Online Appendix for "The Term Structure of Currency Carry Trade Risk Premia" —Not For Publication—

This Online Appendix describes additional empirical and theoretical results on foreign bond returns in U.S. dollars. Section A reports additional results on portfolios of countries sorted by the short-term interest rates. Section B reports similar results for portfolios of countries sorted by the slope of the yield curves. Section C reports additional results obtained with zero-coupon bonds. Section D compares finite to infinite maturity bond returns. Section E reports additional theoretical results on dynamic term structure models, starting with the simple Vasicek (1977) and Cox, Ingersoll, and Ross (1985) one-factor models, before turning to their k-factor extensions and the model studied in Lustig, Roussanov, and Verdelhan (2014).

A Sorting Countries by Interest Rates

This section first focuses on our benchmark sample of G10 countries, and then turn to larger sets of countries. In each case, we consider three different bond holding periods (one, three, and twelve months), and two time windows (12/1950–12/2012 and 12/1971–12/2012).

A.1 Benchmark Sample

Figure 5 plots the composition of the three interest rate-sorted portfolios of the currencies of the benchmark sample, ranked from low to high interest rate currencies. Typically, Switzerland and Japan (after 1970) are funding currencies in Portfolio 1, while Australia and New Zealand are the carry trade investment currencies in Portfolio 3. The other currencies switch between portfolios quite often.

Table 3 reports the annualized moments of log returns on currency and bond markets. As expected [see Lustig and Verdelhan (2007) for a detailed analysis], the average excess returns increase from Portfolio 1 to Portfolio 3. For investment periods of one month, the average excess return on Portfolio 1 is -0.24% per annum, while the average excess return on Portfolio 3 is 3.26%. The spread between Portfolio 1 and Portfolio 3 is 3.51% per annum. The volatility of these returns increases only slightly from the first to the last portfolio. As a result, the Sharpe ratio (annualized) increases from -0.03 on Portfolio 1 to 0.40 on the Portfolio 3. The Sharpe ratio on a long position in Portfolio 3 and a short position in the Portfolio 1 is 0.49 per annum. The results for the post-Bretton-Woods sample are very similar. Hence, the currency carry trade is profitable at the short end of the maturity spectrum.

Recall that the absence of arbitrage implies a negative relationship between the equilibrium risk premium for investing in a currency and the SDF entropy of the corresponding country. Therefore, given the pattern in currency risk premia, high interest rate currencies have low entropy and low interest rate currencies have high entropy. As a result, sorting by interest rates (from low to high) seems equivalent to sorting by pricing kernel entropy (from high to low). In a log-normal world, entropy is just one half of the variance: high interest rate currencies have low pricing kernel variance, while low interest rate currencies have volatile pricing kernels.

Table 3 shows that there is a strong decreasing pattern in local currency bond risk premia. The average excess return on Portfolio 1 is 2.39% per annum and its Sharpe ratio is 0.68. The excess return decreases monotonically to -0.21% on Portfolio 3. Thus, there is a 2.60% spread per annum between Portfolio 1 and Portfolio 3.

If all of the shocks driving currency risk premia were permanent, then there would be no relation between currency risk premia and term premia. To the contrary, we find a very strong negative association between local currency bond risk premia and currency risk premia. Low interest rate currencies tend to produce high local currency bond risk premia, while high interest rate currencies tend to produce low local currency bond risk premia. The decreasing term premia are consistent with the decreasing entropy of the total SDF from low (Portfolio 1) to high interest rates (Portfolio 3) that we had inferred from the foreign currency risk premia. Furthermore, it appears that these are not offset by equivalent decreases in the entropy of the permanent component of the foreign pricing kernel.

The decline in the local currency bond risk premia partly offsets the increase in currency risk premia. As a result, the average excess return on the foreign bond expressed in U.S. dollars measured in Portfolio 3 is only

