The Metaphysics of Quantities and Their Dimensions*

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1 Introductory

Quantities have dimensions. Force, mass, and acceleration are quantities — for now let this just mean that they may be faithfully measured by numbers —, and the dimension of force is "ML/T^2," the dimension of mass is "M," and the dimension of acceleration is "L/T^2," where "M" stands for mass, "L" for length, and "T" for time, or duration. While the notion of a quantity is familiar to those with only a passing acquaintance with physics, the notion of a quantity's dimension is probably not.[1] Even though they are unfamiliar, dimensions play a variety of important roles in physics, and science generally. Perhaps the most important one is the role they play in the technique of dimensional analysis. One of the aims of physics is to discover laws, and laws of physics usually take the form of relations between quantities — Newton's second law, $F = ma$, is a relation between the quantities force, mass, and acceleration. Newton's discovery of this law involved, in part, consulting a lot of observational evidence. Dimensional analysis is a different technique for discovering law-like relations between quantities, one that doesn't involve inferring a generalization from a collection of instances. If you know only that some law or other relates force, mass, and acceleration, but you also know the dimensions of these quantities, and a few other facts, you can use dimensional analysis to conclude

*To appear in *Oxford Studies in Metaphysics* Volume 10.

[1]The notion of the dimension of a quantity has nothing to do with the familiar notion of a dimension of space.

that the law is $F = ma$, even if you don't have any of the data that Newton did. How to use the technique won't matter in what follows.[2] The point is that dimensions can help answer important physical questions. But should metaphysicians care about dimensions? Is the fact that the dimension of acceleration is L/T^2 in any way a metaphysically interesting fact about it? Does this fact, for example, have anything to do with the nature of acceleration?

The physicist R. C. Tolman thought it did; he wrote in 1917 that a statement of the dimension of a quantity is "a shorthand restatement of its definition and hence [is] an expression of its essential physical nature" (quoted in Bridgman 1922: 26). To say that the dimension of acceleration is L/T^2 is certainly to say that there is some connection or other between acceleration and length and duration. Tolman here says that the connection has something to do with the *essence*, or *nature*, of

[2]For the record, and simplifying a little: to apply dimensional analysis we need to know that the force on a body is some function of its mass and acceleration, so that $F = g(m, a)$ for some function g, and we need to know that the function g makes this equation "dimensionally consistent." An equation is dimensionally consistent iff the dimension of the quantity on the left-hand side is the same as the dimension of the combination of quantities on the right. Assuming we know these things, we can reason: force has dimension ML/T^2, so $g(m, a)$ must be a combination of mass and acceleration that also has dimension ML/T^2. Clearly if g is multiplication, so $g(m, a) = ma$, we get this result, since the dimension of ma is the product of the dimension of m and the dimension of a, namely ML/T^2. The Buckingham Π theorem tells us that, in this case, multiplication is the only function you can put in for g to make the equation dimensionally consistent. (For a more detailed explanation of dimensional analysis, including a proof of the Π theorem, see Barenblatt 1996. Technically, applying the technique requires a weaker assumption than dimensionally consistency, sometimes called "dimensional homogeneity"; for more on this distinction see Lange 2009, p. 760.)

Although I have contrasted dimensional analysis with the use of observational data, dimensional analysis is not an a priori technique; the things one must know to apply it, namely the dimensions of the quantities and that the quantities stand in a dimensionally homogeneous relationship, cannot be known a priori. In fact, regarding the example I have used, the usual route to coming to know that the dimension of force is ML/T^2 requires prior knowledge of the law that $F = ma$; those of us who know the dimension of force in this way obviously could not use our knowledge to discover that $F = ma$.

acceleration. Could that be right? It seems right to me, at least initially, but it has had powerful opponents. Percy Bridgman, who won the Nobel Prize in physics in 1946, scorned views like Tolman's, scorn that comes across in his description of those views:

> It is by many considered that a dimensional formula has some esoteric significance connected with the "ultimate nature" of an object, and that we are in some way getting at the ultimate nature of things in writing their dimensional formulas. (1922: 24)

Bridgman went on to give several arguments against the idea that dimensional formulas have "some esoteric significance," or, as we might put it, that they have some metaphysical significance. His view was that dimension formulas just reflect arbitrary choices we have made when setting up our scales of measurement.

I'm going to examine this debate. The idea that the dimension of a quantity has something to do with its nature can be made precise in different ways; I will identify two distinct ways to make it precise, and defend one of them.

This debate is not one that philosophers discuss, or have ever discussed. At least, I have not been able to find anything written on it.[3] The question is, neverthe-

[3]There are certainly articles in philosophy journals that discuss dimensional analysis, for example Marc Lange's "Dimensional Explanation" and Luce's "Similar Systems and Dimensionally Invariant Laws." But I haven't found any that discuss the nature of dimensions, or what dimensions say about the nature of quantities.

This is not to say that philosophers have ignored questions about the nature of quantities. In fact some questions about the nature of quantities are increasingly a focus of attention. Take the property of being 2kg in mass. Is this a fundamental, or perfectly natural, property? If it is non-fundamental, what *are* the fundamental mass properties or relations, and how is *being 2kg mass* defined in terms of them? Mundy (1987) can be understood as holding that the property of being 2kg mass, and all other particular mass properties, are fundamental, while Field's views about quantities (Field 1980) suggest that it is the "mass-less" and "mass-congruence" relations that are fundamental (intuitively, *a mass-less b* holds iff a is less massive than b, and *a, b are mass-congruent to c, d* holds iff the difference in mass between a and b is the same as the difference in mass between c and d). There are other views also (surveyed in Eddon 2013). I think that these questions about the nature less, interesting, worthy of our attention. I think this is obvious; further evidence comes from the fact that physicists like Tolman and Bridgman saw that dimensional analysis raised questions about the metaphysical import of talk of the dimension of a quantity, and proceeded to take those questions up. For this reason my primary interlocutors will be those physicists.

If you have never before been exposed to talk of the dimension of quantity, either through exposure to dimensional analysis or through some other avenue, you might feel that you are in no position at all to think about this debate. It's a little like asking someone who has had no exposure to physics to evaluate hypotheses about the essential properties of electrons. Fortunately, there is widespread agreement about part of the answer to the question of what a quantity's dimension tells us about it. The debate is over whether a quantity's dimension tells us any more than that. (Tolman's claim that it tells us something about the quantity's nature go beyond the agreed-on part.) Seeing the agreed-on claims about dimensions will, I hope, help the uninitiated get a better grip on what the debate is about. So that is what I will start with, in the next section.

2 Quantities, Scales, and Dimension Formulas

Dimensions are had by quantities; so I should start by saying a bit more about what a quantity is. Familiar examples are, again, force, mass, and acceleration, and also velocity, momentum, and energy. I will take a quantity like mass to be a family of properties, the "specific values" of that quantity. So the mass family includes the properties, or values, *having 1 kg mass, having 2 kg mass,* and so on.

There is a distinction between "vector" quantities and "scalar" quantities: all quantities have "magnitude," but vector quantities also have a "direction" associated

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of quantities are orthogonal to the question I take up, the question of what dimensions say about the nature of quantities. For these questions, the questions Mundy's and Field's and others' work is relevant to, are questions "internal" to a given quantity. The question they want to answer about mass—the question of which mass properties or relations are most fundamental—is a question just about mass, not about its relation to any other quantities. But the question I'm asking, the question of what the dimension of a quantity says about its nature, is a question about the relations between that quantity and other quantities, as we will see.

with them. Although many quantities of interest are vector quantities (force and acceleration for example), for the sake of simplicity I will limit my attention to scalar quantities, and often pretend that some vector quantities like force are scalar quantities.

