach discussed above, for the case when the dipole moment is a constant plus a sinousoidally varying function, e.g., $\mathbf{m}(t) = \hat{\mathbf{z}}\, M_o\left[1 + \varepsilon\cos(\frac{2\pi t}{T})\right]$, with $\varepsilon = 0.1$. Figure 7 shows an frame from the animation of the motion of the field line whose equatorial crossing point *for the average dipole moment* $\hat{\mathbf{z}}\, M_o$ is at cT.

This animation is carried out using the approach discussed above, which means that the local velocity of the field line is given by $\mathbf{v} = \mathbf{E}\times\mathbf{B}/B^2$. Unlike the magnetoquasistatic cases discussed above, in some regions the resulting motion of the field lines is non-physical, in that this velocity exceeds the speed of light. However, even though the motion is non-physical, it is useful pedagogically, in that the direction of motion of a field line indicates the direction of energy flow (but not the magnitude of the energy flow, of course), since $\mathbf{v} = \mathbf{E}\times\mathbf{B}/B^2$ is parallel to the Poynting vector.

In particular, in the induction and quasi-static zones, there are regions where the energy flow is outward over part of the period and inward over other parts, corresponding to energy flowing out and being reversibly recovered as the energy in the dipole field is alternately increased and decreased. However, when we move into the radiation zone, the energy flow is always outward, corresponding to the irreversible energy loss of radiation, as is the motion of our field lines in that region. The animation gives an intuitive sense for the fact that there is such a transition, and for where this transition takes place.

Discussion

Let us discuss the objectives of our approach. One of the primary aims of animation is to engender a sense of wonder in the student. The 3D visualizations that we have created and plan to create are visually compelling. They engage the student's imagination because they show the world in a photo-realistic way, including representations of phenomena which heretofore could only be seen in the mind's eye. In large lecture courses in the freshman year one of the purposes is to inspire students to invest the time to pursue quantitative mastery of the subject outside of lecture. Our animations are successful in large degree as long as they arouse interest and excitement by engendering a sense of wonder.

Beyond engendering a basic sense of wonder, what is the central student learning need that we are trying to meet? It is this. Students need an enormous amount of help in understanding the nature of fields. The central learning objective of introductory courses in electromagnetism is to help students understand how fields are generated, how they mediate the interaction of material objects, and how they propagate. Our contention is that in the standard pedagogy this learning objective is not well fulfilled.

Our approach to help remedy this deficiency is to give the fields a more prominent role in the pedagogy, by literally making them more visible. They are thereby made more understandable dynamically, based on students' pre-existing models of the behavior of e.g. strings and rubber bands. The animations continually remind the student that it is the field that mediates interactions between material objects--that the field has as much "reality" as the objects themselves. Ultimately, animations allow students to understand intuitively what is happening dynamically simply by looking at the shape of the field lines, once the eye and the mind are trained to this purpose. It is this intuition that we seek to develop.

Figures

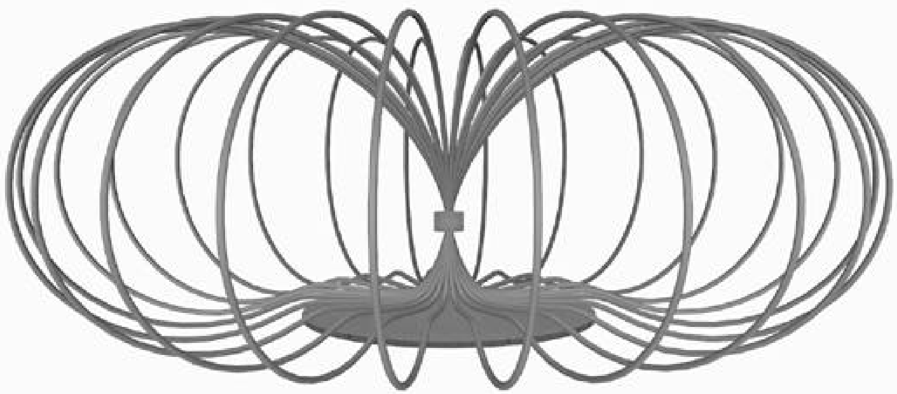


Figure 1

Figure 1: A magnet levitates above a conducting disk with zero resistance. We show a field line rotated many times in azimuth.

