Dynamical catalysis

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If a biomolecular chemical reaction is mass transport limited, the reaction rate can be accelerated by embedding one of the reaction centers in a molecular framework. This framework acts as a lower dimensionality surface along which the other reaction partner can travel. The rate enhancement can be identified with an increase in entropy of activation. A model for this effect is constructed and applied to a reaction center embedded in three molecular framework geometries: a linear chain, a disk, and a polar reactive patch on a sphere. The new result for the reactive patch on a sphere is relevant for enzymatic systems where substrates must find their way to a small reactive region. © *1998 American Institute of Physics.* [S0021-9606(98)01003-4]

I. INTRODUCTION

In the conventional picture of bimolecular reaction kinetics, molecular species A and B reversibly form a reaction complex (A/B) that passes through a transition state or intermediate to form product. The reaction mechanism is

$$A+B \underset{k_{-1}}{\overset{k_1 \quad k_2}{\rightleftharpoons}} (AB) \rightarrow \text{product}. \tag{1.1}$$
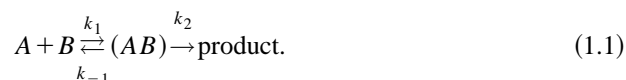
After making the steady state approximation for the reaction complex intermediate, $d[(AB)]/dt=0$, the effective bimolecular rate coefficient k_{eff} is found to be

$$k_{\text{eff}} = \frac{k_1 k_2}{k_{-1}+k_2}. \tag{1.2}$$

Ordinarily the rate-limiting step is the second step in the reaction mechanism Eq. (1.1) because the chemistry of making and breaking bonds is usually slow compared to the dynamics of reactants approaching one another. In this limit $k_{-1} \gg k_2$ and

$$k_{\text{eff}} \rightarrow \frac{k_1}{k_{-1}} k_2. \tag{1.3}$$

The rate coefficient for the chemical step is assumed to have the form:

$$k_2 = k_2^0 \exp[-\Delta F^t/k_b T], \tag{1.4}$$

where k_2^0 is equal to k_{bT}/h according to transition state theory, k_b is Boltzmann's constant, T is the absolute temperature, and ΔF^t is the free energy difference between the reaction complex (AB) and the transition state.

Catalytic activity occurs through a lowering of the transition state free energy relative to the reaction complex intermediate either by a change in the transition state enthalpy ΔH^t or entropy ΔS^t difference.

In the opposing limit, when mass transport is rate limiting, $k_2 \gg k_{-1}$ and

$$k_{\text{eff}} \rightarrow k_1. \tag{1.5}$$

Evidently, if a means is found to modify the rate of approach, $k_1 \rightarrow k_1^+$, then there effectively will be catalytic activity in this mass transport rate determining limit.

In this paper we examine a general mechanism that can increase the rate of encounter and separation of A and B,

$$k_1^+ = k_1(1+g), \quad k_{-1}^+ = k_{-1}(1+g), \tag{1.6}$$

where the quantity g is the increment to the forward and backward rates of encounter. The effective rate coefficient becomes

$$k_{\text{eff}} = \frac{k_1^+ k_2}{k_{-1}^+ + k_2}. \tag{1.7}$$

Formally, the enhancement to the rate coefficient can be expressed as an entropy of activation that we denote as

$$\Delta S_d = k_B \ln(1+g). \tag{1.8}$$

Note that the pre-equilibrium constant

$$K = \frac{k_1}{k_{-1}} = \frac{k_1^+}{k_{-1}^+} \tag{1.9}$$

is unaffected by the enhancement in the forward and reverse rates. This rate enhancement has many of the attributes of a catalytic effect so that we term it a *dynamical catalytic effect*. This dynamical effect is only present when mass transport is the rate-limiting step and, in principle, it involves no modification of the transition state.

The mechanism for enhancing the rate of encounter arises when a reactive partner is embedded in a polymer, a two-dimensional framework, or on the surface of a sphere. The passive framework acts as a lower dimensionality network that more efficiently transports the reacting substrate to the reaction center.

Adam and Dlbruck[1] were the first to point out that there could be a rate enhancement over free diffusion if the mass transport proceeds in successive stages of lower dimensionality. Considerable work has been done on the case of a reactive site embedded in a linear polymer[2–6] with the work of von Hippel and co-workers on the translocation of proteins on DNA indicating the biological relevance of this problem.