Table 3: Interest Rate-Sorted Portfolios: Benchmark Sample

Portfolio		1	2	3	3 − 1	1	2	3	3 − 1	1	2	3	3 − 1
Horizon		1-Month				3-Month				12-Month			
						Panel A: 12/1950–12/2012							
$-\Delta s$	Mean	1.43	0.35	-0.18	-1.62	1.65	0.23	-0.28	-1.93	1.78	0.22	-0.42	-2.20
$f-s$	Mean	-1.68	0.78	3.45	5.13	-1.64	0.81	3.39	5.03	-1.55	0.91	3.19	4.74
rx^{FX}	Mean	-0.24	1.13	3.26	3.51	0.00	1.03	3.11	3.10	0.23	1.12	2.77	2.54
	s.e.	[1.00]	[0.84]	[1.03]	[0.92]	[1.02]	[0.91]	[1.20]	[1.03]	[1.10]	[1.03]	[1.25]	[0.98]
	Std	7.92	6.64	8.10	7.22	8.29	6.91	8.74	7.97	9.20	7.32	9.59	8.03
	SR	-0.03	0.17	0.40	0.49	0.00	0.15	0.36	0.39	0.03	0.15	0.29	0.32
	s.e.	[0.13]	[0.13]	[0.14]	[0.14]	[0.13]	[0.13]	[0.14]	[0.15]	[0.13]	[0.13]	[0.14]	[0.15]
$rx^{(10),*}$	Mean	2.39	1.67	-0.21	-2.60	2.17	1.31	0.41	-1.76	1.98	1.07	0.85	-1.13
	s.e.	[0.45]	[0.47]	[0.57]	[0.57]	[0.50]	[0.53]	[0.64]	[0.61]	[0.58]	[0.65]	[0.72]	[0.65]
	Std	3.50	3.65	4.50	4.47	4.01	4.34	5.10	4.72	4.41	5.22	5.67	5.05
	SR	0.68	0.46	-0.05	-0.58	0.54	0.30	0.08	-0.37	0.45	0.20	0.15	-0.22
	s.e.	[0.13]	[0.13]	[0.13]	[0.13]	[0.14]	[0.13]	[0.13]	[0.12]	[0.15]	[0.14]	[0.13]	[0.12]
$rx^{(10),\$}$	Mean	2.15	2.81	3.06	0.91	2.17	2.34	3.52	1.35	2.21	2.19	3.62	1.41
	s.e.	[1.18]	[0.98]	[1.16]	[1.09]	[1.22]	[1.06]	[1.31]	[1.21]	[1.26]	[1.14]	[1.42]	[1.23]
	Std	9.33	7.70	9.21	8.61	10.00	8.22	9.94	9.32	10.64	8.83	11.08	9.95
	SR	0.23	0.36	0.33	0.11	0.22	0.28	0.35	0.14	0.21	0.25	0.33	0.14
	s.e.	[0.13]	[0.13]	[0.13]	[0.13]	[0.13]	[0.13]	[0.13]	[0.13]	[0.13]	[0.13]	[0.14]	[0.14]
$rx^{(10),\$}-rx^{(10),US}$	Mean	0.63	1.30	1.55	0.91	0.65	0.82	2.00	1.35	0.67	0.65	2.08	1.41
	s.e.	[1.22]	[1.09]	[1.32]	[1.09]	[1.16]	[1.18]	[1.51]	[1.21]	[1.33]	[1.22]	[1.61]	[1.23]
						Panel B: 12/1971–12/2012							
$-\Delta s$	Mean	2.16	0.30	-0.29	-2.45	2.39	0.14	-0.49	-2.89	2.61	0.05	-0.74	-3.35
$f-s$	Mean	-2.03	1.07	3.87	5.90	-1.99	1.11	3.80	5.78	-1.89	1.23	3.60	5.49
rx^{FX}	Mean	0.13	1.38	3.57	3.44	0.41	1.25	3.30	2.90	0.72	1.28	2.86	2.14
	s.e.	[1.52]	[1.26]	[1.50]	[1.35]	[1.56]	[1.37]	[1.75]	[1.52]	[1.67]	[1.55]	[1.82]	[1.42]
	Std	9.66	8.11	9.57	8.46	10.12	8.42	10.46	9.52	11.22	8.88	11.59	9.68
	SR	0.01	0.17	0.37	0.41	0.04	0.15	0.32	0.30	0.06	0.14	0.25	0.22
	s.e.	[0.16]	[0.16]	[0.16]	[0.16]	[0.16]	[0.16]	[0.17]	[0.17]	[0.16]	[0.17]	[0.17]	[0.18]
$rx^{(10),*}$	Mean	2.82	2.12	-0.13	-2.95	2.60	1.66	0.56	-2.05	2.48	1.30	1.01	-1.47
	s.e.	[0.64]	[0.68]	[0.81]	[0.79]	[0.72]	[0.77]	[0.92]	[0.87]	[0.84]	[0.94]	[1.01]	[0.87]
	Std	4.12	4.33	5.17	5.09	4.69	5.12	5.85	5.33	5.17	6.06	6.31	5.45
	SR	0.68	0.49	-0.02	-0.58	0.55	0.32	0.10	-0.38	0.48	0.21	0.16	-0.27
	s.e.	[0.16]	[0.16]	[0.16]	[0.16]	[0.17]	[0.16]	[0.16]	[0.15]	[0.19]	[0.18]	[0.16]	[0.15]
$rx^{(10),\$}$	Mean	2.95	3.49	3.45	0.50	3.01	2.90	3.86	0.85	3.20	2.58	3.87	0.67
	s.e.	[1.78]	[1.46]	[1.71]	[1.59]	[1.85]	[1.60]	[1.89]	[1.77]	[1.91]	[1.66]	[2.03]	[1.75]
	Std	11.33	9.34	10.82	10.06	12.09	9.91	11.68	10.95	12.85	10.44	13.01	11.70
	SR	0.26	0.37	0.32	0.05	0.25	0.29	0.33	0.08	0.25	0.25	0.30	0.06
	s.e.	[0.16]	[0.16]	[0.16]	[0.16]	[0.16]	[0.16]	[0.17]	[0.16]	[0.16]	[0.17]	[0.18]	[0.16]
$rx^{(10),\$}-rx^{(10),US}$	Mean	0.44	0.99	0.94	0.50	0.48	0.37	1.33	0.85	0.64	0.01	1.31	0.67
	s.e.	[1.77]	[1.55]	[1.88]	[1.59]	[1.70]	[1.71]	[2.16]	[1.77]	[1.99]	[1.78]	[2.32]	[1.75]

