Are Shapes Intrinsic?*

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1 Introduction

Are shapes intrinsic? Intuitively, an intrinsic property is a property that characterizes something as it is in itself. What intrinsic properties something has in no way depends on what other things exist (things other than it or its parts) or how it is related to them. With extrinsic properties (properties that are not intrinsic), by contrast, other things can "get in on the act" when it comes to determining whether something has them.

So consider your left hand. Ignore the other things that exist and consider it as it is in itself. Does it have a shape? Or does it, when so considered, become shapeless?

Immediately, "it does have a shape!" seems like the right answer. (Indeed, shapes are often cited as paradigm cases of intrinsic properties.) The proposition that shapes are intrinsic thus has the power of intuition behind it, and that is reason enough to try to defend it. It also figures prominently in well-known philosophical arguments.[1] Lately, though, I have begun to have doubts that shapes really are intrinsic.

What we want is a theory of shape properties according to which shapes are intrinsic. So what is a theory of shape properties? I assume that some spatial relations are basic, or fundamental, and that other spatial relations are analyzed in

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[1]Ted Sider, for example, appeals to the proposition that shapes are intrinsic in his argument for the doctrine of temporal parts [Sider 2001]. (Some other temporary intrinsic property would serve his purposes as well, though.)

terms of them.[2] To give a theory of shape properties, then, is to produce a list of which spatial relations are fundamental, together with the laws those relations obey and (if shape properties are not themselves on the list) an explanation of how shape properties are to be analyzed using those relations. So, for example, we might say that the basic spatial relations are distance relations; using these relations, we could analyze, say, the property of being an equilateral triangle in a familiar way: an equilateral triangle is a three-sided figure such that the distance between any two of its corners is the same. By looking at a shape's analysis we can tell whether it is intrinsic: a shape is intrinsic just in case it can be completely analyzed in terms of the fundamental spatial relations among the parts of things that instantiate it. It seems obvious, at first glance, that something's shape *is* just a matter of the fundamental spatial relations among its parts, and so it seems obvious that shapes are intrinsic. But this obvious claim is hard to defend. In this paper I will examine all the theories of shape properties that I know of. I argue that each of these theories either cannot save the intuition that shapes are intrinsic, or can be shown to be false.

Here is the plan for this paper. In section 2 I say more about what shape properties are and ward off two quick arguments that shape properties are not intrinsic. Then in section 3 I say more about the notions of fundamentality, analysis, and intrinsicness that I am appealing to. The remainder of the paper is devoted to examining theories of shape properties.

(I assume throughout this paper that space is three-dimensional and Euclidean. Unfortunately I do not have room to discuss problems raised for intrinsic shapes by the possibility that space does not have the geometry or the topology of Euclidean space.)

2 Shape Properties

In this section I make a series of clarificatory remarks about shape properties.

Some shapes have simple names: being spherical, being triangular, being square. But for other shapes (the vast majority, in fact) we have no such names.

[2] I include properties as one-place relations. When I want to restrict a claim to relations with more than one argument place I will do so explicitly.

Instead, we refer to the shape by referring to something that has that shape: the shape of the Statue of Liberty, for example, or the shape of the Empire State Building. Even though we have no simple name for the shape of the Statue of Liberty, though, the Statue of Liberty certainly has some shape.

The shapes I'm talking about are *spatial* shapes, not spatiotemporal shapes. Over the course of its life a baseball traces out a four-dimensional region of space-time, a region that is not even roughly spherical. But at any time a baseball occupies a roughly spherical three-dimensional region of space.[3]

Some shapes are maximally specific. The shape property instantiated by all and only the exact, to-scale replicas of the Statue of Liberty is an example. Some are less so. The property of being a four-sided figure is an example: not every four-sided figure has exactly the same shape. I will focus mostly on the maximally specific shapes. They pose more difficulties. It is harder to produce a theory on which they come out intrinsic than one on which less specific shapes come out intrinsic.

Does *every* material object have a shape? There are tough cases. Suppose that the smallest particles are small enough to occupy just one point of space. They have no spatial extent. Are they too small to have a shape? Or again, suppose some material objects (clouds, for example) have vague boundaries. Do they have a shape?

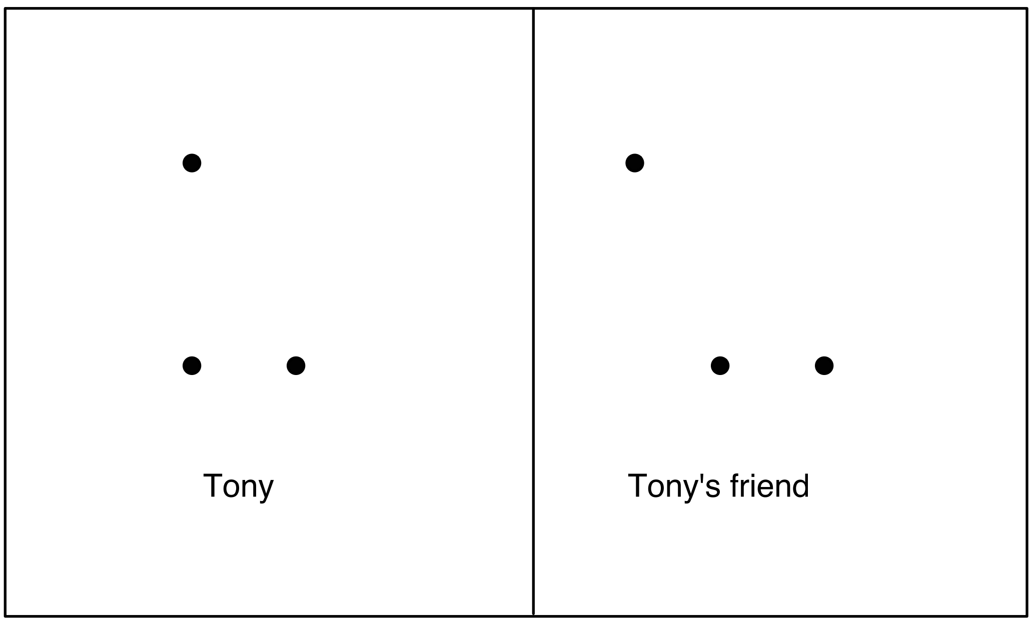
I think that both pointsized material objects and that material objects with vague boundaries have shapes (though in the latter case it is vague just which shape they have). But for my purposes it does not matter what we say about these cases. I will ignore them, and confine my attention to material objects that are not pointsized (though they may have pointsized parts) and do not have vague boundaries.