The case of a reactive site on a spherical macromolecule is perhaps the one of greatest interest because of the relevance to reactions at globular enzymes. For many years it
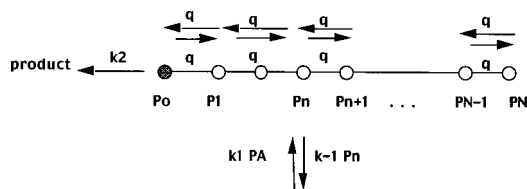
FIG. 1. Kinetic mechanism for one reactive site $B = P_0$ at end of linear polymer chain of N passive sites. The shuttling rate between adjacent sites is q and each site is in equilibrium with adsorption of A from the surrounding solution.

has been recognized that there is a considerable difference between the diffusion-controlled rate to a spherical target molecule that is equally reactive over its entire surface and a spherical target molecule that has only a reactive patch on its surface. Alberty and Hammes[7] first pointed out the significance of this difference for the diffusion rate to an enzyme molecule and to its active site. The kinetics of diffusion-controlled reactions between chemically asymmetric molecules when one or both of the translating and rotating reacting partners possesses an axially symmetric reactive region on a sphere has been investigated by Solc and Stockmayer,[8] Schmitz and Schurr,[9,10] and Temkin and Yakobson.[11]

When the added complications of diffusion to a buried active site[12] are included, it becomes clear that there must be some means beyond free diffusion that permits a small substrate to find the reacting center of an enzyme. The possibility of free diffusion to the surface of the enzyme followed by sliding on the lower dimensionality spherical surface to the active site has been suggested[13] and the role of intermolecular interactions in such a mechanism has been investigated.[14]

II. THE MODEL

The means for realizing this dynamic rate enhancement is to place one of the reacting partners, say B, in a molecular network that accelerates the frequency of encounters. The enhancement will be accomplished if (1) the mobile A species finds the molecular network faster (because it is larger) than the free reacting B center, and (2) the mobile species rapidly diffuse along the molecular network to the reacting center. In sum, the molecular network serves as a lower dimensional pathway for reaction partners to find each other.

A. Reactive center at one end of a linear polymer

In this model the reactive center B is located at one end of a polymer chain of N homogeneous units. Species A binds reversibly from solution to each polymer site ($n = 1,...,N$) and, if bound, A can shuttle to adjoining sites with rate q. If A reaches the end site P_0 where B is located it may react irreversibly to form product. For simplicity, we assume that the on and off rates for A onto polymer sites is the same as the rate of association and deassociation of A and B to the encounter complex (AB) when A and B are free in solution; we also assume that the k_2 for A reacting with B when B is bound in the polymer is the same as when B is free in solution. See Fig. 1.

The kinetic equations for this reaction mechanism are

$$\frac{dP_0}{dt} = k_1 A P + q P_1 - (q + k_{-1} + k_2) P_0, \quad (2.1a)$$

$$\frac{dP_n}{dt} = k_1 A P + q(P_{n-1} + P_{n+1}) - (2q + k_{-1}) P_n,$$

$$n = 1,...,N-1, \quad (2.1b)$$

$$\frac{dP_N}{dt} = k_1 A P + q P_{N-1} - (q + k_{-1}) P_N. \quad (2.1c)$$

In Eq. (2.1) P_n is the concentration of sites at position on the polymer chain that have bound A, $k_1 A P$ is the flux of A from solution that binds to each site, $k_{-1} P_n$ is the dissociation rate of A from site n, and q denotes the shuttling rate between adjacent sites. The quantity P is the concentration of each free site n available for binding A and this is proportional to concentration of B since, by assumption, there is one B per chain located at position $n = 0$; thus $P_0 = B$. Equation (2.1c) can be transformed into the general form of Eq. (2.1b) by introducing the auxiliary quantity P_{N+1} and adopting the boundary condition

$$P_{N+1} = P_N. \quad (2.1d)$$

Rather than solve the set of finite difference equations, we pass to a continuous description of the concentration of A bound along the chain $c(x)$ assuming that the polymer sites are separated by equal distance δ. In addition we introduce the steady state approximation $dP_n/dt = 0$. Thus Eq. (2.1b) becomes

$$0 = k_1 A P + D c''(x) - k_{-1} c(x) \quad (2.2)$$

with $D = q\delta^2$ for $0 < x < L = \delta N$. The boundary condition at the end of the chain Eq. (2.1d) is simply

$$c'(L) = 0. \quad (2.3)$$

The boundary condition at the origin where the reactive center B is located, Eq. (2.1a) becomes

$$0 = k_1 A P + \frac{D}{\delta} c'(0) - (k_{-1} + k_2) c(0), \quad (2.4)$$

which states the steady state flux balance at B: Free A arriving from the solution plus bound A arriving from the polymer chain either reacts or dissociates back into solution.