Notes: The table reports the average change in exchange rates (Δs), the average interest rate difference ($f-s$), the average currency excess return (rx^{FX}), the average foreign bond excess return on 10-year government bond indices in foreign currency ($rx^{(10),*}$) and in U.S. dollars ($rx^{(10),\$}$), as well as the difference between the average foreign bond excess return in U.S. dollars and the average U.S. bond excess return ($rx^{(10),\$}-rx^{US}$). For the excess returns, the table also reports their annualized standard deviation (denoted Std) and their Sharpe ratios (denoted SR). The annualized monthly log returns are realized at date $t+k$, where the horizon k equals 1, 3, and 12 months. The balanced panel consists of Australia, Canada, Japan, Germany, Norway, New Zealand, Sweden, Switzerland, and the U.K. The countries are sorted by the level of their short term interest rates into three portfolios. The standard errors (denoted s.e. and reported between brackets) were generated by bootstrapping 10,000 samples of non-overlapping returns.

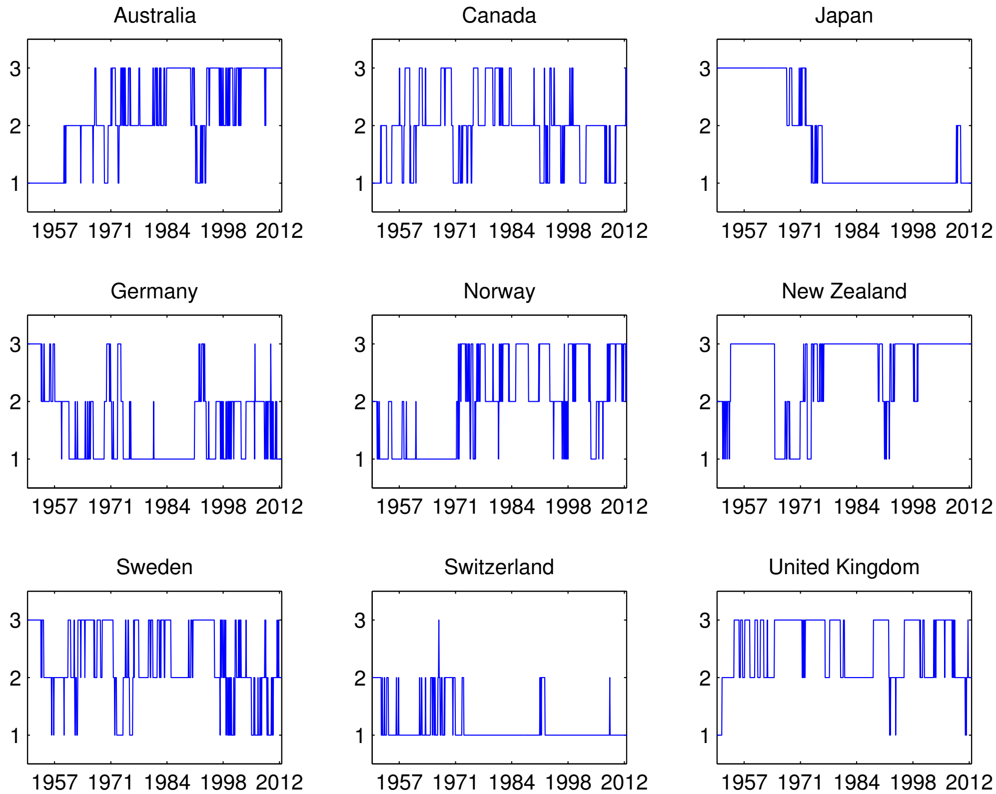

Figure 5: Composition of Interest Rate-Sorted Portfolios — The figure presents the composition of portfolios of 9 currencies sorted by their short-term interest rates. The portfolios are rebalanced monthly. Data are monthly, from 12/1950 to 12/2012.