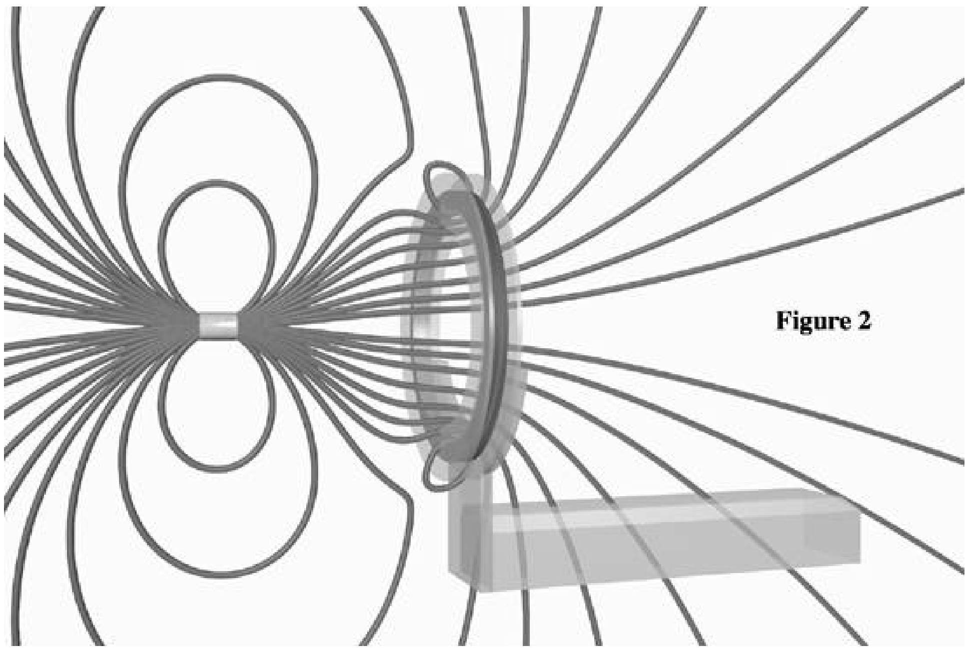


Figure 2: A magnet is pulled away from a conducting coil of wire (cf. Graph 1 and eq. 10). We show many field lines in a single plane. The induced current in the coil tries to keep the number of field lines threading the coil from decreasing, so that the field lines appear to get "hung up" in trying to move through the coil.