There is another kind of tough case. It is not as tough as the two I mentioned, but what we say about cases of this kind does matter for my arguments. (In partic-

[3] I focus on spatial shapes because I think that the intuition that shapes are intrinsic is an intuition about spatial shapes, not about spatiotemporal shapes. The question whether spatiotemporal shapes are intrinsic is also tied up with the debate over the doctrine of temporal parts. McDaniel [2003] argues that endurantists should say that spatiotemporal shapes are extrinsic, even if spatial shapes are intrinsic.

ular, it matters for my arguments in section 6.4 and following.) Do material objects with a finite number of pointsized parts have shapes? Consider Tony and his friend, for example (see figure 1). Both are mereological sums of three pointsized particles. (For obvious reasons I use larger than pointsized dots to represent the particles.) Do they have shapes? I say they do. If it make sense to ask of two things whether they

Figure 1: Tony and his friend



have the same shape, then they must both have shapes. And it does make sense to ask whether Tony and his friend have the same shape—and the correct answer is "no." If someone points to Tony and said "bring me an object just the same shape as this one" it would not be appropriate to bring her Tony's friend.

Still, some may resist the claim that Tony has a shape. What might be the source of this resistance? Maybe they resist because Tony is composed of finitely many pointsized parts that are not in contact. But this should not bar Tony from having a shape. For all we know we are composed of finitely many pointsized parts that are not in contact. And if we learned that we were we would not then say that we have no shape.

Or maybe they find themselves resisting the claim that Tony has a shape because they find themselves doubting that there is any such thing as Tony. There are the three particles, arranged in an "L" configuration; but there is no one thing that has them all as parts to have a shape. Some philosophers do deny that there is any such thing as Tony. Peter van Inwagen does, for example [van Inwagen 1990]. He also denies that there are any taxi cabs, and so denies that anything is taxi-cab-shaped. But even he would admit that there are some things (the things we ordinarily say compose a taxi) that are arranged in a taxi-cab-shaped configuration. And that is enough for my purposes. My discussion works just as well whether we ask whether shapes are intrinsic properties, or whether we ask whether "arrangements" or "configurations" are intrinsic relations.[4] In this paper, though, I set aside any doubts about composition and just assume that things like Tony exist.

Finally, to solidify our grip on shape properties I will quickly dismiss two bad arguments that not all shapes are intrinsic. The first argument focuses on the shapes of hollow spheres:

(1) For anything to be a hollow sphere, it must be completely empty inside.

(2) Whether something is completely empty inside is not just a matter of how it is in itself.

(3) So the property of being shaped like a hollow sphere is not intrinsic.

And if (3) is true, then not all shapes are intrinsic.

This argument is not good. We should distinguish between the property of being a hollow sphere and the property of being shaped like a hollow sphere. This

[4]The notion of an intrinsic relation is a straightforward generalization of the notion of an intrinsic property. Just as an intrinsic property characterizes something as it is in itself, an intrinsic relation (with more than one argument place) characterizes some *things* as they are in *themselves*. For example, *x is as massive as y* is intrinsic: if two things are equally massive, that is a matter of how those two things are in themselves. Whether they instantiate *x is as massive as y* does not depend on what else there is or what those other things are like. (The relation *x is as massive as y* is also internal: it supervenes on the intrinsic properties of its relata. Not all intrinsic relations are internal.)

gives us two readings of premise (1). Premise (1) is true when we read it as concerning the first property: anything that has the property of being a hollow sphere is completely empty inside. So it follows from this and premise (2) that the property of being a hollow sphere is not intrinsic. But this is not the argument's conclusion; the argument's conclusion is about the property of being shaped like a hollow sphere. And regarding this property premise (1) is not true: to be shaped like a hollow sphere a material object need not be completely empty inside. The surface of a baseball is shaped like a hollow sphere, even though it is completely filled by the rest of the baseball. Or, again, every solid block of granite has parts shaped like hollow spheres, even though those hollow spheres are completely surrounded, both inside and out, by other parts of the granite block. We cannot conclude, then, that the property of being shaped like a hollow sphere is not intrinsic.

For the second bad argument that not all shapes are intrinsic, suppose I am holding some sand cupped in my hands. The mereological sum of the grains of sand is then shaped roughly like half a solid sphere. We might argue: the sand only has that shape because I am holding it. Were I to move my hands, or were my hands to cease to exist, the sand would lose its shape and fall to the floor. So its having that shape depends on the existence of something else: namely, my hands. So its shape is not intrinsic.

This argument is not good. We should distinguish between physical dependence and metaphysical dependence. The sand's shape physically depends on the existence of my hands: it is the physical forces my hands exert on the sand (and the sand grains on each other) that gives the sand its shape. But it is fine for an intrinsic property to physically depend in this way on the existence of other things. Intrinsic properties merely need to be metaphysically independent of the existence of other things. And the sand's shape is metaphysically independent, it seems: we can conceive of a world (a world with different laws of nature, to be sure) where the sand has the shape it actually does even though my hands do not hold it in the shape.

3 Fundamentality, Analysis, and Intrinsicness

As I said, I will assume that some spatial relations are fundamental, and that all other spatial relations may be analyzed in terms of the fundamental ones. To analyze a relation is to show how its pattern of instantiation depends on the pattern of instantiation of the fundamental relations. One may analyze a relation R by displaying an open sentence that (1) expresses R and (2) contains predicates that express only fundamental relations. I will sometimes call such an open sentence "R's analysis." To continue the example I used earlier in more detail, if we suppose that the three-place relation *the distance from x to y is r* is fundamental, then the following is an analysis of the three-place relation *x, y, and z are three vertices of an equilateral triangle*:[5]

> $\exists r$(the distance from x to y is r & the distance from y to z is r & the distance from z to x is r)

Again, whether something instantiates an intrinsic property does not at all depend on what other things exist (things other than it and its parts), or what those other things are like. A shape is intrinsic, then, just in case it can be completely analyzed in terms of the fundamental spatial relations among the parts of things that instantiate it. So we can tell whether a shape is intrinsic by inspecting its analysis: a shape property Px is intrinsic just in case every quantifier in its analysis is restricted to x and x's parts.[6]

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[5] I italicize open sentences to form names of the relations those sentences express.

[6] This definition of "intrinsic shape" is related to the general definition of "intrinsic" that David Lewis uses in [Lewis 1986]. He says that Px is an intrinsic property just in case it supervenes on the fundamental properties x and its parts instantiate, as well as the fundamental relations that hold among x's parts.