0.91% per annum higher than the average excess returns measured in Portfolio 1. The Sharpe ratio on a long-short position in bonds of Portfolio 3 and Portfolio 1 is only 0.11. U.S. investors cannot simply combine the currency carry trade with a yield carry trade, because these risk premia roughly offset each other. Interest rates are great predictors of currency excess returns and local currency bond excess returns, but not of dollar excess returns. To receive long-term carry trade returns, investors need to load on differences in the quantity of permanent risk, as shown in Equation (28). The cross-sectional evidence presented here does not lend much support to these differences in permanent risk.

Table 3 shows that the results are essentially unchanged in the post-Bretton-Woods sample. The Sharpe ratio on the currency carry trade is 0.41, achieved by going long in Portfolio 3 and short in Portfolio 1. However, there is a strong decreasing pattern in local currency bond risk premia, from 2.82% per annum in Portfolio 1 to −0.13% in the Portfolio 3. As a result, there is essentially no discernible pattern in dollar bond risk premia.

Figure 6 presents the cumulative one-month log returns on investments in foreign Treasury bills and foreign 10-year bonds. Most of the losses are concentrated in the 1970s and 1980s, and the bond returns do recover in the 1990s. In fact, between 1991 and 2012, the difference is currency risk premia at the one-month horizon between Portfolio 1 and Portfolio 3 is 4.54%, while the difference in the local term premia is only 1.41% per annum. As a result, the un-hedged carry trade in 10-year bonds still earn about 3.13% per annum over this sample. However, this difference of 3.13% per annum has a standard error of 1.77% and, therefore, is not statistically significant.

As we increase holding period k from 1 to 3 and 12 months, the differences in local bond risk premia between portfolios shrink, but so do the differences in currency risk premia. Even at the 12-month horizon, there is no evidence of statistically significant differences in dollar bond risk premia across the currency portfolios.