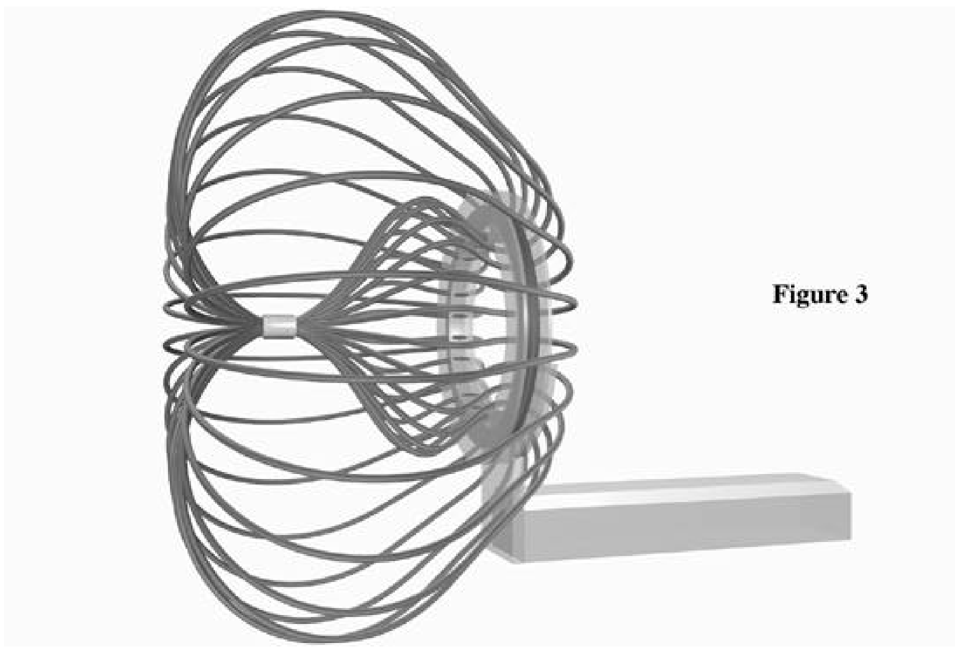
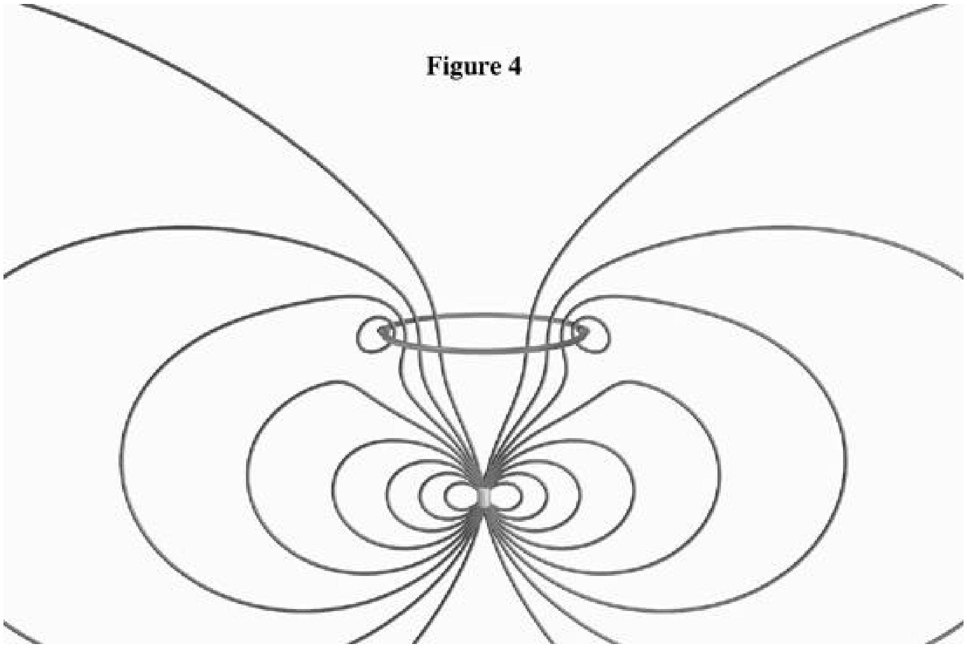


Figure 3: The same as Figure 2, except we have decreased the resistance of the coil by a factor of 3, and we show now only one field line, repeated many times in azimuth about the axis of the coil. This is a better representation of the 3D nature of the field.



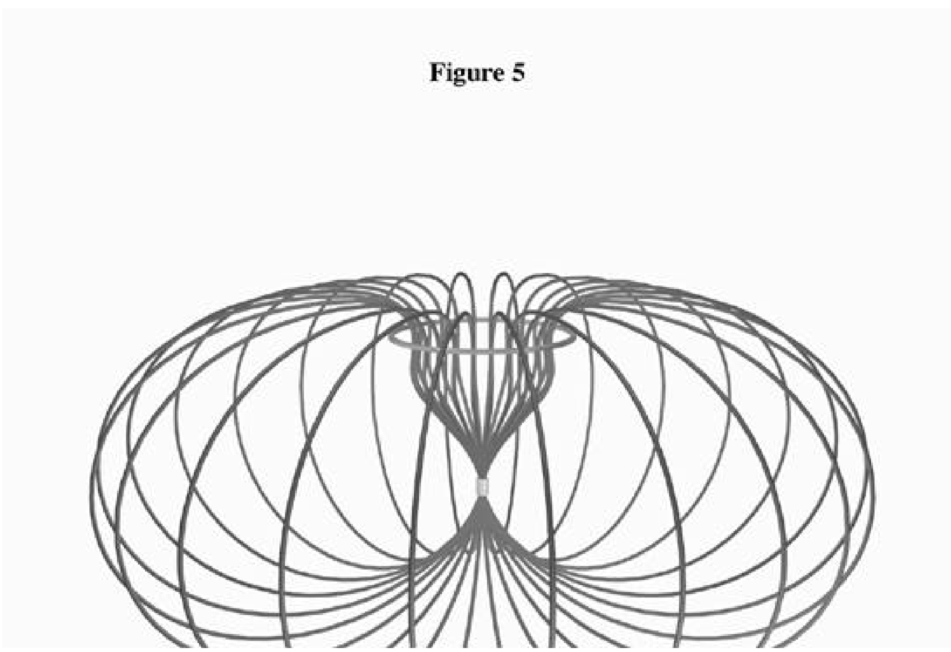


Figure 4 and 5: A magnet falls through a conducting ring with finite resistance (cf. Graph 2 and eq. 14). We show the field lines when the magnet is below the ring, in two views, one with many field lines shown only in a plane, and one with only one field line shown many times in azimuth, to give a feel for the 3D nature of the fields.