There is a problem for Lewis's definition: if (say) God exists necessarily then the property *coexisting with God* supervenes on every set of relations, and so is counted as intrinsic. That is why I demand analysis, instead of just supervenience, in my definition.

It follows from Lewis's definition that the fundamental properties are intrinsic. So his definition is appealing only if we accept this consequence. But I don't think

4 Substantivalism, Supersubstantivalism, and Relationalism

Now that the ground has been cleared I can begin my survey of theories of shape properties. The theories divide up according to which view about the ontology of space they presuppose. There are two views about the ontology of space: substantivalism and relationalism. Substantivalists affirm that space exists. Relationalists deny it. (When relationalists deny that space exists, though, they do not mean to deny that the world is spatial, that it contains material objects that stand in spatial relations to each other. Relationalism is not a form of idealism. Relationalists accept that there are material objects and spatial relations among material objects; they just deny that there are, in addition, points and regions of space that these material objects occupy.)

Substantivalists themselves divide into two camps, depending on their view of the nature of material objects. Some substantivalists (probably most substantivalists) hold that material objects are distinct from space. Since these substantivalists believe that material objects and regions of space are two radically different kinds of concrete thing, I will call them "dualists." Others substantivalists—call them "supersubstantivalists"—hold instead that material objects are identical with regions of space.[7]

Dualism, I take it, paints an attractive picture of the world. There are material objects, on the one hand; there are regions of space, on the other; and there is a fundamental relation, the location relation, that material objects bear to points of space. This view meshes well with an "atoms and the void" way of thinking about the world that is so seductive. Nevertheless, I will argue in the next section that if dualism is true then shapes of material objects are not intrinsic. (This is so no matter which dualist theory of shapes we accept.)

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this consequence should be controversial. It is part of our conception of fundamental properties that they are intrinsic: it is usually said that the fundamental properties make for similarity among their instances, and that they carve nature at its joints [Lewis 1983].

[7]The name "supersubstantivalism" was coined, as far as I know, by Sklar [1974].

5 Dualism and Intrinsic Shapes

Suppose dualism is true. Some material objects are spherical. Some regions of space are spherical. And it is necessary that every spherical material object is located at a spherical region of space. But this can't be just a magical, mysterious necessity, a necessity that must go unexplained. If dualism is true then spherical material objects are spherical *because* they are located at spherical regions of space. That is, dualists should say that material objects have the shapes they do in virtue of the shapes of the regions of space at which they are located. So intuitively, whether something is spherical is not a matter of how it is in itself; it is a matter also of the shape of the region of space at which it is located. Sphericality is not intrinsic.

I can make this argument more precise by looking at dualists' analyses of sphericality. Spherical material objects are spherical because they are located at spherical regions. So the property of being a spherical material object has an analysis that contains a quantifier that ranges over regions of space. Schematically, a partial analysis of *x is a spherical material object* looks like this:

$\exists y$(y is a region of space and x is located at y and y is spherical)

This is, of course, not a complete analysis; to make it complete we need an analysis of *y is spherical* (as a property of regions of space) in terms of the fundamental spatial relations (whatever they may be). But however those details are filled in, we can already see that this analysis contains a quantifier not restricted to the parts of x. So sphericality is not intrinsic. And this argument makes no use of special facts about sphericality; for any shape, a similar argument will show that that shape is not intrinsic.

(I say that dualists should analyze the shapes of material bodies in terms of the shapes of the regions they occupy. But couldn't they just as easily explain the necessary connection in the other direction, by analyzing the shapes of regions of space in terms of the shapes of material bodies?

The answer is "no." Dualists cannot analyze *x is a spherical region of space* as

$\exists y$(y is a spherical material object and y occupies x).

for not all spherical regions are occupied. And they cannot go modal and analyze *x is a spherical region of space* as

$\Diamond\exists y$(y is a spherical material object and y occupies x).

Suppose region *R* is possibly occupied by a spherical material object. I want to know: why is this possible? The best answer is: because *R* is spherical. We explain why *R* has a certain modal property by noting that it has a distinct non-modal property. But on the proposed analysis dualists cannot give this answer. They have no way to explain why *R* is possibly occupied by a spherical material object: for them, this is just a brute modal fact.)

My argument exploits the following fact: if space exists then there are necessary connections between the shapes of material objects and the shapes of the regions of space they inhabit. There are two ways to avoid the problem this fact raises. First, deny that material objects are distinct from the regions of space they inhabit, and embrace supersubstantivalism.[8] Then there is no mystery in the necessary connection between their shapes; it becomes merely the logical truth that

[8]What about a hybrid of dualism and supersubstantivalism? Like dualism, the hybrid theory says that there are things that occupy regions of space. But the hybrid theory denies that those things are material objects. Instead, the hybrid theory says that (i) for any occupied region of space there exists the mereological fusion of that region with the thing that occupies it; and (ii) all and only those fusions are material objects.

My argument that dualism is incompatible with intrinsic shapes does not apply to the hybrid theory: in this theory, the quantifier in the dualist analysis of *x is a spherical material object* can be restricted to *x*'s parts.

I find it hard to take the hybrid theory seriously. Why believe that there are any such things as mereological fusions of things that occupy regions of space with the regions they occupy? And even if there are such fusions, why believe that they are the material objects?

Even if you do not share these doubts, the hybrid theory does not make the world safe for intrinsic shapes. Shapes are intrinsic, on the hybrid theory, only if regions of space have their shapes intrinsically. I argue that no acceptable theory of shapes entails that regions of space have their shapes intrinsically in section 6. (Parsons [forthcoming] discusses the hybrid theory and other objections to it. I thank him and Kris McDaniel for bringing the hybrid theory to my attention and discussing it with me.)

everything has the same shape as itself. Or second, deny that there are any regions of space, and embrace relationalism.

6 Supersubstantivalism

Let us first try being supersubstantivalists. Is the world now safe for intrinsic shapes?

It seems intuitive that the shape of any region of space is a matter of the distances between the points in that region. (So I reject the claim that shapes of regions of space are themselves fundamental properties.) And at first glance, the distance between two points seems to be a matter just of those two points; other things aren't getting into the act. So any property analyzable in terms of distances between points—shape properties among them—will be intrinsic.

But not so fast. Just what fundamental distance properties do we have in mind here? There are two choices.