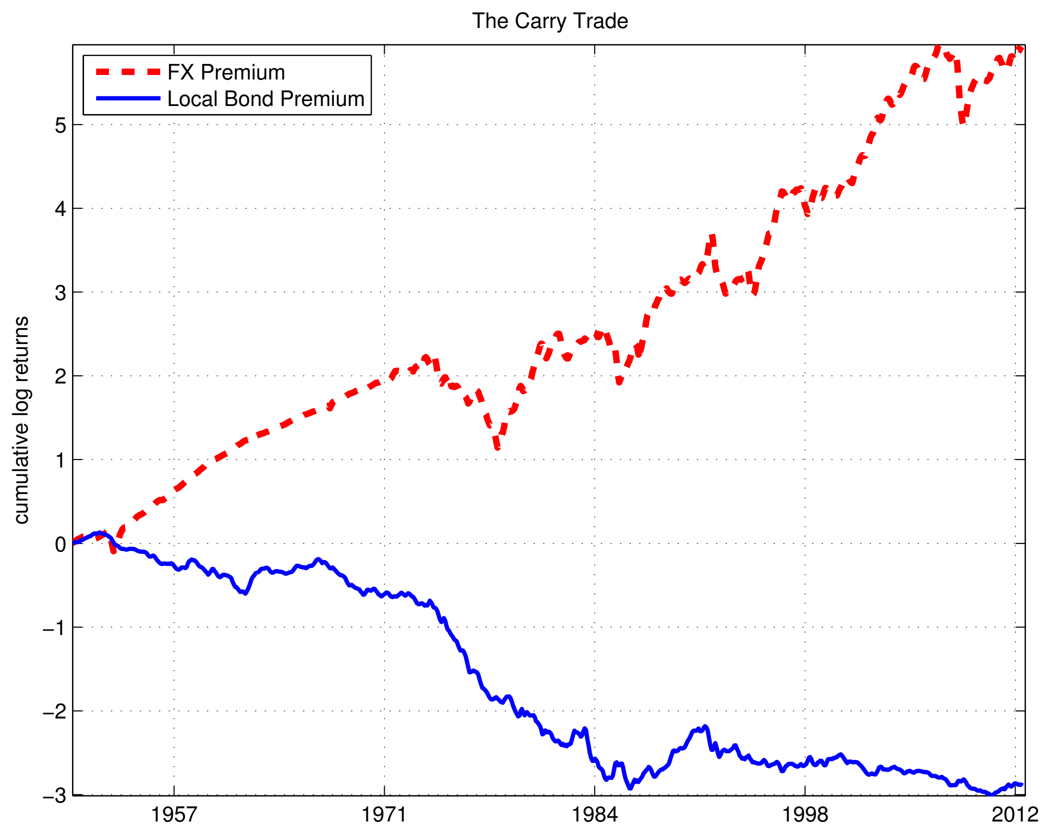

Figure 6: The Carry Trade and Term Premia – The figure presents the cumulative one-month log returns on investments in foreign Treasury bills and foreign 10-year bonds. The benchmark panel of countries includes Australia, Canada, Japan, Germany, Norway, New Zealand, Sweden, Switzerland, and the U.K. Countries are sorted every month by the level of their one-month interest rates into three portfolios. The returns correspond to a strategy going long in the Portfolio 3 and short in Portfolio 1. The sample period is 12/1950–12/2012.

A.2 Developed Countries

Very similar patterns of risk premia emerge using larger sets of countries. In the sample of developed countries, we sort currencies in four portfolios. Figure 7 plots the composition of the four interest rate-sorted currency portfolios. Switzerland and Japan (after 1970) are funding currencies in Portfolio 1, while Australia and New Zealand are carry trade investment currencies in Portfolio 4.

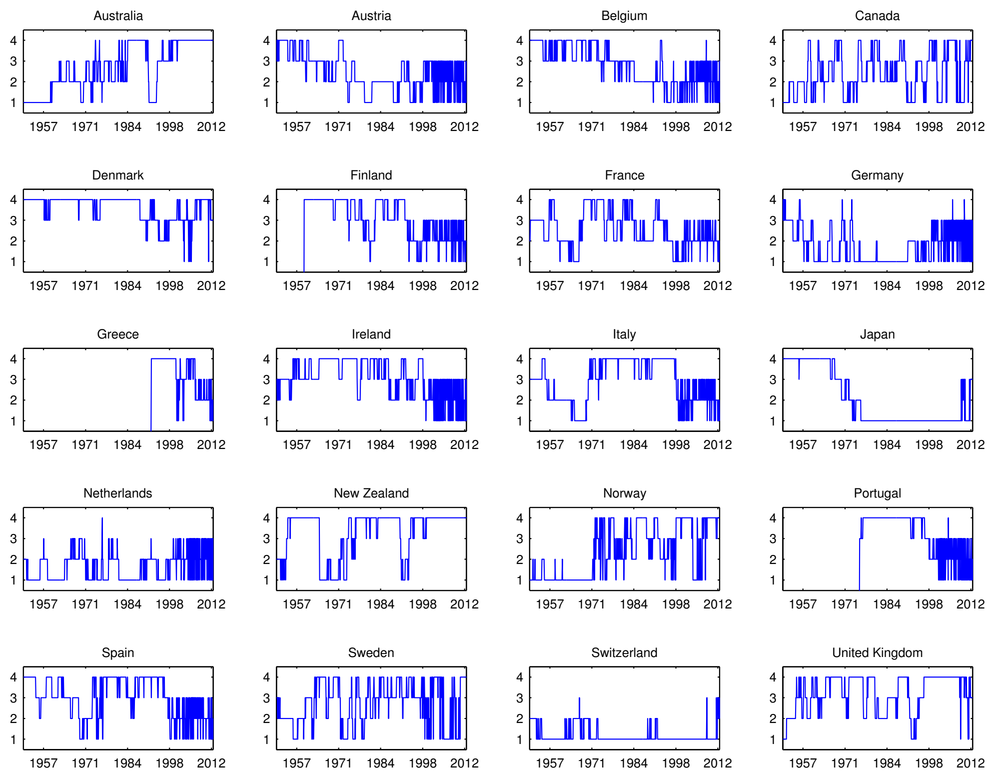

Figure 7: Composition of Interest Rate-Sorted Portfolios — The figure presents the composition of portfolios of 20 currencies sorted by their short-term interest rates. The portfolios are rebalanced monthly. Data are monthly, from 12/1950 to 12/2012.