The general solution to Eq. (2.2) which satisfies the boundary condition Eq. (2.3) is

$$c(x) = a \cosh[(x-L)\lambda] + k_1 A P / k_{-1}, \quad (2.5)$$

where $\lambda = \sqrt{k_{-1}/D}$. The constant ''$a$'' is determined from the boundary condition Eq. (2.3); one finds

$$a \cosh(\lambda L) = \frac{-k_2 k_1 A P / k_{-1}}{k_2 + k_{-1} + \frac{D}{\delta} \lambda \tanh(\lambda L)}. \quad (2.6)$$

The appearance of product is given by

$$\frac{d[\text{product}]}{dt} = k_2 c(0) = k_2 \left[a \cosh(\lambda L) + \frac{k_1 A P}{k_{-1}} \right]. \quad (2.7)$$
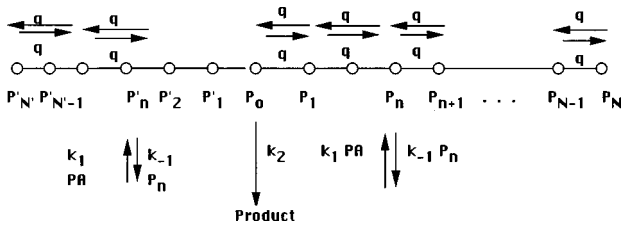
FIG. 2. Kinetic mechanism for one reactive site $B = P_0$ in a polymer chain with one side N passive units and the other side N' passive units. The shutting rate between adjacent sites is q and each site is in equilibrium with adsorption of A from the surrounding solution.
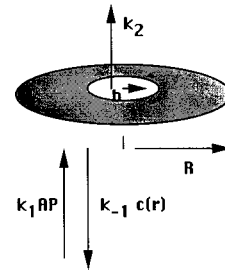

FIG. 3. Kinetic mechanism for reactive region $o < r < b$ in circular disk $b < r < R$. Diffusion occurs on the disk of the adsorbed species A.

This result should be compared with the kinetic equation:

$$\frac{d[\text{product}]}{dt} = k_{\text{eff}} A B. \tag{2.8}$$

Substitution of Eq. (2.6) into Eq. (2.7) and use of the fact that the concentration of P is proportional to B, leads to an expression for k_{eff} of the form of Eqs. (1.6) and (1.7) with

$$g = \frac{L}{\delta} \frac{\tanh(\lambda L)}{(\lambda L)}, \tag{2.9}$$

where $\lambda = \sqrt{k_{-1}/D}$.

In the limit of rapid diffusion along the chain, the maximum enhancement is exhibited. In this limit $\lambda \to 0$, $g \to (L/\delta) = N$, and thus $k_1^+ = k_1(1 + N)$. Diffusion along the chain is so rapid that each polymer site effectively presents a higher concentration of A to the reactive site located at the end of the chain.

In the opposite limit of slow diffusion, $\lambda \to \infty$, $g \to 0$, and $k_1^+ \to k_1$. As expected, the presence of the polymer chain makes no difference to the rate of reaction.

B. Reactive center internal to a linear polymer chain

In this model the reactive center B is located at position $n = 0$, which is attached to two polymer chains of length N_1 and N_2. See Fig. 2. In the continuous limit the appropriate steady state equations for each chain are

$$0 = k_1 A P + D c_i''(x) - k_{-1} c_i(x), \quad i = 1,2, \tag{2.10}$$

with the boundary condition at the chain ends $c_i'(L_i) = 0$, $i = 1,2$. At the origin, where the reactive site is located, the boundary condition is the balance between incoming and outgoing flux:

$$0 = k_1 A P + \frac{D}{\delta} c_1'(0) + \frac{D}{\delta} c_2'(0) - (k_{-1} + k_2) c_1(0) \tag{2.11}$$

with the auxiliary condition $c_1(0) = c_2(0)$ (Fig. 3).