6.1 Mixed Fundamental Distance Relations

Suppose *the distance from x to y is r,* a three-place relation relating two points of space to a number, is the fundamental spatial relation. (If it is a fundamental spatial relation, then it is presumably the only one: presumably (in Euclidean space, at least) any other spatial relation can be analyzed in terms of it.)

If this relation is fundamental then shape properties are not intrinsic. For if this relation is fundamental then a region of space gets to have a shape only because its parts are appropriately related to numbers; its having a shape is not a matter of the way that region of space is, in and of itself.

I will go through this argument in more detail. Look at the analyses of shape properties in this theory. For example, we can analyze the shape that regions composed of three points of space that stand at the vertices of an equilateral triangle in terms of this relation.[9] The analysis looks like this:

[9]I choose this property for the simplicity of its analysis. The shape of solid equilateral triangles can also be analyzed using *the distance from x to y is r.* But the analysis will be much longer and will contain a clause identical to the analysis of

$\exists r \exists y_1 \exists y_2 \exists y_3$($y_1, y_2$, and y_3 are parts of x & the distance from y_1 to y_2 is r & the distance from y_2 to y_3 is r & the distance from y_3 to y_1 is r)

This analysis contains a quantifier that ranges over numbers, and no number is part of any triangular material object. So this property is not intrinsic.

Someone might object that numbers (and abstract objects generally) should receive a special dispensation in the definition of "intrinsic property." That definition bans quantifiers not restricted to the parts of the thing instantiating the property from analyses of intrinsic properties. But perhaps it should allow quantifiers that range over only abstract objects into such analyses; so long as no quantifier ranges over *concrete* things other than the parts of the thing instantiating the property, the property is intrinsic. (The above analysis can easily be rewritten so that the first quantifier is restricted to numbers.)

We should reject this revised definition of "intrinsic." To illustrate why it is wrong, notice that it leads to incorrect results about the intrinsic properties of numbers themselves. If we suppose that numbers exist, then it is plausible to suppose that *x is the successor of y* is a fundamental relation that natural numbers instantiate. (Other relations among the natural numbers—like addition and multiplication—can be defined in terms of successor.) Now the property of being the smallest natural number, like the property of being the shortest person in the room, is not intrinsic. Whether a number has this property depends on what other numbers there are, and whether it is the successor of any of them. But this property's analysis—"$\neg\exists y$(y is a number and x is the successor of y)"—contains just one quantifier restricted to numbers, and so the revised definition of "intrinsic" says that it is intrinsic.

Say that relations which relate abstract objects to concrete objects (*x is n years old* is an example) are "mixed" relations; other relations—relations that relate only abstract objects, or only concrete objects—are unmixed, or "pure." The relation *the distance from x to y is r* is mixed; and that is the problem with it. The above argument generalizes to show that if shapes are intrinsic then they cannot be analyzed in terms of mixed fundamental spatial relations. What we need, if we want to save the claim that shapes are intrinsic, is a theory of shapes according to which the fundamental spatial relations are pure.

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the simpler property.

I mention this because, if one were going to chose a mixed spatial relation to be a fundamental relation, one would not be likely to choose *the distance from x to y is r.* It has seemed to many that this relation isn't really fundamental, but is analyzed in terms of some other fundamental mixed spatial relations. Three alternative theories are as follows:

(1) The fundamental spatial relation concerns distance ratios, rather than distances. That is, there is just one five-place fundamental spatial relation, *x and y are r times as far apart as z and w.* Distances between points are then analyzed in terms of the ratios of their distances to the distance between two reference objects (say, the ends of the standard meter).

(2) The fundamental spatial relation is *the length of x is r*, a relation that assigns lengths to curves in space. Distances between points are then analyzed in terms of lengths of curves: the distance between *x* and *y* is the length of the shortest path from *x* to *y*.

(3) This theory reads its account of the fundamental spatial relations off of standard presentations of differential geometry. According to these presentations, the structure of space is given by two things: an atlas (for differential and topological structure) and a metric (for metric structure). An atlas is a collection of functions from subregions of space to open sets in $\mathbb{R}^3$. (Each function gives coordinates to the points of space on which it is defined.) A metric is a function from points of space to functions from pairs of tangent vectors to numbers. Lengths of curves can be analyzed using the metric: the metric tells us the lengths of tangent vectors, and the length of a curve is obtained by adding up (really, integrating) the lengths of vectors tangent to that curve. Distances between points can then be analyzed in terms of lengths of curves, as in (2).[10]

Each of these theories has its virtues. Those who think that there are no absolute facts about distance, but only facts about distance ratios, will like the first theory.

[10]In his [1993] Bricker defends this third theory.

And the second and third theories fit with the way the distance between two points is defined in differential geometry.

But for all their virtues, these alternative theories do just as poorly when it comes to saving the view that shapes are intrinsic.

6.2 Pure Fundamental Relations: Multiple Distance Relations

So let us consider a theory of shapes according to which pure distance relations are fundamental. According to this theory, there are infinitely fundamental distance relations. They are two-place relations, with names like "*the distance from x to y is one,*" "*the distance from x to y is two,*" and so on. We should not be misled by the names we give these relations. These names do not contain any semantic structure. Although the name of one of these relations contains the letters "t-h-e d-i-s-t-a-n-c-e" and also contains the letters "o-n-e," we are not to think of these as denoting phrases that refer to a number. These relations are not derived from the single mixed three-place relation *the distance from x to y is r* by filling one of its argument places with a number. Instead, they are themselves pure fundamental relations, relating only two points of space to each other.[11]

In the previous section I gave an analysis of the shape property that regions shaped like the vertices of an equilateral triangle instantiate in terms of the single relation *the distance from x to y is r*. It is easy to convert this into an analysis of the same shape property in terms of this infinite family of two-place distance relations:

> $\exists y_1 \exists y_2 \exists y_3 (y_1, y_2$, and y_3 are parts of x & either (the distance from y_1 to y_2 is one & the distance from y_2 to y_3 is one & the distance from y_3 to y_1 is one) or (the distance from y_1 to y_2 is two (and so on))

This analysis does not contain quantifiers ranging over numbers. So on this view shapes are intrinsic.

Although shapes are intrinsic on this view, it has other problems which make it unacceptable. If these relations are fundamental then there are necessary truths that we must think are brute and inexplicable, but which we ought to be able to

[11]This seems to be the view Melia prefers at the end of his [Melia 1998].

explain. For example, it is necessary that if two things instantiate *the distance from x to y is one* then they do not also instantiate *the distance from x to y is two*. And not only is this necessary; each distance relation excludes the other distance relations in this way. Or again, it is necessary that if *a* and *b* instantiate *the distance from x to y is two* and *b* and *c* instantiate *the distance from x to y is two* then *a* and *c* do not instantiate *the distance from x to y is five.* And not only is this necessary; other instances of the triangle inequality are necessary as well.

