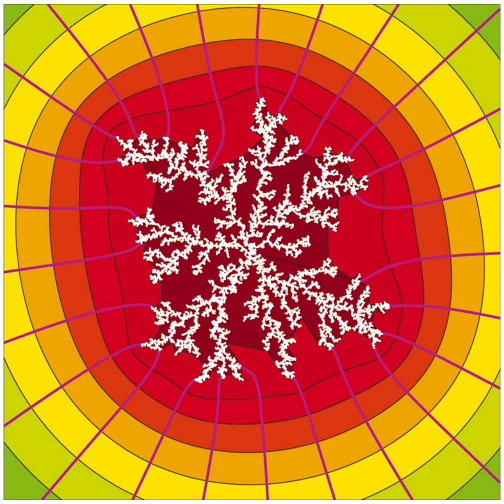

FIG. 1.

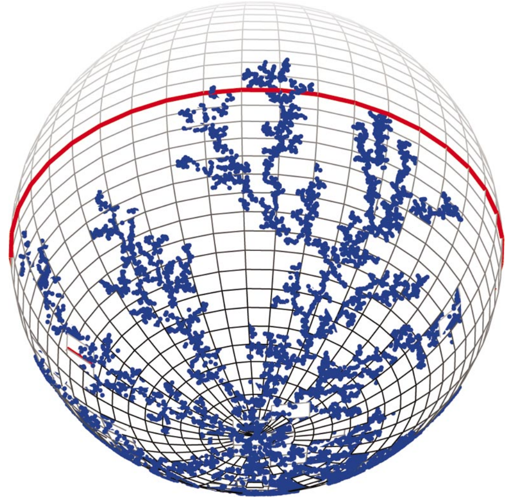

FIG. 2.

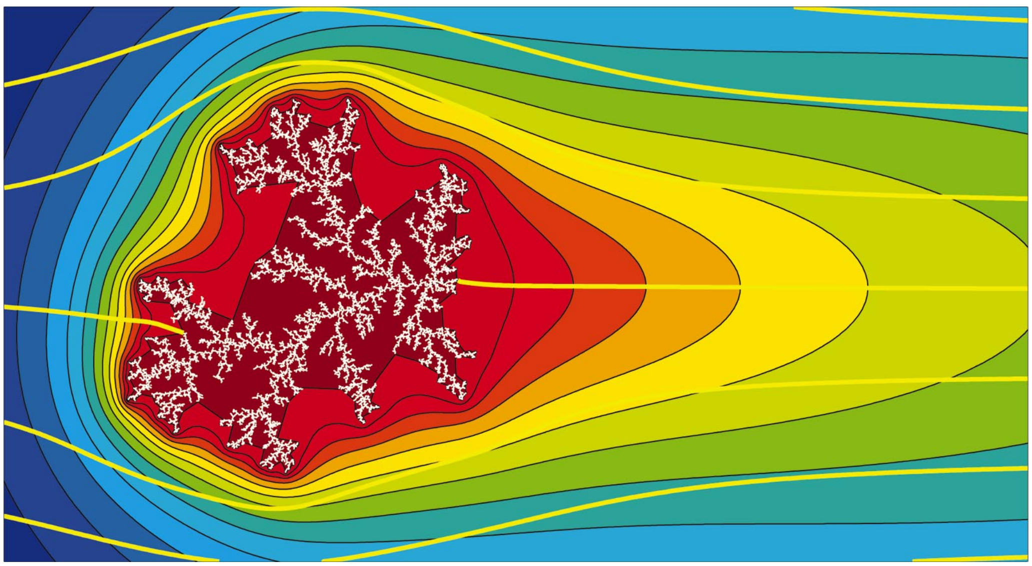

FIG. 3.

Transport-limited aggregation

Martin Z. Bazant and Jaehyuk Choi
Massachusetts Institute of Technology, Cambridge, Massachusetts 02139-4307
Benny Davidovitch
Harvard University, Cambridge, Massachusetts 02138
Darren Crowdy
Imperial College, London, United Kingdom

(Received 20 September 2004; published online 23 November 2004)
[DOI: 10.1063/1.1821713]

Diffusion-limited aggregation (DLA) and its variants provide the simplest models of fractal patterns, such as colloidal clusters, electrodeposits, and lightning strikes. The original model involves random walkers sticking to a growing cluster,[1] but recently DLA (in the plane) has been reformulated in terms of stochastic conformal maps.[2] This fruitful new perspective provides the exact Laplacian concentration field (colors) and diffusive flux lines (purple) around a growing DLA cluster (Fig. 1) and other analytical insights.

Here, we exploit a conformal-invariance principle for non-Laplacian fields[3] to extend the method to more general, transport-limited growth models,[4] on flat and curved surfaces, such as DLA on a sphere (Fig. 2) and advection-diffusion-limited aggregation (Fig. 3 and video) in a fluid flow (yellow streamlines). ADLA clusters grow preferentially upstream (to the left), and yet their fractal dimension remains that of DLA, since diffusion dominates over advection at small scales.

[1]T. A. Witten and L. M. Sander, “Diffusion-limited aggregation: A kinetic critical phenomenon,” Phys. Rev. Lett. **47**, 1400 (1981).
[2]M. B. Hastings and L. S. Levitov, “Laplacian growth as one-dimensional turbulence,” Physica D **116**, 244 (1998).
[3]M. Z. Bazant, “Conformal mapping of some non-harmonic functions in transport theory,” Proc. R. Soc. London, Ser. A **460**, 1433 (2004).
[4]M. Z. Bazant, J. Choi, and B. Davidovitch, “Dynamics of conformal maps for a class of non-Laplacian growth phenomena,” Phys. Rev. Lett. **91**, 045503 (2003).