The general solution for the concentration of A particles on each chain is of the following form:

$$c_i(x) = a_i \cosh[(x - L_i)\lambda] + k_1 A P / k_{-1}. \tag{2.12}$$

The constants a_1 and a_2 are determined from the boundary condition at the origin, Eq. (2.11) in a manner exactly similar to the procedure used in the previous section. One obtains a result for k_{eff} of the general form Eqs. (1.6) and (1.7) with

$$g = \frac{L_1}{\delta} \frac{\tanh(\lambda L_1)}{\lambda L_1} + \frac{L_2}{\delta} \frac{\tanh(\lambda L_2)}{\lambda L_2}. \tag{2.13}$$

Again the maximum enhancement is found for rapid diffusion along the chain. In this limit $\lambda \to 0$, $g \to (L_1 + L_2)/\delta$, and $k_1^+ = k_1(1 + N_1 + N_2)$.

C. Reactive site at the center of a disk

The dynamical catalytic effect will be exhibited when the reactive site is embedded in two- or three-dimensional structures which facilitate mass transport. As a preliminary example we consider a circular reactive region of radius b containing B embedded in a disk $b < r < R$ that contains particles P. The substrate A binds reversible to P (on both sides) and may diffuse to the central reactive region. See Fig. 3.

The diffusion equation governing the distribution of A at steady state on the disk is

$$0 = k_1 A P + \frac{D}{r} \frac{d}{dr} r \frac{dc(r)}{dr} - k_{-1} c(r), \quad b < r < R, \tag{2.14}$$

with a reflecting boundary condition at the outer edge $c'(R) = 0$.

At the inner boundary the total flux of arriving A particles is balanced by the flux of A particles that depart due to reaction or disassociation back to the solution:

$$0 = \pi b^2 k_1 A P + 2\pi b D c'(b) - (k_{-1} + k_2) c(b) \pi b^2. \tag{2.15}$$

For simplicity we have assumed that the concentration of A in the reactive region is uniform and equal to the value at the boundary between the reactive region and the disk.

The solution to the diffusion equation [Eq. (2.14)] that satisfies the reflecting boundary condition at $r = R$ is

$$c(r) = a \left[I_0(\lambda r) - \frac{I_0'(\lambda R)}{K_0'(\lambda R)} K_0(\lambda r) \right] + \frac{k_1 A P}{k_{-1}},$$

$$b < r < R, \tag{2.16}$$

where $K_0(z)$ and $I_0(z)$ are modified Bessel functions and, as before, $\lambda = \sqrt{k_{-1}/D}$. The constant ''$a$'' is determined by the condition at the reactive boundary $r = b$, Eq. (2.15) and
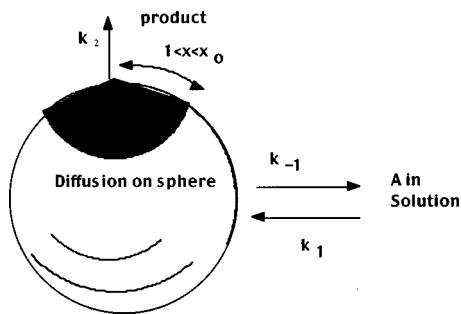
FIG. 4. Kinetic mechanism for the reactive region θ on the sphere. Diffusion of species A absorbed on the surface to the reactive region.

k_{eff} is identified from the reactive term $k_2 c(b)$. One finds k_{eff} of the form given in Eqs. (1.6) and (1.7) with

$$g = -\frac{2D\lambda}{bk_{-1}} \left[\frac{I_0'(\lambda b)K_0'(\lambda R) - I_0'(\lambda R)K_0'(\lambda b)}{I_0(\lambda b)K_0'(\lambda R) - I_0'(\lambda R)K_0(\lambda b)} \right]. \quad (2.17)$$

In the limit of rapid diffusion $\lambda \to 0$ and the modified Bessel functions approach

$$I_0(z) \to \left(1 - \frac{z^2}{4}\right), \quad K_0(z) \to -\ln(z). \quad (2.18)$$

Thus in this limit one finds $g \to [(R^2 - b^2)/b^2]$ and accordingly

$$k_1^+ = k_1 \left[1 + \frac{(R^2 - b^2)}{b^2} \right]. \quad (2.19)$$

Here again the maximum rate enhancement can be expressed as $k_1^+ = k_1[1 + N_{eff}]$, where N_{eff} is the effective number of passive molecular sites available to transport A to the reactive region containing B. The effective number N_{eff} is simply the ratio of the passive molecular area to the reactive area πb^2 since the sites are assumed to be uniformly distributed.