An important notion for us is that of a scale of measurement of a quantity. A scale associates a number with each value of that quantity; the kilogram scale, for example, associates the number 2 with the property of having 2 kg mass. Now of course there are tons of ways to associate numbers with mass values, many of them useless — it would be useless to associate the number 3 with every mass value. What I care about are "faithful" scales, scales that adequately reflect the intrinsic structure of the quantities they are scales for. I won't go into what that intrinsic structure is in a great deal of detail; it will be enough for our purposes to note that the property of having 4 kg mass is, in some sense, "double" the property of having 2 kg mass, and that it is in virtue of this that a scale is faithful only if the number it assigns the first property is double the number it assigns the second. In general, just as the ratio of the property of having 4 kg mass to the property of having 2 kg mass is 2, any two values of a given quantity stand in a definite numerical ratio, and a faithful scale will assign to those values numbers that stand in the same ratio.

It follows that to set up a scale for measuring a quantity it is enough to choose a unit — the value v of that quantity that shall be associated with the number 1. The number the scale assigns to any other value u is then determined by the ratio of u to v: if the ratio of u to v is n, then the scale must assign n to u, if it is to be faithful.

It follows, further, that any two faithful scales for a given quantity, like mass, differ from each other by a positive multiplicative constant: if we write $S(v) = n$ to mean that scale S assigns quantity value v the number n, then if S_1 and S_2 are faithful scales for a quantity, there is a fixed positive number K such that $S_2(v) = KS_1(v)$ for every value v.[4] For example, the numbers the centimeter scale assigns to

[4] I should note that this claim only applies to scales for quantities with a certain structure, which might be described as "scalar quantities with a ratio structure and a lower bound." These are the only kinds of quantities I will concerned with in this paper.

You might worry that temperature is a counterexample to the claim I make in the text about scalar quantities. The numbers the Kelvin scale assigns to temperatures lengths are all 100 times larger than the numbers the meter scale assigns to lengths.

Now at the beginning of this paper I said what the dimensions of some quantities (mass, acceleration) were, and quoted Tolman's claim that the dimension of quantity has something to do with its nature or definition. This talk of the "dimension" of a quantity is, however, a little nebulous, and it is time now to introduce more precise terminology. The term we need is "the dimension formula of a quantity." Formulas are in the same ontological category as sentences. The dimension formula of mass is "M," a capital "m," and the dimension formula of acceleration is "L/T^2," a capital "l" followed by a slash etc. Just as we can ask what a sentence means, we can ask what a formula means. The most uncontroversial claim about dimension formulas says this about their meanings:

(1) The dimension formula of a quantity defines that quantity's *dimension function*.

Great; so what's a dimension function? Here is the answer:

(2) A quantity's dimension function tells you how the numbers assigned to values

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are not a fixed multiple of the numbers the Celcius scale assigns to temperatures (the temperature value to which Celcius assigns the number 0, Kelvin assigns 273.15, but there is no positive K with $0 = K \cdot 273.15$). This is not inconsistent with my claim, though, since the Celcius scale is not faithful.

The short explanation of why it is not faithful is that it fails to adequately reflect that temperature has a lower bound — to do so it would have to assign 0 to the lower bound. The longer, and true, explanation is more complicated. In fact temperature does not have a lower bound. It is possible for a thermodynamic system to have a negative temperature on the Kelvin scale. Still, even though temperature does not have a lower bound, the Celcius scale is not faithful — and neither is the Kelvin scale. Systems with negative temperatures (on the Kelvin scale) are *hotter* than systems with positive temperatures (on the Kelvin scale) — the first kind of system is disposed to transfer energy to the second kind. But faithful scale should assign higher numbers to hotter things (this is a problem for the Celcius scale, since it assigns negative numbers to the temperatures to which the Kelvin scale assigns negative numbers).

Since temperature is not, in the end, a quantity with a lower bound, it technically falls outside the scope of my claim that any two scales for a quantity differ by a positive multiple.

> of that quantity change, when you change from using one *system of scales of measurement* to another (in the same class).

This needs a lot of unpacking, of course; we will want to know what a system of scales is, what it is to for two systems to be in the same class, and how the dimension function tells you the cited fact — we will want to know what the function's inputs and outputs are. But (1) and (2) at least makes one of the dimension formula's roles clear. If we switch from using the meter scale for length to using the centimeter scale for length, the numbers assigned to length values all go up by a factor of 100; somehow this function that the dimension formula for length defines, the dimension function for length, is going to deliver up to us this factor.

To a philosopher, the job-description in (1) that dimension formulas answer to may not look very interesting. The question is going to be, the interesting debate is going to be over, whether there is any other, metaphysically interesting, job that dimension formulas do.

Before getting to that question I should spell out how dimension formulas define dimension functions, and how dimension functions work.[5] A *system of scales of measurement* is a set of related scales for measuring a set of quantities. A system of scales designates some quantities as "primary" and some as "secondary." The primary quantities are those measured by scales in which the unit is not chosen for its relation to the unit of any other scale. The secondary quantities are those measured by scales in which the unit is chosen for its relation to the units of other scales. For example, we usually think in terms of a system in which speed is a secondary quantity, and length and duration are primary. The unit for speed is chosen by reference to the units for length and duration: we say that the unit for speed is the speed at which something travels when it covers one unit of distance in one unit of time.

That's one way to choose a unit for speed, given units for other quantities. It is not the only way — we could choose the unit to be the speed at which something travels when it covers *two* units of distance in one unit of time. In general, there is

[5]Much of the material to follow comes from chapter 1 of (Barenblatt 1996), though my terminology is in some places slightly different.

always more than one way to choose a unit for a secondary quantity, given units for the primary quantities.[6]

To set up a system of scales, then, we divide the quantities we will measure into those that will be primary and those that will be secondary, select units for the primary quantities, and specify how the units for the secondary quantities are determined by the units for the primary ones.

Suppose two systems of scales X_1 and X_2 measure all the same quantities, that the set of quantities designated as primary is the same in both systems, and that the units for secondary quantities are defined by reference to the units for primary quantities in the same way. So all differences between X_1 and X_2 flow from differences over which values of the primary quantities are the units. Then X_1 and X_2 belong to the same *class* of systems.

Again, when we change from using a scale S_1 for a given quantity Q to using a scale S_2, the numbers assigned to values of Q all get multiplied by some constant K. If S_1 and S_2 belong to systems of scales that are in the same class, then even if Q is a *secondary* quantity, the constant K is determined by the scales for the *primary* quantities:

(3) If S_1 (the "old" scale) and S_2 (the "new" scale) are scales that measure the same quantity Q, and they belong to systems of scales X_1 and X_2 that are in the same class, then the constant K satisfying $S_2(v) = KS_1(v)$ (where v is a value of Q) is a function of the ratios of the old units to the new units of the scales for the **primary** quantities.

This function, which takes some numbers (the ratios of the old units to the new units) and outputs K, is the dimension function of Q.