The problem is not that these necessary truths are not logical necessities. I do not claim that all necessities involving fundamental relations are logical (as some versions of the combinatorial theory of possibility do). The problem is that these necessities exhibit a striking pattern, and we ought to be able to explain why they exhibit this pattern. For example, we want to say that the necessities divide into subclasses, where all members of a given subclass share a common form. But we cannot. The theory does not have the resources to say in what their shared form consists.[12]

This argument generalizes to other attempts to turn theories of mixed fundamental spatial relations into theories of pure fundamental spatial relations by substituting infinite families of pure $(n-1)$-place relations for a single n-place mixed relation. A theory that says that relations like *x and y are two times as far apart as z and w* are fundamental fails for the same reasons; a theory that says that properties like *the length of x is three* (where *x* is a curve in space) are fundamental fails for the same reasons.[13]

[12]In his [1989] Field argues that we would not want to accept a physical theory that used two-place *predicates* like "the distance from *x* to *y* is one" as semantic primitives, because it would be unlearnable (since it contains infinitely many primitive predicates) and unusable (since it contains infinitely many axioms that cannot be captured in a finite number of axiom schemas). But this is not enough to show that we cannot accept that two-place relations like *the distance from x to y is one* are fundamental: for the proponent of this metaphysical view may deny that the physical theory scientists learn and calculate with needs to have semantic primitives that express fundamental relations.

[13]I give theories according to which distance along a path is the fundamental notion of distance little attention in this paper, despite their importance in differential geometry, the geometry of curved spaces, and (so) in general relativity. Partly this

6.3 Pure Fundamental Relations: The Higher-Order Theory

Suppose that in addition to the infinitely many fundamental distance relations there are also a small number of fundamental second-order relations that the first-order distance relations instantiate. Suppose, for example, that there are two fundamental three-place relations: R_1 *is the sum of* R_2 *and* R_3 and R_1 *is the product of* R_2 *and* R_3; and one fundamental two-place relation: R_1 *is less than* R_2. These second-order relations give structure to the set of first-order distance relations. If they are instantiated in the right pattern, they give the first-order relations the same structure as the (non-negative) real numbers.[14]

This second-order theory avoids the objection I gave in the previous section to the view that only the first-order pure distance relations are fundamental. It is necessary that if *a* and *b* instantiate *the distance from x to y is one* then they do not also instantiate *the distance from x to y is two*, and likewise it is necessary that if *a* and *b* instantiate *the distance from x to y is four* then they do not also instantiate *the distance from x to y is six.* According to the higher-order theory, these are both instances of the following necessity:

(α) $\forall R \forall G \forall x \forall y$(if R and G are distance relations[15] and x and y instantiate R then they do not also instantiate G).

It is also necessary that if *a* and *b* instantiate *the distance from x to y is two* and *b* and *c* instantiate *the distance from x to y is two* then *a* and *c* do not instantiate *the distance from x to y is five.* But this necessity is not an instance of (α). So this

is for reasons I give in the text: the only ways I know of to formulate such a theory fall prey to objections I raise against other views. But even if there is a theory that avoids those objections, it will fail to save the claim that shapes are intrinsic. Bricker [1993] gives the arguments.

[14] On this choice of second-order relations it makes sense to say that two things are distance one apart even before we choose a unit of measurement. Those who dislike this feature may prefer some other second-order theory that lacks it. ([Mundy 1987] is an example of such a theory.) But the arguments I give below against this second-order theory apply (mutatis mutandis) to these other theories as well.

[15] In the second-order theory you can analyze *R is a distance relation* as $\exists G \exists H$(*R is the sum of G and H).*

theory can express what the "distance exclusion" necessities have in common, and how they differ from the "triangle inequality" necessities.

I do not like the higher-order theory. I do not like it because I don't really believe there are any (properties or) relations. Deniers of the existence of relations can, of course, make sense of most relation-talk (including the relation-talk in all other theories of shapes discussed in this paper) by being fictionalists.[16] One thing that fictionalists cannot do, though, is make sense of talk of fundamental second-order relations.

But enough about me. Are shapes intrinsic, according to the higher-order theory? It looks, at first, like they are. After all, the higher-order theory agrees (we are told) with the first-order theory that the first-order distance relations are fundamental, and I admitted that if those relations are fundamental then shapes are intrinsic.

But this is not right. The second-order theory cannot say that the second-order relations *and* the first-order distance relations are fundamental. For the set containing all those relations is redundant. It does not form a minimal supervenience base for characterizing reality: a language adequate for giving a complete description of every possible world need contain basic predicates that express only a proper subset of it. But (I say) the fundamental relations do form a minimal supervenience base.[17]

[16]That is, all their utterances containing words like "relation" are silently prefixed with some operator like "According to the fiction that there are relations,..." Dorr [2002] discusses just what the fiction of relations might look like.

[17]So I agree with [Lewis 1983]. Lewis calls on the fundamental relations to do a lot of work for him. To name two roles they play, they ensure that our words have relatively determinate meanings and they help distinguish laws from accidents. It may be that the fundamental relations cannot both form a minimal supervenience base and play all these roles. (Here is one example where the roles come apart. Sider [2001] claims that the property of existence is fundamental. That way it helps secure a determinate meaning for "exists" and ensures that people who disagree about ontological questions mean the same thing by their quantifiers. But the property of existence is not a member of any minimal supervenience base.) Faced with this problem, some (Sider [2004] for example) are willing to give up on the idea that the fundamental relations form a minimal supervenience base. I, on the other hand, give up on the idea that the fundamental relations can play all the other roles Lewis wanted them to.

Why is it redundant to include the second-order relations? Consider the following two open sentences:

($P1$) the distance from x to y is one

($P2$) $\exists R$(R equals R divided by R and x and y instantiate R)[18]

If the second-order relations are fundamental then ($P1$) and ($P2$) contain only predicates that express fundamental relations.[19] But ($P1$) and ($P2$) express relations that are necessarily coinstantiated. So *the distance from x to y is one* supervenes on the higher-order relations: a set containing it and the higher-order relations is redundant.