In the opposite limit of slow diffusion one may show $g \to 0$ and $k_1^+ \to k_1$, as expected.

III. REACTIVE SITE ON A SPHERE

The dynamical catalytic effect under consideration here is the rate enhancement due to sliding along the unreactive portion of the sphere to the chemically reactive region[13] compared to the situation where the unreactive portion of the sphere simply acts as a reflecting barrier.[7,8]

We treat the diffusion of particles A *to* the sphere and dissociation of particles A from the sphere as a kinetic equilibrium step and focus attention of the diffusion of particles A *on* the sphere surface to the reactive region occupied by particles B. See Fig. 4. The combined problem of diffusion *to* and *on* the sphere that has a chemically reactive region can be analyzed directly but requires numerical computation.

The steady state diffusion equation for the concentration of particles A on the surface of the sphere of radius R is

$$0 = k_1 AP + \frac{D}{R^2} \frac{1}{\sin\vartheta} \frac{d}{d\vartheta} \sin\vartheta \frac{dc(\vartheta)}{d\vartheta}$$

$$- k_{-1} c(\vartheta), \quad \vartheta_0 < \vartheta < \pi, \quad (3.1)$$

where the chemically reactive patch occupies the north polar region $0 < \vartheta < \vartheta_0$. In terms of the variable $x = \cos\vartheta$, Eq. (3.1) is

$$0 = k_1 AP + \frac{D}{R^2} \frac{d}{dx} (1 - x^2) \frac{dc(x)}{dx} - k_{-1} c(x),$$

$$x_0 < k < -1. \quad (3.2)$$

The boundary condition at the south polar region $x = -1$ must be

$$\lim_{x \to -1} (1 - x^2) c'(x) \to 0. \quad (3.3)$$

Obtaining a solution for this case is a bit more complicated than the cases previously considered because diffusion is taking place on a curved surface. We find a solution in terms of solutions to the Legendre equation[15]

$$(1 - x^2) w_\nu''(x) - 2x w_\nu'(x) + \nu(\nu+1) w_\nu(x) = 0. \quad (3.4)$$

This equation has two solutions on the cut $-1 < x < +1$ that are denoted $P_\nu(x)$ and $Q_\nu(x)$, Legendre functions of degree ν and order zero of the first and second kind. A solution to Eq. (3.2) is obtained as a linear combination of these Legendre functions:

$$c(x) = a_1(\nu) P_\nu(x) + a_2(\nu) Q_\nu(x) + \frac{k_1 AP}{k_{-1}}, \quad (3.5)$$

provided that

$$\nu(\nu+1) = -\frac{k_{-1} R^2}{D}. \quad (3.6)$$

The particular linear combination must be taken to satisfy the boundary condition Eq. (3.3). Note that ν can be complex. For the region where ν is real

$$0 < \frac{k_{-1} R^2}{D} < \frac{1}{4}, \quad (3.7)$$

we select the positive square root for n:

$$\nu = -\frac{1}{2} + \frac{1}{2} \sqrt{1 - \frac{4k_{-1} R^2}{D}}. \quad (3.8)$$

Since $P_\nu(x)$ and $Q_\nu(x)$ are singular $[x]$ as $x \to -1$:

$$P_\nu(x) \to \frac{\sin(\nu\pi)}{\pi} \left[\ln\left(\frac{1+x}{2}\right) + \pi \cot(\nu\pi) + \text{const} \right],$$

$$Q_\nu(x) \to \frac{\cos(\nu\pi)}{2} \left[\ln\left(\frac{1+x}{2}\right) - \pi \tan(\nu\pi) + \text{const} \right], \quad (3.9)$$

one must choose a particular linear combination that cancels the singularity in order to satisfy Eq. (3.3).[16] We choose the particular combination

$$c(x) = a \left[\frac{\sin(\nu\pi)}{\nu\pi} Q_\nu(x) - \frac{\cos(\nu\pi)}{2\nu} P_\nu(x) \right] + \frac{k_1 AP}{k_{-1}}, \quad (3.10)$$

which one can verify satisfies Eq. (3.3) for all $\nu \neq 0$.