To know how the dimension function is defined we look to the dimension formula for Q (recall (1)), and interpret the letters that appear in the dimension

[6] Here is a more interesting example: suppose we want force to be a secondary quantity, and length, mass, and duration to be primary. The usual way to choose a unit for force is to say that it is the amount of force required to produce unit acceleration in something with unit mass. But we could choose a unit for force by saying that the unit force is the strength of the gravitational force between two bodies of unit mass that are one unit of distance apart.

formula to denote the numbers that are the ratios of the old units to the new units of the relevant quantities (and interpret the other symbols in the dimension formula, like "/," to denote the usual mathematical operations). To see how all this works, let us assume that we are working in a class of systems of scales in which only length, duration, and mass are primary, and in which the unit for speed, a secondary quantity, is defined in the usual way. In this case, there are three primary quantities, and so, since dimension functions take as input the ratios of old to new units of the primary quantities, all dimension functions take three arguments. The dimension formulas that define these functions contain the letters "M," "L," and "T," and no other letters; when a dimension formula is used to define a dimension function, "M" is interpreted to name the ratio of the old unit for mass to the new unit, "L" the ratio of the old unit for length to the new unit, and "T" the ratio of the old unit for duration to the new unit. When Q is a quantity, $[Q]$ is its dimension function, and the conversion factor K is given by $K = [Q](M, L, T)$, the application of the function $[Q]$ to the arguments M, L, T.[7,8] This may be clearer if I work through some examples:

> Example 1: Mass. The dimension formula for mass, at least in the class of systems of scales we are using, is, again, "M." So $[\text{mass}](M, L, T) =$

[7]I should note that the claim that the conversion factor for a quantity in this class is a function, in the mathematical sense, of three arguments, is not the same as the claim that the conversion factor for a given quantity *depends on* each of those arguments. The function defined by the expression $f(x, y, z) = x^2$ is a function of three arguments, but its value only depends on the first argument. The examples below illustrate this distinction.

[8]I said that everything in this section is common ground in the debate over the nature of dimensions. This may not quite be right. Bridgman says that on his view "the symbols in the dimensional formula [are] reminders of the rules of operation which we used physically in getting the numerical measure of the quantity," they do not represent "the facts used in changing from one set of units to another" (30). But he also says that his view "cannot be distinguished" from the one I am describing "as far as any results go." I am not entirely sure what Bridgman means when he says we should regard dimension formulas as reminders. Barenblatt clearly endorses the claim that dimension formulas just serve to define dimension functions, and that the letters "M" and so on denote ratios of old units to new (1996: 31-32).

M, and M is the number that is the ratio of the old unit for mass to the new unit for mass. So suppose we are changing from a system of scales that uses grams to measure mass, to a system that is otherwise the same except that it uses kilograms to measure mass. The old unit for mass then is the gram, and the new unit is the kilogram. The ratio of the gram to the kilogram is $1/100$ (a gram is $1/100$th of a kilogram). So in this case $M = 1/100$. Then if the old scale, the gram scale, assigns the number n to a mass value v, we compute that the new scale, the kilogram scale, assigns the number $[\text{mass}](M, L, T)n = Mn = (1/100)n$ to v. The result is familiar: the numbers assigned to masses change by a factor of $1/100$ when we change from using grams to using kilograms.

Example 2: Speed. The dimension formula for speed, at least in the class of systems of scales we are using, is "L/T." So the dimension function for speed is defined by the equation $[\text{speed}](M, L, T) = L/T$. If we change from using meters to using centimeters, and use the same scales for the other primary quantities, then L, the ratio of the old unit for length to the new unit, is equal to 100, while $M = T = 1$. So if the old scale for speed assigns the number n to a speed value v, we compute that the new scale assigns the number $(L/T)n = 100n$ to v. Again the result is familiar: the numbers assigned to speeds change by a factor of 100 when we change from using the "meters per second scale" to using the "centimeters per second" scale.

One thing about dimension functions is worth emphasizing. The dimension function of a quantity gives the factor by which the scale for measuring it changes when we pass from one system of scales to another system *in a given class of scales*. So dimension functions, and dimension formulas too, are class relative.

For example, we usually take force to be a secondary quantity, and mass a primary one; the dimension formula for force is then ML/T^2. But we could instead use a class of systems of scales in which mass is secondary, and force primary; the dimension formula for force is then just F, while the dimension formula for mass is

FT^2/L.[9] The class-relativity of dimension formulas (and dimension functions) will be important in the next section.

3 The Positivist Theory of Dimensions, and its Opponents

Here are the teams. On the one side, Tolman and others, who think that the dimension of a quantity has something to do with its nature. On the other, Bridgman and his allies, who deny this. I will call the view of Bridgman and his allies The Positivist Theory of Dimensions;[10] to have it set out explicitly, if a bit vaguely, the view is this:

> The Positivist Theory: The dimension formula of a quantity defines the dimension function of that quantity (relative to some class of systems of scale). It does not (in addition) have anything to do with the nature or essence of that quantity.

So what, exactly, do those who oppose the positivist theory believe? I will distinguish between two distinct anti-positivist theses. These theses in turn share two presuppositions, so before stating the theses I want to articulate those presuppositions.

The first presupposition is that there is a "natural" way to divide quantities

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[9] To figure this out, we assume that the dimension of the right-hand side of $F = ma$ is the same as the dimension of the left-hand side. Acceleration is still secondary, and its unit still chosen in the customary way, so its dimension formula is still L/T^2. So the dimension formula Z for mass must satisfy $F = Z \cdot L/T^2$, where what can go in for Z are products of powers of the primary quantities L, T, and F. The only solution is FT^2/L. (Why products of powers? I say something about this in the appendix.)

[10] The appearance of "positivist" is meant to be suggestive — Bridgman was an operationalist, the worst kind of positivist — but should not be taken too seriously. Other proponents include Max Plank: "to inquire into the 'real' dimension of a quantity has no more meaning than to inquire into the 'real' nature of an object" (1932: 46; see also page 8); Barenblatt (1996); (Langhaar 1951: 5); and (Palacios 1964: xiv). These people may not all subscribe to the positivist view exactly as I state it, but my interest is in the view I state, not the interpretive question of whether, and if so just where, its allies' actual views deviate from it.

into primary and secondary. Return to the two systems of scales I ended the last section with. One designated mass as primary and force as secondary, the other did the reverse. These systems belong to different classes (for, again, a class of systems of scales is partly determined by a choice of which quantities are to be regarded as primary and which secondary). Now there are various criteria we might use to evaluate these classes of scales. One of them might, for example, be better than another if we are intersted in making certain kinds of measurements. Anti-positivists think that we can also evaluate them with respect to whether they designated as primary only quantities that *really are* primary. The presupposition here is this:

> Non-Relativity: Quantities are not just primary, or secondary, relative to this or that class of systems of scales; they are also primary, or secondary, in a non-relative way.

The positivist theory rejects Non-Relativity. However, despite the positivist theory's opposition to it, I think Non-Relativity is a quite plausible thing to believe. Non-Relativity can appear even more tempting if we use a different word in place of "primary": doesn't it seem right to say that some quantities are *fundamental* in a non-relative way?

The idea that some *properties* are fundamental and others are derivative has great currency in metaphysics.[11] I certainly think we should accept it. Since quantities are just families of properties, we should also accept that some quantities are fundamental and others derivative.[12] This gives us a way to "argue" for Non-Relativity. Some quantities are fundamental, and a quantity is primary simpliciter[13] iff it is fundamental; so some quantities are primary simpliciter. I don't

[11] Its popularity among contemporary metaphysicians traces back to (Lewis 1983).

[12] There are details to work out about the relationship between "fundamental" as a predicate of properties and "fundamental" as a predicate of quantities. We could say that a quantity is fundamental iff every one of it values is a fundamental property; or iff the disjunction of its values is a fundamental property; and there are other options. I will not try to work out these details here.