One way to make the set of fundamental relations non-redundant is to return to the first-order theory. I have already argued against that view. So to avoid having a redundant set of fundamental relations the higher-order theory must say that distance relations like *the distance from x to y is one* are not, after all, fundamental.

In the higher-order theory, then, relations like *the distance from x to y is one* have analyses in terms of the higher-order relations. (($P2$), for example, is an analysis of *the distance from x to y is one.*) It's a weird thing to analyze first-order relations in terms of second-order relations; but that is where the higher-order theory leads.

If relations like *the distance from x to y is one* are not fundamental, then the higher-order theory does not automatically inherit the first-order theory's solution to the problem of intrinsic shapes. An analysis of a shape property in the first-order theory is not automatically an analysis of that property in the higher-order theory. We must ask, from the beginning, what the analyses of shape properties look like in the higher-order theory, and whether those analyses show those properties to be intrinsic.

[18] Actually the higher-order relation *R equals G divided by H* is not fundamental, but it can be defined in terms of *R is the product of G and H* (and the other fundamental relations), just as the division operation on real numbers can be defined in terms of multiplication.

[19] If some higher-order relations are fundamental then our canonical language for describing the world at a fundamental level should (as the language in which ($P2$) is written does) contain quantifiers over properties and the predicate "instantiates."

How are shape properties analyzed in the higher-order theory? We can generate an analysis of any shape property by looking at its analysis in the first-order theory and replacing predicates that express first-order distance relations with the analyses of those relations in the higher-order theory. For example, the shape property that regions shaped like the verticies of an equilateral triangle with sides length one looks like this:

> $\exists y_1 \exists y_2 \exists y_3(y_1, y_2$, and y_3 are parts of x and $\exists R(R$ equals R divided by R and y_1 and y_2 instantiate R and y_2 and y_3 instantiate R and y_3 and y_1 instantiate R)).

This analysis contains a quantifier that ranges over something other than x and its parts (namely, the second-order quantifier over first-order distance relations). So the shape property it analyses is not intrinsic.[20]

6.4 Pure Fundamental Relations: Betweenness and Congruence

We have not succeeded in finding an acceptable theory of shapes that takes distance relations as fundamental and saves the intuition that shapes are intrinsic. But if the fundamental spatial relations are not distance relations, what are they? To answer, we might look to synthetic axiomatizations of geometry. There is a synthetic axiomatization of Euclidean geometry using just two primitive predicates of points of space, "*x*, *y* are congruent to *z*, *w*" (or "*x* and *y* are as far apart as *z* and *w*") and "*x* is between *y* and *z*." (Actually there is one such axiomatization for two-dimensional Euclidean geometry, and one for three-dimensional Euclidean geometry, and so on.) These two predicates clearly express pure spatial relations. So this is a theory according to which distance relations are not fundamental (or, at least, according to which the only fundamental distance relation is congruence—the

[20] Armstrong [1978] says that (monadic, non-relational) properties are non-spatiotemporal parts of the things that instantiate them. So on his view, the property of instantiating two fundamental properties is intrinsic, even though its analysis quantifies over properties. But I don't see how the higher-order theory can be saved by adopting Armstrong's view: what sense is there in the claim that a relation is a non-spatiotemporal part of the things it relates?

relation of equidistance.) Distance relations, on this theory, have an analysis.[21]

This theory allows that the geometry of space as a whole is intrinsic. (The property of having a Euclidean geometry, then, is intrinsic.) But what about smaller regions of space? Are their shapes intrinsic?

A region's shape is intrinsic, on this theory, just in case its shape can be completely analyzed in terms of the betweenness and congruence relations among the points in that region. That is, a region's shape is intrinsic just in case any other region that is isomorphic to it with regard to the betweenness and congruence relations among its parts has the same shape.

Not all shapes are intrinsic according to this theory. Consider again Tony and his friend.[22] Tony's parts and his friend's parts instantiate betweenness and congruence in the same pattern: betweenness is nowhere instantiated; congruence is nowhere instantiated.[23] So any property of Tony's that can be analyzed just in terms of the betweenness and congruence relations among Tony's parts is a property that Tony's friend also has. But Tony and his friend have different shapes; so Tony's shape is not intrinsic, according to this theory.

(Tony's shape does have an analysis in terms of betweenness and congruence, on this theory. But that analysis will quantify over points of space that are not part of Tony.)

Someone might object to my argument by denying that Tony and his friend differ in shape. He might reason as follows: "I claim that shapes are intrinsic, and that betweenness and congruence are the only fundamental spatial relations. We should follow where this theory leads, and it leads us to some surprising claims about which regions have the same shape. Tony and his friend instantiate between-

[21]There is a representation theorem that establishes that, given a choice of unit, there is a unique way to assign numerical distances to pairs of points in this theory. Tarski sketches a proof in [Tarski 1959].

[22]I introduced "Tony" as a name for a material object, and use it here as a name for a region of space. Since we are now assuming that supersubstantivalism is true, these uses are compatible.

[23]I omit trivial facts about the instantiation of betweenness and congruence, facts the universal generalizations of which are theorems of Euclidean geometry. For example: if a and b are two of Tony's parts, then a, b are congruent to a, b.

ness and congruence in the same pattern; things that are isomorphic with respect to betweenness and congruence have the same shape; so Tony and his friend have the same shape. We were initially inclined to think that they did not; but we were mistaken."

Now, I am willing to make some revisions to my ordinary beliefs about which material objects are the same shape. For example, I am initially inclined to think that my left hand and my right hand have different shapes. In the face of powerful arguments, I may give up this inclination, and accept that they have the same shape but differ in orientation.[24] But this objection goes too far. With my hands there is a way to "soften the blow" of the counterintuitive claim that they have the same shape: there is a way to put my hands' parts in correspondence so that the distances between corresponding pairs of parts are the same. And we can explain why my hands seem to have different shapes by appealing to their different orientations in space. There is no analogous way to make plausible the claim that Tony and his friend have the same shape.

Like the other theories of shapes I've looked at, then, this theory is not compatible with the claim that all shapes are intrinsic.

6.5 Non-Intrinsic Shapes

In the last section I addressed a defender of the betweenness and congruence theory who asked us to save the intuition that shapes are intrinsic by revising our opinions about which things have the same shape. I declined to do this. There is another way to try to defend the betweenness and congruence theory. I look at it in this section.