We must be concerned with $\nu = 0$. The limit $\nu \to 0$ arises when $k_{-1} \to 0$; in this limit,

$$c(x) \to a\left[Q_0(x) - \frac{1}{2\nu}P_0(x) - \frac{1}{2}\left(\frac{dP_\nu(x)}{d\nu}\right)_{\nu=0}\right] + \frac{k_1}{k_{-1}}. \tag{3.11}$$

Since $P_0(x) = 1$ and[15]

$$\left(\frac{dP_\nu(x)}{d\nu}\right)_{\nu=0} = \ln\left(\frac{1+x}{2}\right) \tag{3.12}$$

it is necessary to choose $a = -(2k_1APR^2/D)$ to cancel the singularity at $n = k_{-1} = 0$. Thus, in the limit $\nu \to 0$, the general solution Eq. (3.10) approaches

$$c(x) = \frac{k_1APR^2}{D}\left[\ln\left(\frac{1+x}{2}\right) - 2Q_0(x)\right] \tag{3.13}$$

with $Q_0(x) = \frac{1}{2}\ln[(1+x)/(1-x)]$ and $c(-1) = 0$. One may verify that Eq. (3.13) is correct by direct integration of the original differential equation for $c(x)$, Eq. (3.2), with k_{-1} set equal to zero.

In the general case when k_{-1} is not equal to zero, the constant a in Eq. (3.10) must be determined from the boundary condition at $x = x_0$. As usual, the condition at the reactive boundary is found by equating the flux of A into the region $0 < x < x_0$ with the flux leaving by reaction and dissociation:

$$0 = k_1APS(x_0) - 2\pi D(1-x_0^2)c'(x_0)$$
$$- (k_{-1} + k_2)c(x_0)S(x_0), \tag{3.14}$$

where $S(x_0)$ is the area of the reactive region $S(x_0) = 2\pi R^2(1-x_0)$ and again we have assumed that the concentration of A in the reactive region is uniform and equal to the value at the boundary.

Once the constant "a" is determined k_{eff} is identified from the reactive flux $k_2 c(x_0)$. The effective rate coefficient is of the form of Eqs. (1.6) and (1.7) and the quantity g is

$$g_1 = \frac{(1+x_0)D}{k_{-1}R^2}\left[\frac{(2/\pi)\tan(\nu\pi)Q'_\nu(x_0) - P'_\nu(x_0)}{(2/\pi)\tan(\nu\pi)Q_\nu(x_0) - P_\nu(x_0)}\right], \tag{3.15}$$

with the complex quantity ν given by Eq. (3.8) in the regime $0 < (k_{-1}R^2/D) < (1/4)$. Alternatively, using Eq. (3.6) one finds

$$g_1 = -\frac{(1+x_0)}{\nu(\nu+1)}\left[\frac{(2/\pi)\tan(\nu\pi)Q'_\nu(x_0) - P'_\nu(x_0)}{(2/\pi)\tan(\nu\pi)Q_\nu(x_0) - P_\nu(x_0)}\right]. \tag{3.16}$$

In the limit $k_{-1} \to 0$ ($\nu \to 0$), the quantity g approaches

$$g_1 \to (1+x_0)\left[2Q'_0(x_0) - \frac{1}{(1+x_0)}\right], \tag{3.17}$$

where use has been made of Eq. (3.11). From the definition of $Q_0(x)$, one finds

$$g_1 \to \frac{1+x_0}{1-x_0}, \quad k_1^+ \to k_1\left[1 + \frac{1+x_0}{1-x_0}\right]. \tag{3.18}$$

Again, the maximum rate enhancement can be expressed in terms of N_{eff}, the effective number of passive molecular sites available to transport A to the reactive region occupied by B:

$$N_{\text{eff}} = \frac{4\pi R^2 - S(x_0)}{S(x_0)} = \frac{1+x_0}{1-x_0}. \tag{3.19}$$

We now seek a solution for the concentration $c(x)$ when $(k_{-1}R^2/D) > (1/4)$ and the value of the degree ν of the Legendre functions becomes complex. We display the imaginary part of this parameter explicitly:

$$\nu = \tfrac{1}{2} \pm i\sqrt{d},$$

where

$$\frac{k_{-1}R^2}{D} = \frac{1}{4} + d. \tag{3.20}$$

In this regime we choose for $c(x)$ a linear combination of the Legendre functions of the second kind which is real and satisfies the boundary condition Eq. (3.3) that $c(x)$ is not singular at the south pole:

$$c(x) = a\{Q_{-1/2+i\sqrt{d}}(x_0) + Q_{-1/2-i\sqrt{d}}(x_0)\} + \frac{k_1AP}{K_{-1}}. \tag{3.21}$$

