18.325 Topics in Applied Mathematics, Spring 2001, Prof. Bazant.

Problem Set 1b

Due at lecture on Tu Feb 27 (along with PS 1a).

1. (a) Derive the first correction to the intermediate asymptotics of a random walk with a nonzero "skewness" $D_3 \neq 0$, assuming all other moments D_n are finite. Also, for simplicity, assume that there is no drift $D_1 = 0$, which can always be achieved by a change of variables (if D_1 is finite!). **Extra Credit:** Derive the second correction when both $D_3 \neq 0$ and $D_4 \neq 0$ (which reduces to the case done in class when $D_3 = 0$).

   (b) Consider a simple *asymmetric* walk in one dimension, which has probability $0 < p < 1$ of stepping to the right and $q = 1 - p$ of stepping left with a step length of a in time intervals τ, i.e. the transition probability is

$$p_\tau(y) = p\delta(x - a) + q\delta(x + a). \tag{1}$$

   Use your results from part (a) to derive the intermediate asymptotics of the PDF of the position of the walker correct to order $(\tau/t)^{1/2} = n^{-1/2}$. (Remember to get rid of the drift first...) **Extra Credit:** Compute the next term at order $\tau/t = 1/n$.

2. If you did not already do so in problem 2 on homework 1a, express $c(x, t)$ as a similarity solution involving three dimensionless groups. (Note that the background concentration c_o needs to be "subtracted out" to see the exact self-similarity arising due the lack of a natural length scale.) Derive the ODE for the similarity function and check that your solution satisfies it.