In the previous section I argued that according to the betweenness and congruence theory, Tony's shape is not intrinsic. So according to this theory not all shapes are intrinsic. But this theory seems to be doing better than the other theories I looked at that failed to save the claim that shapes are intrinsic. I argued that according to those theories, no shape is intrinsic. But according to the betweenness and congruence theory, while not all shapes are intrinsic, some are.

[24]Kant famously appeals to the claim that the property of being a right hand is not intrinsic to argue for substantivalism. Earman [1989] and Van Cleve [1987] discuss Kant's argument.

For example, according to the betweenness and congruence theory, the shape of a region of space containing just three points that stand at the verticies of an equilateral triangle is intrinsic. It's shape can be analyzed just in terms of the congruence relations among its parts: any two points in the region are congruent to any other two. Any other region with three points that satisfies this description is also shaped like the verticies of an equilateral triangle.

Seizing on this fact, we might try to defend the betweenness and congruence theory as follows: we began with the intuition that all shapes are intrinsic. But maybe we only thought that because we were on a small diet of examples. Maybe it is right to say that only some shapes are intrinsic, and others are extrinsic, and moreover that the betweenness and congruence theory correctly distinguishes between them.

Here is an example to help motivate the claim that not all shapes are intrinsic. Consider a straight line. A straight line, it is often said, is the shortest path between two points. Perhaps *what it is* to be a straight line is to be the shortest path between two points.[25] This suggests that whether a line is straight depends on what other paths there are, and on how long those paths are. And then it looks like the property of being shaped like a straight line is not intrinsic.

This example may motivate the claim that some shapes are not intrinsic. It does not make the betweenness and congruence theory more attractive, though. For according to that theory the property of being a straight line *is* intrinsic. (On that theory, betweenness is a fundamental spatial relation, and the property of being a straight line can be analyzed in terms of the betweenness relations among the points on that line; so it is an intrinsic property.[26]) This example motivates, instead,

[25] Or, more carefully, perhaps what it is for x to be a straight line is for the following to be true: for any two points on x the part of x connecting them is the shortest path between them.

[26] A region is a straight line just in case given three of its points one is between the other two, and it has "no holes." The claim that a region that is "almost a line"—a region such that for any three points in it one is between the other two—has no holes can be spelled out in more detail without quantifying over things not in the region. The claim amounts to the continuity axiom: for any partition of the points in that region into two sets, if no member of either of the sets is between any members

a theory according to which *the length of x is r*, a relation that assigns lengths to paths, is a fundamental spatial relation. But I have already given reasons to reject this theory.

Still, let us set aside this example and consider the possibility that not all shapes are intrinsic on its own merits. Does the distinction the betweenness and congruence theory draws between intrinsic and extrinsic shapes seem at all intuitive? I don't think it does. According to this theory neither Tony's nor Tony's friend's shape is intrinsic, while the shape of a region containing just three points at the verticies of an equilateral triangle is. It seems to me that these shapes are on a par: if one is intrinsic then so is the other.

6.6 Other Pure Fundamental Relations

Betweenness and congruence are only one of several choices of primitives for synthetic Euclidean geometry. For each choice of primitives there is a theory of shape properties according to which the relations those primitives express are the fundamental spatial relations.[27] Do any of these other choices do better at making shapes intrinsic?

There are two questions we can ask about these theories, corresponding to two questions I have discussed about the betweenness and congruence theory:

(1) Does the theory save the intuition that shapes are intrinsic?

(2) If it does not, does it draw an intuitive distinction between intrinsic and extrinsic shapes?

In this section I briefly look at two of these other theories. In both cases, the answer to questions (1) and (2) is "no."

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of the other, then there is a point v in that region that "divides" the sets: given a point from one set and a point from the other, v is between them.

[27] [Royden 1959] is a standard reference for the varieties of primitives that may be used to axiomatize Euclidean geometry.

6.6.1 Congruence Alone

There is an axiomatization of Euclidean geometry that takes congruence alone as primitive, instead of congruence and betweenness. In this theory betweenness is defined in terms of congruence, and every definition of "x is between y and z" contains quantifiers that range over points of space other than x, y, and z. Let "the congruence theory" be the theory of shapes that reads its account of the fundamental spatial relations off of this axiomatization: according to this theory, congruence alone is the fundamental spatial relation.

A region's shape is intrinsic, on this theory, just in case its shape can be completely analyzed in terms of the congruence relations among the points in that region.

The set of shapes that are intrinsic according to the congruence theory is a subset of the set of shapes that are intrinsic according to the betweenness and congruence theory. If the congruence relations among the points in that region are enough to fix the distances between those points, then the congruence relations *and* the betweenness relations are certainly also enough to fix the distances between those points. So, for example, the shape of a region that is composed of the vertices of an equilateral triangle is intrinsic according to both theories.

But the converse is false. The property of being a straight line, for example, is intrinsic according to the betweenness and congruence theory, but not according to the congruence theory.

How does the congruence theory fare with respect to questions (1) and (2)? The answer to question (1) is "no." The congruence theory does not save the intuition that shapes are intrinsic. In fact the congruence theory declares fewer shapes intrinsic than does the betweenness theory. The answer to question (2) is also "no." The congruence theory says that the shape of an equilateral triangle is intrinsic, while the shape of any other triangle is not; yet these shapes seem to be on a par.

6.6.2 The Right Angle Theory

There is an axiomatization of Euclidean geometry that uses "x, y, and z form a right angle at y" as its only primitive. In this theory both betweenness and congruence are defined predicates. Every definition of "x is between y and z" contains quantifiers that range over points of space other than x, y, and z, and similarly for every definition of "x, y are congruent to z, w." Let "the right angle theory" be the theory of shapes that reads its account of the fundamental spatial relations off of this axiomatization: according to this theory, *x, y, and z form a right angle at y* is the only fundamental spatial relation.

A region's shape is intrinsic, on this theory, just in case its shape can be completely analyzed in terms of the "right-angle" relations among the points in that region.

The right angle theory disagrees with both the congruence theory and the betweenness and congruence theory about which shapes are intrinsic. The property of being composed of the vertices of an equilateral triangle, for example, is intrinsic according to the latter two, but is not intrinsic according to the first. I do not know of a maximally specific shape that is intrinsic according to the right angle theory. But the property of being composed of the vertices of a right triangle, which is a relatively specific shape property (and a shape property that Tony instantiates), is intrinsic according to the right angle theory, but is not intrinsic according to either other theory.