[13] "Primary simpliciter" is just a short way to write "primary in a non-relative way."

expect this argument to convince any skeptics — that's why "argue" is in scare-quotes — but it at least draws a connection between being primary simpliciter and a notion, fundamentality, that is already well understood.[14]

I do not mean to suggest that Non-Relativity is a completely unproblematic thesis. If we accept that some quantities are primary simpliciter, we are going to want to know which quantities those are. How do we go about figuring this out? *Can* we know which quantities are primary simpliciter, or are we bound to remain ignorant?

I will only say something brief about these questions. The same questions arise with respect to fundamental properties. In that case, one common suggestion is that we take our best physical theories as guides to which properties are fundamental — if a property appears in one of those theories (is expressed by a predicate in one of those theories), that is good evidence it is fundamental. To the extent that this is a good answer to the epistemic questions about fundamental properties, an analogous suggestion is a good answer to the epistemic questions about primary quantities: if physicists, when their aims involve no practical computations, but

[14] A referee objected: Gross Domestic Product (GDP) is surely not a fundamental quantity; so I must hold that it is a secondary quantity, and therefore that its dimension formula, in a class of systems of scales that designates as primary the quantities that really are primary, is some combination of "M," "L," "T," and so on (assuming that mass, length, and duration are primary). This, however, is implausible.

I am not entirely sure what to think about GDP, or other quantities that appear in economics and other special sciences, but I am tempted by this reply. Consider the quantity frequency. The unit for frequency is chosen to be the value of frequency had when the relevant system completes one "cycle" in one unit time. But "cycles" is not a quantity: one does not choose a unit for measuring cycles, one just counts cycles. That's why the dimension formula for frequency contains "T" (the formula is "$1/T$") but no letters that have anything to do with cycles. The same, I suspect, goes for GDP. We (in the US) usually measure GDP in dollars per year, but (my suspicion is) we should not regard the dollar as a unit for measuring value, but as a thing we count, like cycles. (If the dollar and the euro were both units for measuring value, then one dollar should always be some fixed number of euros. But it's not; the exchange rate fluctuates.) If this is right, then it is not after all implausible that GDP's dimension formula contains only letters for fundamental physical quantities: its dimension formula is the same as that of frequency, "$1/T$."

only the formulation of the laws, use a system of scales in which a quantity is primary, that is good evidence that that quantity is primary simpliciter.

A lot more could be said about the epistemology of primary quantities, and how it does or does not recapitulate the epistemology of fundamental properties, but since these epistemic questions are not relevant to what follows, I will leave the topic here.

Anti-positivists presuppose Non-Relativity because they presuppose that each quantity has a "true" dimension formula, and a quantity cannot have a "true" dimension formula unless Non-Relativity holds. For recall that a given quantity has tons of dimension formulas, one for each class of systems of scales that measures that quantity. One of those dimension formulas can only be its "true" dimension formula if nature singles out exactly one class of systems of scales as the "true" class; a quantity's true dimension formula is then its dimension formula in the true class. What would nature have to do to single out exactly one class of systems of scales as the true class? It certainly must single out one set of quantities as those that are primary simpliciter, so it can say that the true class has the property of designated as primary all and only the quantities that are primary simpliciter.

But it turns out — and I kind of wish this weren't so — that accepting Non-Relativity is not sufficient for defining "true class," and so is not sufficient for defining "true dimension formula." As I mentioned above, a class of systems of scales is determined, not just by a choice of which quantities to designate as primary, but also by a way of choosing the units of the secondary quantities, given units for the primary quantities. And there is always more than one way to choose a unit for a secondary quantity. So anti-positivists need a second assumption,

> Right Units: for any quantity that is secondary simpliciter, only one way of choosing a unit for that quantity given units for the primary quantities is the "right" way.

If Right Units is correct, then the true class of systems of scales can be defined as the class that designates as primary exactly the quantities are are primary simpliciter, and chooses the units for the secondary quantities in the right way.

Right Units is, I think, harder to swallow than Non-Relativity. We usually choose the unit for speed to be the value had by something that moves unit distance in unit time. But, as I said earlier, we could choose it to be the value had something that moves two times the unit distance in unit time. Which is the "right" way for defining a unit for speed? Some might find it hard to believe that this question has an answer. Different people might find it hard for different reasons; one reason might be that, if it does have an answer, it is hard to see how we could know what that answer is. This particular example, however, doesn't lead me to doubt Right Units; the usual choice for a unit of speed, the choice that designates as the unit the speed something has when it moves unit distance in unit time, is, at least in some sense, more natural than any alternative. But other examples are harder.[15] Still, I will not take an in-depth look at the cases for and against Right Units here. I will say, though, that one of the theses I will be interested in, which I will shortly name Definitional Connection, can get by without assuming Right Units, even though the most natural statement of if does require that assumption. (I will explain how it can avoid Right Units in section 6.) So Definitional Connection is still something you could believe, even if you can't stomach Right Units.

Having flagging that the anti-positivist theses to be discussed make presuppositions which could easily be challenged, let's press on. Tolman's idea was that the dimension of quantity has something to do with its essential nature, that it is a "shorthand restatement of its definition." Now Kit Fine famously linked essences

[15]It is tempting to say that the right way for choosing a unit for a secondary quantity never uses "arbitrary constants" (as the second way for choosing a unit for speed uses the number 2). But this is not the end of the story. Some quantities, like the curl of a vector field, cannot be defined in terms of more fundamental quantities in any way without using something like an arbitrary constant (in this case, an arbitrary choice of an orientation for space). One thing to say about this case is that curl is not a scalar quantity, so is of a kind of quantity that I am ignoring. A more substantive response is to say that curl is not a "real" quantity, since from a four-dimensional perspective one never needs to use the curl of a vector field to do physics. (Given the theory of relativity, speed is not a real quantity either, but my argument in the paper is compatible with this; speed, even in section 5, serves only as an example.) Further problems come from very simple quantities. Here are two ways to select a unit for area: let it be the area value had by a square with unit-length sides; let it be the area value had by an equilateral triangle with unit-length sides. Which is "right"? Neither uses an arbitrary constant. If I had to take a stand, I would say that the square is right, but I don't know how to justify this answer.

to definitions in his 1994 paper "Essence and Modality." The essence of a thing is its "real definition"; it says "what a thing is," as a "nominal" definition says what a word means. I will read Tolman's reference to a quantity's "definition" as talk of its real definition, and so also as talk of its essence. For the sake of precision it is worth regimenting all such talk using the same locution, so I will use the operator "It is definitional of X that..." Using this notion, just what does Tolman mean when he says that the dimension of a quantity is a restatement of its definition? For now I will only give a vague answer; I will take up the task of making it more precise in section 5:

> Definitional Connection: Let P1,...,Pn be the symbols for the primary quantities that appear in quantity Q's dimension formula in the true class of systems of scales. Then some relationship between Q and the primary quantities associated with the symbols P1,...,Pn is definitional of Q.

That's very abstract, so it will help to see an instance. Suppose the speed is a secondary quantity, and length and duration are primary, and that the dimension formula for speed in the true class is "L/T." Then, if Definitional Connection is right, some relationship between speed, length, and time is definitional of speed. Just what this relationship might be, I will say more about in section 5.