Figure 6

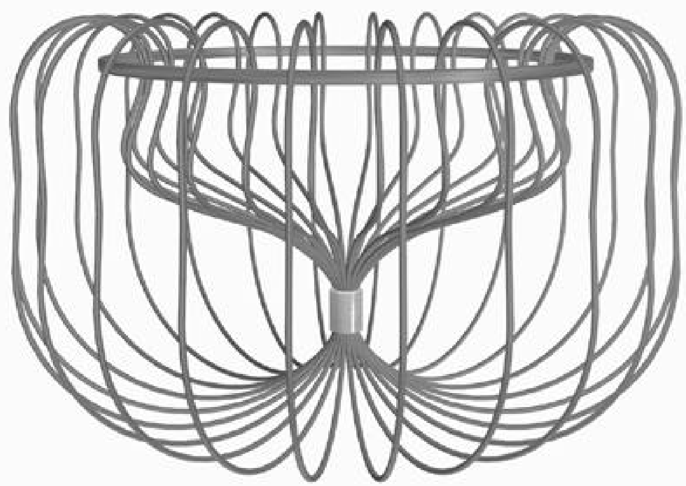


Figure 6: A magnet suspended below a conducting ring with zero resistance (see Graph 5 and eq. 14). The magnet initially was at rest in the center of the ring, with zero current in the ring. As the magnet falls under gravity, the induced current in the ring tries to keep the flux through the ring from changing, producing the field configuration shown.

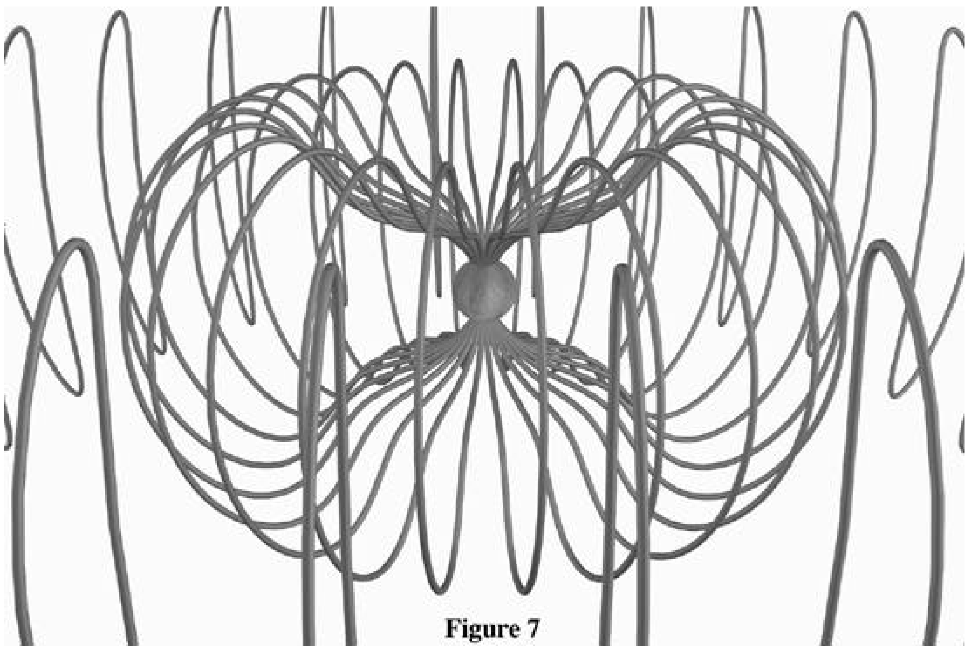


Figure 7: A field line during magnetic dipole radiation for a dipole whose dipole moment is varying sinusiodally ±10% in amplitude with a period of T. The field line shown has an equatorial crossing distance of cT *for the average dipole moment*. We show the induction and radiation zones.