The constant "a" and k_{eff} is determined exactly as before. The quantity g_2, in this region, is found to be

$$g_2 = \frac{(1+x_0)D}{k_{-1}R^2}\left[\frac{Q'_{-1/2+i\sqrt{d}}(x_0) + Q'_{-1/2-i\sqrt{d}}(x_0)}{Q_{-1/2+i\sqrt{d}}(x_0) + Q_{-1/2-i\sqrt{d}}(x_0)}\right]. \tag{3.22}$$

Accordingly we find for the rate enhancement factor:

$$g = \begin{cases} g_1 = \text{Eq. (3.15)} & 0 < (k_{-1}R^2/D) < (1/4) \\ g_2 = \text{Eq. (3.22)} & (k_{-1}R^2/D) > (1/4) \end{cases}. \tag{3.23}$$
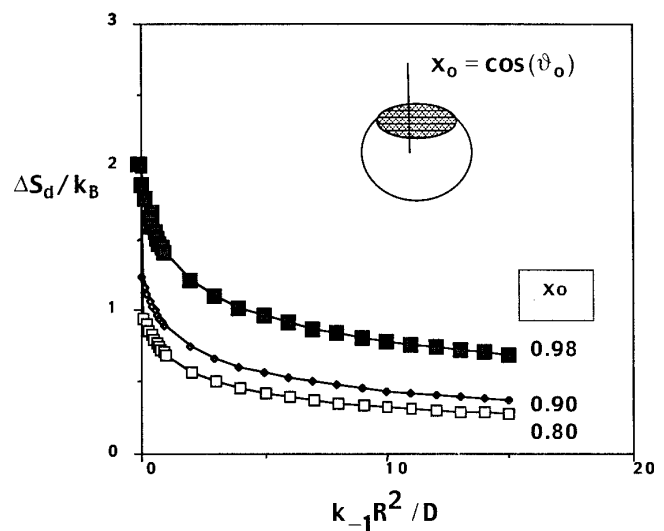

FIG. 5. Kinetic rate enhancement expressed as change in entropy of activation ΔS_d versus the kinetic parameter $k_{-1}R^2/D$ for the case of a sphere with reactive region $S(x_0) = \pi R^2(1-x_0)$ for $x_0 = \cos\vartheta_0 = 0.98, 0.9, 0.8$.
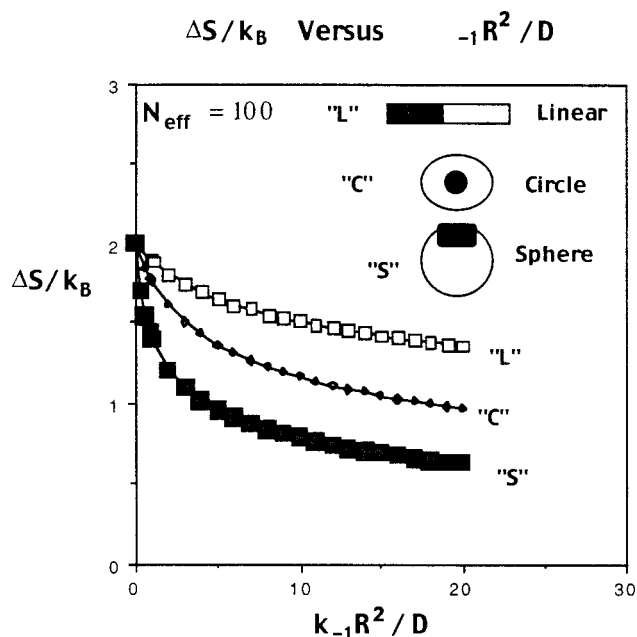
FIG. 6. Kinetic rate enhancement expressed as change in entropy of activation ΔS_d versus the kinetic parameter $k_{-1}R^2/D$ for the case of a reactive region in a linear polymer "L," in the center of a circular disk "C," and at the north polar region of a sphere "S." In each case the ratio of reactive sites to total sites is $N_{\text{eff}}=100$.

IV. DISCUSSION OF RESULTS

The contribution of the present work is first, the development of a model that permits one to assess on a common basis the phenomena of "dynamical catalysis" and second, the application to different molecular framework geometries. The two most interesting geometries that we have considered are the linear polymer chain (where our results agree with prior work of von Hippel and Berg[2–4]) and the new result for a sphere with the reactive patch; this latter case is of considerable interest in biophysical applications where a substrate molecule is seeking a reactive site on a larger macromolecule, usually a protein.