Before spending more time on Definitional Connection, I want to get another anti-positivist idea about dimensions on the table. The other one is, in fact, more popular. It is suggested by another expression of anti-positivism, due to W. W. Williams:

> The dimensional formulae may be taken as representing the physical identities of the various quantities, as indicating, in fact, how our conceptions of their physical nature...are formed, just as the formula of a chemical substance indicates its composition and chemical identity....The question then arises, what is the test of the identity of a physical quantity? Plainly it is the manner in which the unit of that quantity is built up (ultimately) from the fundamental units L, M, and T [...]. (Quoted in Bridgman 1922: 26; the paper quoted from was published in 1892)

Some of what Williams says is close to Definitional Connection. But Williams also draws an analogy between dimensional formulas and chemical formulas. The idea seems to be that, just as the chemical formula for glucose, "$C_6H_{12}O_6$," tells us that a glucose molecule is built out of six carbon atoms, twelve hydrogen atoms, and six oxygen atoms, the dimension formula for acceleration, "L/T^2," tells us that, and how, acceleration values are "built up from" lengths and durations. This idea identifies a second role dimensional formulas might play, in addition to that of defining dimension functions.[16] Generalizing gives us our second anti-positivist thesis:

> Construction: Let Q be any quantity, and D its dimension formula in the true class of systems of scales. If Q is non-primary, then D exhibits the way in which values of Q are constructed, or built out of, values of primary quantities.

The picture here is that there are several different ways in which two quantity values may be combined to create a third. Two quantity values may be multiplied together; or one may be divided by the other; or one quantity value may be raised to some power. For example, if we assume that speed is a secondary quantity in the true class, and that length and duration are primary, then the dimension formula of speed in the true class is "L/T." In this formula the symbols for length and duration flank the division sign. According to Construction, this indicates that speed values are constructed by taking length values and dividing them by duration values. The meter per second, for example, which is a speed value, is constructed by dividing the meter (the value of length assigned the number 1 when we choose the meter as our unit) by the second.

It is worth emphasizing that Construction says that some quantity *values* — properties — are built, by operations with names like "multiplication" and "division," out of other *values*. Some resistance to Construction is based on failing to

[16] A formula can define a function without indicating how anything is built up from other things: the formula "$x - y$" defines the subtraction function, but even though $12 = 10 - 2$, the formula doesn't indicate that 12 is built up from 10 and 2 in any sense. Conversely, chemical formulas indicate how molecules are built from atoms, but do not define any functions.

appreciate this fact. Bridgman complains of "treat[ing] the dimensional formula as if it expressed operations actually performed on physical entities, as if we took a certain number of feet and divided them by a certain number of seconds" (29). Later on the page he asserts that "We cannot perform algebraic operations on physical lengths, just as we can never divide anything by a physical time" (29).[17] If Bridgman is saying that the following instruction makes no sense, then he is absolutely right:

> You've got a meter stick (which is a "physical entity") in your left hand. Please take it and divided it by the stretch of time during which this table exists.[18]

Division just is not an operation defined on meter sticks and stretches of time. But this observation does not touch CONSTRUCTION, which makes no claims or presuppositions about dividing physical things by each other.

One could try to make Bridgman-like observations about quantity values. Pounding the table one could say, "Multiplication is an algebraic operation defined on numbers, not on quantity values!" But this is only kind of true. Mathematicians happily talk of multiplying functions together, and of dividing one group by another. Of course the operations going under the names "multiplication" and "division" here are different from those going under those names when we are working with numbers. But the operations are closely enough related that we use the same name for them. I see no reason why a set of quantity values could not have an algebraic structure like that had by sets of numbers, functions, and groups, so that there were operations on sets of quantity values that deserved to be called multiplication and division.

We now have two anti-positivist theses on display, DEFINITIONAL CONNECTION and CONSTRUCTION (which share the presuppositions NON-RELATIVITY and RIGHT UNITS). The first thing I want to emphasize about them is that they are different theses. Granted, there is probably a way of understanding talk of one property being

[17]Barenblatt echoes this argument (1996: 30).

[18]Suppose substantivalism about time is true, so that there are such entities as stretches of time.

constructed out of others on which Definitional Connection, or something quite close to it, follows from Construction. But Definitional Connection does not entail Construction; it is logically weaker. You can think that dimension formulas tell us something about the natures of quantities without thinking that quantities are constructed out of other quantities.

My main aim in this paper is to say something in favor of Definitional Connection. Distinguishing between Definitional Connection and Construction is an important part of my argument, since many of the arguments proponents of the positivist view give make contact only with Construction, and leave Definitional Connection untouched.

Before discussing Definitional Connection explicitly I want to look at the best of those arguments, partly to illustrate its irrelevance to Definitional Connection, but also because it is worth, at least briefly, coming to some conclusion about Construction. My conclusion will be that it is defensible, though I myself see no good motivation for defending it.

4 Building Some Quantities From Others?

One argument against Construction is especially popular; I myself have endorsed it in the past.[19] Here is a statement of it from Barenblatt's book *Scaling, Self-Similarity, and Intermediate Asymptotics*:[20]

> [I]f the relations for the derived units [that is, units for the secondary quantities] mentioned above were actually to make sense as products or quotients of the fundamental [that is, primary] units, they would have to be independent of what we mean when carrying out the multiplication or division. For example, according to the above definition [which I will not quote, but which entails something like Construction], kgf m is the derived unit for the moment of a force as well as for work; m^2/s is the derived unit for the stream function as well as the kinematic vis-

[19] In my (2012), and section 7.4 of my (2015).

[20] Something like this argument also appears in (Palacios 1964: xiii) and (Duncan 1953: 123).

> cosity, etc. But it is not implied that the stream function is measured in multiples of a basic amount of kinematic viscosity or that the moment of a force is measured in multiples of a basic amount of work! In contrast, using our definition [of dimension formulas, namely the positivist theory], the fact that the dimensions of two physical quantities of different nature are identical does not seem unnatural. (1996: 33)

I am not entirely sure what Barenblatt's argument here is. But I can find *an* interesting argument in here. Let's examine it and not worry about whether it is exactly what Barenblatt had in mind.

Barenblatt's central observation is that there are distinct quantities with identical dimension formulas (in some given class of system of scales). He mentions kinematic viscosity and the stream function. Roughly speaking, the kinematic viscosity of a fluid is a measure of how hard each bit of the fluid "pulls" against its neighbors when its neighbors move past it. In a highly viscous fluid, when a bit of fluid neighboring *B* moves past it, *B* pulls rather hard against that neighbor. (Honey has higher viscosity than water. As a practical matter, how viscous a fluid is tells you how hard it will be for you to move through it: as you move you pull the bits of the fluid that touch you along with you, and this is more difficult to the extent that those bits' neighbors pull back.) As for the stream function: if you imagine a fluid (technically an incompressible fluid) flowing in two dimensions, the value of the stream function at a point *P* tells you how much fluid flows across the line from *P* to an arbitrarily given point *A* in unit time. Clearly the stream function and kinematic viscosity are distinct quantities. A blob of honey, all at the same temperature, has the same viscosity everywhere. The stream function, on the other hand, is a property, not of a thing like a blob of honey, but of an event, the motion of a fluid. It varies from one point in the flow to another. Nevertheless, in any class of systems of scales that designates length and duration as primary quantities (and designates the stream function and kinematic viscosity as secondary), the stream function and kinematic viscosity have the same dimension formula, L^2/T.

Why might this be trouble for Construction? Here is one way to spell out a simple argument against Construction that uses Barenblatt's observation as a premise:

(P1) If Construction is true, then there cannot be distinct quantities with the same dimension formula (in the true class of systems of scales).

(P2) But there are distinct quantities with the same dimension formula.

(C) Therefore, Construction is false.