Table 4 reports the results of sorting the developed country currencies into portfolios based on the level of their interest rate, ranked from low to high interest rate currencies. Essentially, the results are very similar to those obtained on the benchmark sample of developed countries. There is no economically or statistically significant carry trade premium at longer maturities. The 2.98% spread in the currency risk premia is offset by the negative 3.03% spread in local term premia at the one-month horizon against the carry trade currencies.

A.3 Whole Sample

Finally, Table 5 reports the results of sorting all the currencies in our sample, including those of emerging countries, into portfolios according to the level of their interest rate, ranked from low to high interest rate currencies. In the sample of developed and emerging countries, the pattern in returns is strikingly similar, but the differences are larger. At the one-month horizon, the 6.66% spread in the currency risk premia is offset by a 5.15% spread in local term premia. A long-short position in foreign bonds delivers an excess return of 1.51% per annum, which is not statistically significantly different from zero. At longer horizons, the differences in local bond risk premia are much smaller, but so are the carry trade returns.

Table 4: Interest Rate Sorted Portfolios: Developed sample

Portfolio		1	2	3	4	4 − 1	1	2	3	4	4 − 1	1	2	3	4	4 − 1
Horizon		1-month					3-month					12-month				
		Panel A: 1950-2012														
$-\Delta s$	Mean	1.30	0.58	0.06	-1.17	-2.47	1.40	0.37	0.19	-1.23	-2.63	1.54	0.38	-0.13	-1.13	-2.68
$f-s$	Mean	-1.41	0.39	1.51	4.03	5.44	-1.38	0.42	1.52	3.97	5.35	-1.26	0.53	1.56	3.79	5.05
r^{FX}	Mean	-0.11	0.97	1.56	2.86	2.98	0.02	0.79	1.71	2.74	2.72	0.28	0.91	1.43	2.65	2.37
	s.e.	[1.02]	[1.04]	[1.02]	[0.97]	[0.62]	[1.06]	[1.10]	[1.11]	[1.10]	[0.65]	[1.12]	[1.17]	[1.11]	[1.22]	[0.66]
	Std	8.02	8.26	7.96	7.67	4.87	8.36	8.43	8.28	8.18	5.25	9.27	8.76	8.79	9.24	5.36
	SR	-0.01	0.12	0.20	0.37	0.61	0.00	0.09	0.21	0.33	0.52	0.03	0.10	0.16	0.29	0.44
	s.e.	[0.13]	[0.13]	[0.13]	[0.13]	[0.14]	[0.13]	[0.13]	[0.13]	[0.13]	[0.15]	[0.13]	[0.13]	[0.13]	[0.13]	[0.17]
$rx^{(10),*}$	Mean	3.00	1.90	1.05	-0.02	-3.03	2.46	1.59	1.08	0.48	-1.98	1.98	1.02	0.93	1.03	-0.95
	s.e.	[0.53]	[0.56]	[0.56]	[0.53]	[0.62]	[0.60]	[0.62]	[0.64]	[0.62]	[0.63]	[0.71]	[0.89]	[0.80]	[0.76]	[0.61]
	Std	4.15	4.40	4.41	4.14	4.81	4.53	4.84	5.06	4.95	4.91	5.00	5.81	6.24	5.88	4.52
	SR	0.72	0.43	0.24	-0.01	-0.63	0.54	0.33	0.21	0.10	-0.40	0.39	0.18	0.15	0.18	-0.21
	s.e.	[0.12]	[0.14]	[0.13]	[0.13]	[0.11]	[0.13]	[0.13]	[0.13]	[0.13]	[0.12]	[0.13]	[0.13]	[0.13]	[0.13]	[0.12]
$rx^{(10),\$}$	Mean	2.89	2.87	2.62	2.84	-0.05	2.48	2.38	2.79	3.22	0.74	2.26	1.93	2.36	3.68	1.42
	s.e.	[1.22]	[1.24]	[1.18]	[1.09]	[0.91]	[1.29]	[1.28]	[1.26]	[1.19]	[0.93]	[1.32]	[1.47]	[1.34]	[1.40]	[0.88]
