Equation (50)

I think we have an issue of clarity, which actually comes from the equation above, eq (49). Here, the meaning of the i index is unclear. Because g is a function of all components, i, the $\sum_i$ refers to a sum over components, which is most clearly seen with the $\sum_i \mu_i^\Theta c_i$ term. In the next term, because κ is indexed by i and j, they take on meaning of the coordinate directions there. However, there should still be concentration gradient terms from each component, i. A more clear way to write this equation is

$$g = \bar{g}\left(\{g(c_i)\}\right) + \sum_i \left(\mu_i^\Theta c_i + \frac{1}{2} \sum_j \sum_k \left(\partial_j \tilde{c}_i\right) \kappa_{jk} \left(\partial_k \tilde{c}_i\right) \right) \tag{1}$$

where i indexes components and j and k index directions and the partial derivative with respect to direction j is denoted ∂_j.

Then, when we want the chemical potential of a particular component ℓ, we have that

$$\frac{\partial g}{\partial c_\ell} = \frac{\partial \bar{g}}{\partial c_\ell} + \frac{\partial}{\partial c_\ell} \sum_i \mu_i^\Theta c_i = \frac{\partial \bar{g}}{\partial c_\ell} + \mu_\ell^\Theta. \tag{2}$$

Then, when we differentiate with respect to the gradient of c_ℓ, we have

$$\frac{\partial}{\partial \boldsymbol{\nabla} c_\ell} = \frac{1}{c_\ell^\Theta} \frac{\partial}{\partial \boldsymbol{\nabla} \tilde{c}_\ell}. \tag{3}$$

The only term in g which contains gradients of c (or $\tilde{c}$) is the final term, and when $\ell \neq i$, the derivative is zero, so the differentiation selects only the ℓth component. Then, what we will end up with (explanation of the factor of 2 follows below) is

$$\frac{\partial g}{\partial \boldsymbol{\nabla} c_\ell} = \frac{1}{c_\ell^\Theta} \sum_k \kappa_{jk} \partial_k \tilde{c}_\ell \tag{4}$$

which is a rank-1 tensor indexed over directions by j. Then

$$-\boldsymbol{\nabla} \cdot \frac{\partial g}{\partial \boldsymbol{\nabla} c_\ell} = -\frac{1}{c_\ell^\Theta} \boldsymbol{\nabla} \cdot \sum_k \kappa_{jk} \partial_k \tilde{c}_\ell \tag{5}$$

and replacing ℓ with i,

$$\mu_i - \mu_i^\Theta = \frac{\partial \bar{g}}{\partial c_i} - \boldsymbol{\nabla} \cdot \sum_k \kappa_{jk} \partial_k \frac{\tilde{c}_i}{c_i^\Theta} \tag{6}$$

$$= \frac{\partial \bar{g}}{\partial c_i} - \sum_j \sum_k \partial_j \kappa_{jk} \partial_k \frac{\tilde{c}_i}{c_i^\Theta} \tag{7}$$

which is similar to what is in the paper, albeit with more clear summations distinguishing between species and directions.

Equation (57)

It would be a reasonable to replace this with

$$\xi = \frac{c - c_A}{c_B - c_A}, \tag{8}$$

and this might have been a more appropriate choice for the paper. Nevertheless, whether ξ varies between 0 and 1 or 0 to -1 is not particularly important to the results. The primary goal of that section was to demonstrate the similarity between the developed model and the more familiar notation of an Allen-Cahn style equation. So yes, the negative of the term proposed might make more sense, but the results are identical.

Equation (60)

This seems to be a typesetting error that slipped by us. The arXiv print is correct on this one. The last term in the correct equation should be

$$-\frac{\kappa}{c_s}\nabla^2\tilde{c} \tag{9}$$

not

$$-\frac{\kappa}{c_s}\tilde{\nabla}^2\tilde{c} \tag{10}$$

Equation (75)

The second term on the right hand side is incorrect. The equation should be

$$c_0 k_B T \ln a_+ = W\tilde{c} + k_B T(\ln\tilde{c}_+ + 1) - \frac{\partial\varepsilon_p}{\partial\tilde{c}_+}\left|\boldsymbol{\nabla}\phi\right|^2. \tag{11}$$

Equation (85)

This should be the stress-free strain

$$\tilde{\mu} = \ln\frac{\tilde{c}}{1-\tilde{c}} + \tilde{\Omega}(1-2\tilde{c}) - \tilde{\kappa}\tilde{\nabla}^2\tilde{c} - \tilde{\sigma} : \tilde{\varepsilon}^0 \tag{12}$$