A reason to believe (P1) is not hard to find. If Construction is true then values of kinematic viscosity are built out of values of length and time. Specifically, if Construction is true, then you get a value of kinematic viscosity by taking a value of length, multiplying it by itself, and then dividing the result by a value of duration. But if performing these operations on values of length and duration gives you a value of kinematic viscosity, then performing those operations on those same values cannot *also* give you a value of a distinct quantity like the stream function. In general: distinct quantities with the same dimension formula would have to be built up from the primary quantities in the same way. But it cannot happen that distinct quantities are built up from the same primary quantities in the same way.

This might sound convincing, but it goes beyond what Construction officially says. There are two ways a proponent of Construction may resist (P1).

First Way. Construction does not say that it is impossible for distinct quantity values to be built up from the same basic values in the same way. The argument above for (P1) smuggled in an extra premise, a "uniqueness" premise. Here is one way to put it:

> Uniqueness: For any way W of building a quantity value out of other quantity values $q1, \ldots, qn$, the application of W to $q1, \ldots, qn$ is unique.

If Uniqueness is true, and if multiplication and division are ways of building quantity values, then we do have a good argument for (P1). But Uniqueness may be rejected. True, multiplication and division of numbers satisfies uniqueness. But there is plenty of precedent for non-mathematical analogues of mathematical operations to fail to have all the properties of their mathematical counterparts. Take summation. Lots of metaphysicians happily embrace the thesis that a collection of material things may have more than one sum. Plenty think that you can have a bunch of clay particles that have two sums: one of the sums is a lump of clay, the other sum is a statue. The observation that distinct quantities can have the same dimension formula, then, does not refute CONSTRUCTION.[21]

Second Way.[22] Barenblatt's argument may refute CONSTRUCTION when it is interpreted one way, but there is a better way to interpret it that escapes the argument. Officially CONSTRUCTION just says that Q's dimension formula "exhibits the way in which values of Q are constructed, or built out of, values of primary quantities." I have been interpreting this to mean that, if a quantity (like viscosity) has for its dimension formula "L^2/T" then each of its values is completely specified by saying that it is the square of a given length value, divided by a given value of duration. On this interpretation, the dimension formula contains complete information about how a quantity's values are built up from the basic values. But there is a weaker way to interpret CONSTRUCTION: interpret it to say that a quantity's dimension formula gives *only partial* information about how the quantity values are built up. This weaker interpretation becomes more plausible when you look at the precise (nominal) definitions of "kinematic viscosity" and of "the stream function" (they are rather involved, so I won't state them here): the quantities invoked in the two definitions are quite different. On the weaker interpretation, CONSTRUCTION does not say that quantities with the same dimension formula are built up in the same way, so even if UNIQUENESS is true, (P1) is not.

Still, even though CONSTRUCTION is defensible, I am not tempted to endorse it, and I agree with Bridgman that the main temptation to endorse it is based on a mistake. What is that temptation? Consider a law relating different quantities, for example Newton's second law. What is Newton's second law? Of course everyone knows what it is: $F = ma$. But what does this string of symbols mean? How should

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[21]This is not the end of the matter. Fans of uniqueness of summation say their opponents face a "grounding problem": given that the statue and the clay are sums of the same particles, how do they manage to have different properties? Fans of UNIQUENESS might mount a similar argument. Following this thread of the dialectic would take me too far away from my main line of argument. My guess is that it would parallel the dialectic in the case of summation, in which the denial of uniqueness remains a viable position. See (Bennett 2004) for more.

[22]Thanks to Martin Glazier and Karen Bennett, who (separately) proposed this way to me.

it be translated into English? Here is a common translation:

(4) The net force on any material body is equal to the result of multiplying that body's mass and that body's acceleration.

But if *this* is right then it must make sense to multiply a mass and an acceleration. And surely mass and acceleration are not special here. So the truth of Newton's second law supports the thesis that quantity values can be multiplied together. Again, surely multiplication is not special here; if quantity values can be multiplied together then one value can also be divided by another. Once we're comfortable with this idea it is not a great leap to start thinking of multiplication and division as ways of constructing new quantity values from old ones, of thinking, for example, of speed values as got by dividing length values by duration values.[23]

But I do not think that (4) is what $F = ma$ means in English. In this denial I follow Bridgman, and I also follow him in his views about what equations like $F = ma$ do mean. He wrote that[24]

> ...x_1 [in an equation relating some quantities] might stand for the number which is the measure of a speed, x_2 the number which is the measure of a viscosity, etc. By a sort of shorthand method of statement we may abbreviate this long-winded description into saying that x_1 is a speed, but of course it really is not, but is only a number which measures speed. (17-18)

What goes for speed goes for force, mass, and acceleration. Bridgman might just as well have said that while the "a" in "$F = ma$" stands for the number which is the measure of a value of acceleration, we might as a sort of shorthand say that a is a value of the quantity acceleration, or even that a is an acceleration. But those who did not know this was shorthand would be misled about what "a" denotes.[25] (4) is

[23] For a recent endorsement of this argument, see (Van Inwagen 2014: 104-05).

[24] See also (Duncan 1953: 6) and (Palacios 1964: 18).

[25] See also page 41, where Bridgman complains about people thinking that something like (4) is the reason why all true laws can be written so that the dimensions of the left side and the right side are the same.

not true when read literally, it is only true when read as shorthand. When we read it that way, it does not support CONSTRUCTION.

What, exactly, is the correct and fully explicit translation of $F = ma$ into English? Let Z be the class of systems of scales to which the system we currently use, the SI system, belongs; then, I think, the correct and fully explicit translation of $F = ma$ is

(5) For any material body B, and for any system of scales of measurement S in Z, the number S assigns to the net force on B is equal to the result of multiplying the number S assigns to B's mass and the number S assigns to B's acceleration.[26]

As I said, I am not tempted to endorse CONSTRUCTION. Even though I have not, in this section, defended any arguments that it is false, there is an argument against it that I like. But since my main aim in this paper is to defend DEFINITIONAL CONNECTION, not to refute CONSTRUCTION, and since I want to get on to that main aim now, I will save that argument for a later section (section 6).

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[26]As I have been, I am pretending that force and acceleration are scalars. It is straightforward to write down the correct vectorial version of (5), but brings in unnecessary complications.

One might concede that (5) is the translation of "$F = ma$" into English and try to put together a more complicated argument for CONSTRUCTION. (5) makes reference to numbers. One might argue that the truth of (5) must be grounded in some facts that do not involve numbers. And perhaps those facts are facts about the result of multiplying mass values and acceleration values. But this argument does not succeed. One need not accept CONSTRUCTION to have facts available to ground the truth of (5). Instead one could say that the facts that ground (5) are facts like these: if the force on any material body were doubled, while its mass (and everything else) remained the same, then its acceleration would double; if the mass of any material body were halved, while the force on it (and everything else) remained the same, then its acceleration would double; and so on. In general: a body's acceleration is directly proportional to the force on it, indirectly proportional to its mass, and independent of everything else. (In fact Newton first formulated his second law of motion in terms of proportions; see (Newton 1999: 416).)

5 Defending a Definitional Connection

Now that Construction has had its day in the sun I want to turn to Definitional Connection, which has not received nearly as much attention. Here it is again:

> Definitional Connection: Let P1,...,Pn be the symbols for the primary quantities that appear in quantity Q's dimension formula in the true class of systems of scales. Then some relationship between Q and the primary quantities associated with the symbols P1,...,Pn is definitional of Q.