	Std	9.59	9.86	9.26	8.62	7.13	10.24	10.05	9.74	9.22	7.45	10.74	10.66	10.91	10.60	7.60
	SR	0.30	0.29	0.28	0.33	-0.01	0.24	0.24	0.29	0.35	0.10	0.21	0.18	0.22	0.35	0.19
	s.e.	[0.13]	[0.13]	[0.13]	[0.13]	[0.13]	[0.13]	[0.13]	[0.13]	[0.13]	[0.13]	[0.13]	[0.13]	[0.14]	[0.12]	[0.14]
$rx^{(10),\$} - rx^{(10),US}$	Mean	1.38	1.36	1.11	1.33	-0.05	0.96	0.86	1.27	1.70	0.74	0.71	0.38	0.81	2.14	1.42
	s.e.	[1.27]	[1.31]	[1.21]	[1.24]	[0.91]	[1.23]	[1.31]	[1.30]	[1.34]	[0.93]	[1.33]	[1.51]	[1.39]	[1.46]	[0.88]
		Panel B: 1971-2012														
$-\Delta s$	Mean	1.86	0.68	0.28	-1.41	-3.27	1.95	0.40	0.36	-1.49	-3.44	2.18	0.41	-0.17	-1.42	-3.60
$f-s$	Mean	-1.64	0.47	1.66	4.63	6.27	-1.59	0.51	1.68	4.56	6.15	-1.46	0.66	1.74	4.35	5.81
r^{FX}	Mean	0.22	1.16	1.94	3.22	3.00	0.36	0.91	2.04	3.07	2.71	0.71	1.07	1.57	2.93	2.22
	s.e.	[1.55]	[1.59]	[1.50]	[1.46]	[0.91]	[1.58]	[1.63]	[1.63]	[1.61]	[0.95]	[1.70]	[1.76]	[1.62]	[1.84]	[0.97]
	Std	9.80	10.13	9.54	9.30	5.83	10.23	10.32	9.98	9.89	6.26	11.33	10.69	10.65	11.16	6.45
	SR	0.02	0.11	0.20	0.35	0.51	0.03	0.09	0.20	0.31	0.43	0.06	0.10	0.15	0.26	0.34
	s.e.	[0.16]	[0.16]	[0.16]	[0.16]	[0.16]	[0.16]	[0.16]	[0.16]	[0.16]	[0.17]	[0.16]	[0.17]	[0.16]	[0.17]	[0.19]
$rx^{(10),*}$	Mean	3.67	2.58	1.20	0.44	-3.23	2.95	2.16	1.25	1.07	-1.88	2.46	1.25	1.05	1.67	-0.80
	s.e.	[0.79]	[0.83]	[0.81]	[0.75]	[0.89]	[0.87]	[0.90]	[0.92]	[0.88]	[0.91]	[1.04]	[1.31]	[1.16]	[1.09]	[0.89]
	Std	4.98	5.29	5.17	4.77	5.62	5.40	5.75	5.95	5.76	5.76	5.92	6.82	7.39	6.79	5.28
	SR	0.74	0.49	0.23	0.09	-0.57	0.55	0.38	0.21	0.19	-0.33	0.42	0.18	0.14	0.25	-0.15
	s.e.	[0.15]	[0.17]	[0.16]	[0.16]	[0.14]	[0.16]	[0.16]	[0.16]	[0.15]	[0.15]	[0.16]	[0.16]	[0.16]	[0.17]	[0.15]
$rx^{(10),\$}$	Mean	3.89	3.73	3.14	3.66	-0.23	3.31	3.07	3.29	4.14	0.83	3.18	2.32	2.61	4.60	1.42
	s.e.	[1.85]	[1.87]	[1.74]	[1.62]	[1.33]	[1.92]	[1.90]	[1.83]	[1.73]	[1.35]	[1.96]	[2.17]	[1.92]	[2.03]	[1.31]
	Std	11.67	12.04	11.04	10.30	8.44	12.43	12.21	11.62	10.92	8.81	12.96	12.74	13.02	12.46	9.04
	SR	0.33	0.31	0.28	0.36	-0.03	0.27	0.25	0.28	0.38	0.09	0.25	0.18	0.20	0.37	0.16
	s.e.	[0.16]	[0.16]	[0.16]	[0.16]	[0.16]	[0.16]	[0.16]	[0.16]	[0.16]	[0.16]	[0.16]	[0.17]	[0.17]	[0.16]	[0.17]
$rx^{(10),\$} - rx^{(10),US}$	Mean	1.38	1.23	0.63	1.15	-0.23	0.78	0.53	0.76	1.61	0.83	0.61	-0.24	0.05	2.03	1.42
	s.e.	[1.86]	[1.91]	[1.72]	[1.80]	[1.33]	[1.78]	[1.89]	[1.84]	[1.91]	[1.35]	[1.97]	[2.24]	[1.99]	[2.15]	[1.31]

Annualized monthly log returns realized at $t+k$ on 10-year Bond Index and T-bills for k from 1 month to 12 months. Portfolios of 21 currencies sorted every month by T-bill rate at t. The unbalanced panel consists of Australia, Austria, Belgium, Canada, Denmark, Finland, France, Germany, Greece, Ireland, Italy, Japan, the Netherlands, New Zealand, Norway, Portugal, Spain, Sweden, Switzerland, and the United Kingdom.