In Fig. 5 we present results for the sphere of radius R with a reactive patch of variable size. The enhanced rate coefficient is plotted in terms of the increased entropy of activation ΔS_d versus the parameter $k_{-1}R^2/D$ that determines the balance between diffusion along the sphere to the reactive patch and dissociation. Three patch sizes are displayed, corresponding to $\cos \vartheta_0 = x_0$ equal to 0.98, 0.90, and 0.80; the intercept for each curve is N_{eff} as given by Eq. (3.19), 99, 19, and 9, respectively. Note that the three curves have a similar decay pattern and decay relatively slowly.

In Fig. 6 we compare results for three different geometries, the linear polymer Eq. (2.9), the circular patch Eq. (2.17), and the spherical patch Eq. (3.23), when the ratio of

reactive sites to passive sites available to transport substrate is $N_{\text{eff}}=100$. Note the decay is slowest for the linear array because in the lower dimensionality situation, a slower diffusion rate (relative to desorption) will still permit the active species to reach the reactive region.

In this paper we have stressed the analogy of this dynamical effect to catalysis. The analogy is not perfect, however, since the "catalytic" effect arises in comparing the reaction
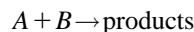
$$A + B \rightarrow \text{products}$$

in free solution with the reaction

$$A + B/\text{network} \rightarrow \text{products}.$$
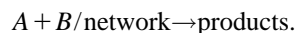
The reactants are not identical and the products may not separate from the reaction complex at the same rate. Nevertheless, the analogy with catalytic activity is instructive, especially because the situation corresponds closely to an active site embedded in a macromolecular framework that is encountered so often in biological applications.

A situation that is more familiar from the perspective of heterogeneous catalysis is where A and B react by one of two pathways: (1) either directly in solution or (2) by first diffusing to a solid network and then diffusing along the network until there is a reactive encounter or dissociation back into the solution.

If the product species easily separate from the network and if diffusion along the network for A and B is fast compared to free diffusion in solution, then the system will exhibit a rate enhancement. This catalytic effect is due to diffusion along the chemically passive heterogenous network which provides the acceleration only through mass transport. This model is also under study.

[1] G. Adam and M. Delbrook, in *Structural Chemistry and Molecular Biology*, edited by A. Rich and N. Davidson (Freeman, San Francisco, 1968).
[2] O. G. Berg, R. B. Winter, and P. H. von Hippel, Biochemistry **20**, 6929 (1981); R. B. Winter and P. H. von Hippel, *ibid.* **20**, 6948 (1981); R. B. Winter, O. G. Berg, and P. H. von Hippel, *ibid.* **20**, 6961 (1981).
[3] O. G. Berg, Chem. Phys. **31**, 47 (1978); Biopolymers **18**, 2161 (1979).
[4] O. G. Berg and C. Bomberg, Biophys. Chem. **4**, 367 (1976); **7**, 33 (1977); **8**, 271 (1978); **9**, 415 (1979).
[5] J. M. Schurr, Biophys. Chem. **9**, 413 (1979).
[6] R. Schranner and P. H. Richter, Biophys. Chem. **8**, 135 (1978).
[7] R. A. Alberty and G. G. Hammes, J. Phys. Chem. **62**, 154 (1958).
[8] K. Solc and W. H. Stockmayer, J. Chem. Phys. **54**, 2981 (1971); Int. J. Chem. Kinet. **5**, 733 (1973).
[9] K. S. Schmitz and J. M. Schurr, J. Phys. Chem. **76**, 534 (1972).
[10] J. M. Schurr and K. S. Schmitz, J. Phys. Chem. **80**, 1934 (1976).
[11] S. I. Temkin and B. I. Yakobson, J. Phys. Chem. **88**, 2679 (1984).
[12] R. Samson and J. M. Deutch, J. Chem. Phys. **68**, 285 (1978).
[13] C. Kuo-Chen and J. Shou-Ping, Sci. Sin. **17**, 664 (1974).
[14] P. H. Richter and M. Eigen, Biophys. Chem. **2**, 255 (1974).
[15] M. Abramowitz and I. A. Stegun, *Handbook of Mathematical Functions* (National Bureau of Standards, Washington D.C., June 1964), Chap. 8.
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