The first thing to do is to make Definitional Connection more specific: what relationship between Q and the primary quantities is definitional of Q? Unfortunately, I do not know how to answer this in a completely general way. I know how to do it for particular quantities, just not for an arbitrary quantity. So let's focus on a claim about one particular quantity that is supposed to follow — given an assumption I'm about to state — from the more-specific-but-unstated version of Definitional Connection.

The assumption (one I've made many times) is that speed is a secondary quantity, and length and duration primary. If this is true, then Definitional Connection is meant to entail the following claims about the essences of particular values of speed:

- It is definitional of the property of moving at 1 m/s that anything that has this property throughout some temporal interval that is 1 second long moves along a curve in space that is 1 meter long.
- It is definitional of the property of moving at 1 m/s that anything that has this property throughout some temporal interval that is 2 seconds long moves along a curve in space that is 2 meters long.
- It is definitional of the property of moving at 4 m/s that anything that has this property throughout some temporal interval that is 2 seconds long moves along a curve in space that is 8 meters long.[27]

[27] In all of these claims, occurrences of descriptions of properties are to be read *de re*. That descriptive content, which contains reference to numbers, is not part of the definitions of these properties.

Using the dimension formula for speed (in the true class of systems of scales) we can wrap these and all similar claims into one generalization:

> (Definitional Connection — Speed): Let a, b, and c be any positive real numbers with the following property: if you interpret "L" to denote b, and "T" to denote c, in the dimension formula for speed, and "compute the result," you get a. (Since the dimension formula for speed is "L/T," the requirement is that $a = b/c$.) Let S be any system of scales in the true class, and let r be the speed value that S assigns the number a, l the length value that S assigns the number b, and d the duration value that S assigns the number c; then it is definitional of r that anything that has r during a temporal interval that has d moves along a curve in space that has l.

For every secondary quantity Q (in the true class), there is a generalization like (Definitional Connection — Speed), in which Q's dimension formula in the true class appears, that advocates of Definitional Connection will accept. Definitional Connection is to be understood to be equivalent to the conjunction of these generalizations.[28]

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[28] In any class, including the true class, some quantities are *dimensionless*: their dimension function is the constant $[Q] = 1$. This means that every system of scales in the given class assigns the same numbers to values of Q — no matter what units those scales choose for the primary quantities. Clearly if a quantity is dimensionless in the true class, then Definitional Connection says nothing interesting about what relations between that quantity and the primary quantities are definitional of that quantity. Is the existence of dimensionless quantities any kind of problem for Definitional Connection?

Well, if every secondary quantity were dimensionless (only secondary quantities can be dimensionless), Definitional Connection would be empty. But it is false that every secondary quantity is dimensionless — obviously, if there are primary quantities, which there are, then some secondary quantities are not dimensionless (for example the "squares" of the primary quantities — if length is primary, area is its square — are not dimensionless).

One idea I have heard is that it may turn out, some hope that it turns out, that the quantities that appear in the laws of physics are all dimensionless. If this is right, and if the only "important" quantities are the ones that appear in the laws, then Definitional Connection concerns only unimportant quantities. This might be

As a reminder, all of this is premised on the assumption that speed is a secondary quantity. I made that assumption just so I could write down a model for the generalizations that Definitional Connection is the conjunction of. If speed is not a secondary quantity, then advocates of Definitional Connection will not accept (Definitional Connection — Speed), and will not understand Definitional Connection to entail it.

Is Definitional Connection correct? I don't have a knock-down argument for it here. My goal is to offer it for your consideration, and try to say a few things to make it plausible. As I said earlier, just isolating it from Construction is some defense of it; for a lot of the controversy around Construction does not apply to Definitional Connection.

The other main thing I have to say in defense of Definitional Connection is this: if you like the at-at theory of motion, then you should like Definitional Connection.

Why this is so emerges from trying to get clear on just what the at-at theory of motion says. Let's start by looking at a classic statement of the theory, namely Bertrand Russell's:

> motion consists *merely* in the occupation [by some given material thing] of different places at different times. (1937: 473; my emphasis)

Other presentations of the theory use similar language; Frank Arntzenius, for example, states the theory as the view that "there is *nothing more* to motion than

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thought to diminish its importance. But I find it hard to imagine how the laws could be stated in purely dimensionless terms: surely acceleration will continue to appear in the laws, and surely there is no way to make acceleration dimensionless? Nor do I see why one should hope that the laws can be stated using only dimensionless quantities. The usual thought is that then the laws will be independent of our scales of measurement. But even laws involving dimensionful quantities, like $F = ma$, can be stated in a way that is independent of our scales of measurement (see footnote 26).

In passing: though Definitional Connection says nothing about what is definitional of a dimensionless quantity, it does not follow that dimensionless quantities lack real definitions. It is just that their dimension formulas play no role in stating those definitions.

the occupation of different locations at different times" (2000: 189; my emphasis). What is the "merely" doing in the Russell passage, and the "nothing more" in the Arntzenius? A natural interpretation has these words serving to mark that the theory is stronger than

> Weak At-At: Necessarily, for any body B, B is in motion during temporal interval I iff B occupies different places at different times in I.

Weak At-At is consistent with the idea that motion, location, and time are "independent" things that are necessarily connected. But the language Russell and Arntzenius use suggests that this idea is not supposed to be consistent with the at-at theory. We can express a stronger version of the at-at theory (stronger than Weak At-At) using talk of real definitions.

Talk of real definitions gives us a way to make sense of the idea of things that are independent but necessarily connected. For then we can construe "independent" not as "modally independent" but as "have independent defintitions": two things have independent definitions iff neither's real definition places any constraints on what is true of the other. It is part of the point of recognizing the legitimacy of talk of real definitions, or essences, or natures, that it makes at least conceptual room for necessary connections between things that do not follow from the natures of those things. This idea is the root of Kit Fine's well-known arguments against modal theories of essence in (Fine 1994).

So one way to do justice to Russell's and Arntzenius's talk of what motion "consists merely" in, and of what motion is "nothing more" than, is to interpret them as asserting that some connection between motion, location, and time, is definitional of motion. More specifically, a better (maybe the best) statement of the at-at theory looks like this:

> Strong At-At: It is definitional of motion that, for any body B, B is in motion during temporal interval I iff B occupies different places at different times in I.

Now the at-at theory is not supposed to be just a theory of the non-quantitative notion of motion; it is also a theory of the quantitative notions, speed and velocity. So what does the analogue of Strong At-At for speed look like? It must say, for every speed, that it is definitional of that speed that something cover a certain distance in a certain time. But what exactly does the theory say? Which "certain distance" and "certain time" appear in the definition of a given speed? The answer comes from having the theory assert (Definitional Connection — Speed), which I repeat here:

> (Definitional Connection — Speed): Let a, b, and c be any positive real numbers with the following property: if you interpret "L" to denote b, and "T" to denote c, in the dimension formula for speed, and "compute the result," you get a. (Since the dimension formula for speed is "L/T," the requirement is that $a = b/c$.) Let S be any system of scales in the true class, and let r be the speed value that S assigns the number a, l the length value that S assigns the number b, and d the duration value that S assigns the number c; then it is definitional of r that anything that has r during a temporal interval that has d moves along a curve in space that has l.

The thesis Definitional Connection — Speed is part of the at-at theory of motion, properly understood.

My argument has been that if you like the at-at theory, you should like Definitional Connection. That's a bit vague so let me be more specific. One way to like the at-at theory is to think that it is true; and if you think it is true, you should definitely like Definitional Connection, for the quantitative version of the at-at theory is just part of Definitional Connection. Of course, one can consistently accept (Definitional Connection — Speed) but reject the other parts of Definitional Connection. But presumably if you are drawn to the at-at theory of motion, you will also be drawn to theories of other secondary quantities (energy, for example, if it is a secondary quantity) that are similar in spirit. That similarity in spirit amounts to this: they also assert that it is definitional of values of the secondary quantity in question that they are related to certain values of the primary quantities.