So the answer to question (1), when asked about the right angle theory, is "no." Similarly for question (2). The right angle theory distinguishes the triangles with intrinsic shapes from those with extrinsic shapes differently than either previous theory. (As I said, according to the right angle theory only right triangles have intrinsic shapes; all others have extrinsic shapes.) But it still says that some do and some do not have intrinsic shapes. And (again) this division does not seem intuitive.

7 Relationalism

I've surveyed several theories of shape properties that presuppose substantivalism. None of them allow for intrinsic shapes. Can relationalism do better?

Substantivalists think that the fundamental spatial relations relate points of space (or, perhaps, points of space and numbers). Relationalists deny that there are any points of space; they say instead that the fundamental spatial relations relate material objects (or, perhaps, material objects and numbers). This difference aside, the choices relationalists have for the fundamental spatial relations are much the same as the choices substantivalists have.

Some of those choices are ruled out for relationalists for the same reason they are ruled out for substantivalists. The argument from section 6.1 that the fundamental spatial relations are not mixed works just as well if we are assuming relationalism to be true. So does the argument from section 6.2 that we cannot take infinite families of pure distance relations like *the distance from x to y is two* as fundamental. What, then, is left?

The relationalist could follow the supersubstantivalist and characterize the spatial structure of the world using two pure fundamental relations, betweenness and congruence. (He will regard these as relations that material objects instantiate, instead of as relations that points of space instantiate.) But he looks to be no better off than the supersubstantivalist. For many (perhaps all) regions of space there could be a material object with the same shape as that region. If a supersubstantivalist who thinks that betweenness and congruence are fundamental cannot say that the shape of the region of space is intrinsic, then a relationalist who believes the same will not be able to say that the shape of that material object is intrinsic either. (So, for example, if a supersubstantivalist who thinks that only betweenness and congruence are fundamental cannot say that Tony's shape is intrinsic, then neither can a relationalist who uses the same two relations.) After all, the material object and the region are isomorphic with respect to betweenness and congruence. Are there any other options?

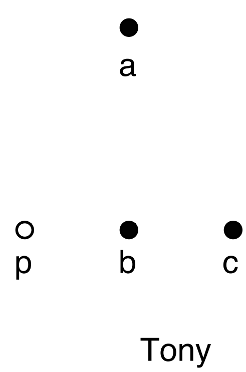
In responding to some objections to their view relationalists often appeal to modal facts. For example, sometimes substantivalists challenge relationalists to distinguish between a world with a Euclidean spatial structure and a world that is Euclidean in most places but in which (as a substantivalist would say) some empty regions of space have a non-Euclidean geometry. (So in the second world space is flat everywhere except for a few empty regions; in those empty regions space is curved.) Pairs of worlds like this can be described that are indistinguishable with regard to the spatial relations among the material objects in them. How can a relationalist distinguish them? One option is to say that they differ in modal facts:

in one, but not the other, there *could be* a material object the parts of which stand in spatial relations that do not obey Euclidean laws.[28]

Can modality help save the thesis that shapes are intrinsic? Return to Tony and his friend. Suppose that betweenness and congruence are the only non-modal fundamental spatial relations. Tony and his friend instantiate betweenness and congruence in the same pattern. Perhaps Tony has some modal spatial property (or his parts instantiate some modal spatial relation) that his friend does not. If this modal property is intrinsic, then Tony and his friend are not intrinsic duplicates, and for all that has been said Tony's shape might be intrinsic.

What might this modal property be? Letting "*a*," "*b*," and "*c*" name Tony's parts (as in figure 2 below) one candidate is the modal property expressed by the following open sentence:

Figure 2: Tony again



(β) It is possible that there is a particle p such that b is between p and c; p, b are congruent to b, c; and p, a are congruent to a, c.

Only if a, b, and c form a right angle at b can there be a particle like p. So Tony does, and his friend does not, have this property. But is this modal property intrinsic?

[28]Leibniz, the arch-relationalist, invokes modality to explain relationalism: "space denotes, in terms of possibility, an order of things that exist at the same time" [Leibniz and Clarke 2000: page 14]. Among many others, Sklar [1974], Earman [1989], and Belot [2000] discusses relationalist appeals to modality.

Let's start by trying to get a grip on the distinction between intrinsic and extrinsic modal properties in general, before we try to decide whether (β) is intrinsic. There are clear examples of modal properties that are not intrinsic. Perhaps I instantiate the modal property *necessarily, x's parents are Skow's actual parents.* But this modal property is not intrinsic: there could have been an intrinsic duplicate of me who had different parents. And maybe the property of being possibly taller is an example of a modal property that is intrinsic. I certainly have the property of being possibly taller; and it seems that an intrinsic duplicate of me would also have this property.

I know of no arguments that establish or refute the claim that (β) is intrinsic. But I myself do not think that (β) is intrinsic. Intrinsic modal properties fall into two categories: those that supervene on the non-modal intrinsic properties of things that have them, and those that do not. The property of being possibly taller is, I think, in the first category. (β), on the other hand, is in the second category. Tony and his friend are isomorphic with respect to betweenness and congruence, and so share all their non-modal intrinsic properties (we can assume that the particles that compose Tony and his friend are all intrinsic duplicates); yet by assumption Tony does and his friend do not instantiate (β).

It is because (β) is in the second category that I do not think it is intrinsic. Intrinsic modal properties that fall in the second category are strange and suspicious things. I'd rather not believe in them.

(β) was just one candidate modal property that distinguishes Tony from other triangles. But I do not see that any other candidates will fare better when it comes to arguing that they are intrinsic. I conclude, then, that relationalism cannot save the thesis that shapes are intrinsic.

8 Conclusion

This concludes my survey of the theories of shape properties. None of the acceptable ones save the intuition that shapes are intrinsic; so I do not know how to save this intuition.

Although I cling to my intuition that shapes are intrinsic, it is worth exploring what we might say if we do decide that this intuition is wrong. If, driven by philosophical argument, we deny that shapes are intrinsic, is there any way to explain any the intuitions to the contrary? I can see the outlines of an explanation of why we have these intuitions that someone who does reject them might give:

> Before we've done the metaphysics, we think that (two-place) distance relations like *the distance from x to y is three* are fundamental. And this leads us to think that shapes are intrinsic: since shapes are analyzable in terms of these relations, if these relations are fundamental then shapes are intrinsic. But there are good arguments that relations like *the distance from x to y is three* are not fundamental, and all other choices of fundamental spatial relations render shapes extrinsic. Our intuitions, then, are based on a faulty presupposition about which spatial relations are fundamental.

I do not endorse this explanation, but it is one that I now take more seriously.[29]

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