There is, however, another way to like the at-at theory, one that does not require you to believe it is true. This is important, since the at-at theory is mildly controversial even in the context of pre-relativistic and pre-quantum physics, and, I think, more controversial when relativity and quantum mechanics are taken into account.[29] You can like the at-at theory even while thinking it false by thinking that it has the right form. What form is that? You could think that the at-at theory is right to presume that the correct theory of any secondary quantity will assert that it is definitional of that secondary quantity that it is related in a certain way to the primary quantities. You could think that where the at-at theory goes wrong is (merely) in holding that speed is a secondary quantity.[30]

6 Definitional Connection Without Right Units

The main work of this paper is done, but there is a loose end I want to tie up. Definitional Connection, as I stated it, presupposes Right Units, a presupposition which might seem implausible. I said that Definitional Connection could be formulated without this presupposition; in this section I will show how to do this.

The fact that Definitional Connection can be formulated without presupposing Right Units is also relevant to evaluating Construction. Construction, as far as I can tell, must presuppose Right Units. So if Right Units is objectionable, its being objectionable provides a reason to believe Definitional Connection rather than Construction.

Without Right Units we cannot say that choosing the unit for area to be the value for area had by a square with unit length sides is an "objectively better" choice than choosing the unit to be the value had by an equilateral triangle with unit length sides. It might seem that Definitional Connection requires Right Units, because stating the generalizations that appear in Definitional Connection requires referring to the true class of systems of scales, and "true class" is defined in terms of right units. But in fact this is not so. Here are two ways of stating the definitional connection between area and length without referring to the true class:

- Let m and n be any two positive reals with $n = m^2$. Let S be a system of scales that designates length as primary and chooses the unit for area to be

[29]For arguments against the theory see (Arntzenius 2000) and (Lange 2005).

[30]The standard alternative to the at-at theory of motion says that motion is not defined by reference to location and time. The analogous view of speed has it that speed is not defined by reference to length and duration, and so that speed is a primary quantity. This is compatible with Definitional Connection.

the area value had by a square with unit length sides. And let a be the area value S assigns n, and l the length value S assigns m. Then it is definitional of a that anything that has a has the same area as a square with sides of length l.

- Let m and n be any two positive reals with $n = \frac{\sqrt{3}}{4}m^2$. Let S be a system of scales that designates length as primary and chooses the unit for area to be the area value had by an equilateral triangle with unit length sides. And let a be the area value S assigns n, and l the length value S assigns m. Then it is definitional of a that anything that has a has the same area as an equilateral triangle with sides of length l.

We can avoid assuming Right Units by taking either (or both!) of these to be conjuncts in Definitional Connection, and taking the other conjuncts to be stated in a similar way.

Definitional Connection still presupposes Non-Relativity, for the quantities that Definitional Connection concerns are meant to be all and only the quantities that are secondary simpliciter. And dimension formulas are still used to state Definitional Connection; $n = m^2$ just is (an instance of) the dimension formula for length (in the given system), and $n = \frac{\sqrt{3}}{4}m^2$ is the dimension formula multiplied by a constant.

It is harder to see how Construction could do without Right Units. Is the product of the property of being 1 meter with itself identical to the area property had by things that are the same area as a square with meter-long sides, or is it identical to the area property had by things that are the same area as an equilateral triangle with meter-long sides? It cannot be identical to both; picking one goes hand-in-hand with privileging one way of choosing a unit for area over another. If Right Units makes you a lot more nervous than Non-Relativity, you should be more drawn to Definitional Connection than to Construction.

7 Conclusion

My goals in this paper have been modest. I have not set out to prove that Definitional Connection is true from premises that are in any interesting sense independent of it. Instead, my aims with respect to this thesis were these. First, to get DEFINITIONAL CONNECTION on the table, and on the agenda. Something close to DEFINITIONAL CONNECTION has surfaced in discussions of the metaphysics of dimensions, but it has not until now been clearly distinguished from CONSTRUCTION. And clearly distinguishing it is a prerequisite to reaching a clear-headed conclusion about whether it is true. A second aim was to say something in defense of DEFINITIONAL CONNECTION. I myself found that appreciating the relation between DEFINITIONAL CONNECTION and the at-at theory of motion increased my confidence in it, and I suspect it may do the same for others.

I will not pretend that this amounts to a comprehensive treatment of DEFINITIONAL CONNECTION — here, as elsewhere in metaphysics, it is hard to find good arguments. I hope, instead, that it is a good start on a worthy project.

In the appendix that follows, I pursue a question that CONSTRUCTION raises that did not fit into the paper's main line of argument.

Appendix: A Bit More on CONSTRUCTION

CONSTRUCTION, on one reading at least, entails that some quantity values are got by "multiplying" other quantity values together, and some are got by "dividing" one quantity value by another, and some are got by "raising a quantity value to a power." For example, if speed is a secondary quantity, the view is that the speed value *4 meters per second* is the result of dividing *being 4 meters long* by *being 1 second in duration*.

Here is a pointed question for fans of CONSTRUCTION: why are these the only three operations?[31] Why can't you construct a new quantity by "taking the sine of a length value," or "subtracting the natural logarithm of a duration value from a mass value"?

It might look at first like this question should embarrass proponents of CONSTRUCTION. Surely they'll either have to say that it is just a brute fact that these are the only operations; or they'll have to say that there is a quantity got by taking the

[31]Really we only need two; division can be defined in terms of multiplication and exponentiation.

sine of length, it is just nowhere instantiated in the actual world (or of no interest to physicists).

But proponents of CONSTRUCTION do not need to say either of these things. There is a proof in most treatments of dimensional analysis that can be used to explain why the only operations are multiplication, division, and exponentiation (for Bridgman's version see (Bridgman 1922: 21-22)). Here is the theorem proved:

> Let X_1 and X_2 be two systems of scales of measurement in the same class. Suppose also that X_1 and X_2 agree on the ratios of numbers assigned to quantity values. That is, if the quotient of the numbers X_1 assigns to values v and u of some given quantity is r, then the quotient of the numbers X_2 assigns to those values is also r.
>
> Then for any quantity Q measured by X_1 and X_2, the dimension formula for Q is a "power law monomial," that is, it has the form "$KP_1^m P_2^n \ldots$," where symbols for the primary quantities go in for "P_1" and so on, and symbols denoting real numbers go in for "K," "m," and so on.

How is this relevant? Suppose some "quantity" were, say, the sine of length. Then its dimension formula (in the true class) would not be a power law monomial. So there would be systems X_1 and X_2 in the true class which did not agree on the ratios of numbers assigned to quantity values. Since these systems are faithful to the intrinsic structure of this "quantity" (we have always restricted ourselves to faithful scales), there must not be scale-independent facts about the ratios of values of this "quantity." But that is definitional of quantities! This contradiction shows that no quantity can be the sine of length, and more generally, that quantities can only be constructed from other quantities using multiplication, division, and exponentiation. (I leave it open whether those who like CONSTRUCTION will want to say that there is *something* (which is not a quantity) that is the sine of length.)[32]

[32]Thanks to Martin Glazier, an audience at the University of Rochester, and an audience at NYU. Thanks especially to Karen Bennett, without whose large investment of editorial efforts this paper would not be what it is.

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