As in the previous samples, the rate at which the high interest rate currencies depreciate (2.99% per annum) is not high enough to offset the interest rate difference of 6.55%. Similarly, the rate at which the low interest rate currencies appreciate (0.43% per annum) is not high enough to offset the low interest rates (3.52% lower than the U.S. interest rate). Uncovered interest rate parity fails in the cross-section. However, the bond return differences (in local currency) are closer to being offset by the rate of depreciation. The bond return spread is 4.63% per annum for the last portfolio, compared to an annual depreciation rate of 6.55%, while the spread on the first portfolio is −0.29%, compared to depreciation of −0.43%.

B Sorting Currencies by the Slope of the Yield Curve

This section presents additional evidence on slope-sorted portfolios, again considering first our benchmark sample of G10 countries before turning to larger sets of developed and emerging countries.

In the sample of developed countries, the steep-slope (low yielding) currencies are typically countries like Germany, the Netherlands, Japan, and Switzerland, while the flat-slope (high-yielding) currencies are typically Australia, New Zealand, Denmark and the U.K.

At the one-month horizon, the 2.4% spread in currency excess returns obtained in this sample is more than offset by the 5.9% spread in local term premia. This produces a statistically significant 3.5% return on a position that is long in the low yielding, high slope currencies and short in the high yielding, low slope currencies. These results are essentially unchanged in the post-Bretton-Woods sample. At longer horizons, the currency excess returns and the local risk premia almost fully offset each other.

In the entire sample of countries, including the emerging market countries, the difference in currency risk premia at the one-month horizon is 3.04% per annum, which is more than offset by a 8.37% difference in local term premia. As a result, investors earn 5.33% per annum on a long-short position in foreign bond portfolios of slope-sorted currencies. As before, this involves shorting the flat-yield-curve currencies, typically high interest rate currencies, and going long in the steep-slope currencies, typically the low interest rate ones. The annualized Sharpe ratio on this long-short strategy is 0.60.

B.1 Benchmark Sample

Figure 8 presents the composition over time of portfolios of the 9 currencies of the benchmark sample sorted by the slope of the yield curve.

Consistent with this distribution of interest rates, average currency excess returns decrease across portfolios. Table 6 reports the annualized moments of log returns on the three slope-sorted portfolios. Average currency excess returns decline from 3.0% per annum on Portfolio 1 to 0.5% per annum on the Portfolio 3 over the last 60 years. Therefore, a long-short position of investing in steep-yield-curve currencies and shorting flat-yield-curve currencies delivers a currency excess return of −2.5% per annum and a Sharpe ratio of −0.4. Our findings confirm those of Ang and Chen (2010). The slope of the yield curve predicts currency excess returns very well. However, note that this result is not mechanical; the spread in the slopes (reported on the third line) is much smaller than the spread in excess returns. Deviations from long-term U.I.P are again small and by construction imprecisely estimated.

Turning to the holding period returns on local bonds, average bond excess returns increase across portfolios. Portfolio 1 produces negative bond excess returns of −0.9% per annum, compared to 3.3% per annum on Portfolio 3. Importantly, this strategy involves long positions in bonds issued by countries like Germany and Japan. These are countries with fairly liquid bond markets and low sovereign credit risk. As a result, credit and liquidity risk differences are unlikely candidate explanations for the return differences. Here again, the bond and currency excess returns move in opposite directions across portfolios.

Turing to the returns on foreign bonds in U.S. dollars, we do not obtain significant differences across portfolios. Average bond excess returns in U.S. dollars tend to increase from the first (flat-yield-curve) portfolio to the last (steep-yield-curve), but a long-short strategy does not deliver a significant excess return. Local bond and currency risk premier offset each other. We get similar findings when we restrict our analysis to the post-Bretton Woods sample.

As a robustness check, Table 7 reports the results of sorting on the yield curve slope on the benchmark G10 sample using different holding periods (one, three, and 12 months).

Table 5: Interest Sorted Portfolios: Whole sample

Portfolio		1	2	3	4	5	5 − 1	1	2	3	4	5	5 − 1	1	2	3	4	5	5 − 1
Horizon		1-month						3-month						12-month					
Panel A: 1950-2012																			
$-\Delta s$	Mean	0.43	-0.05	0.49	-0.63	-2.99	-3.41	0.64	0.05	0.29	-0.67	-3.09	-3.73	0.87	0.04	-0.06	-0.71	-2.95	-3.82
$f-s$	Mean	-1.81	-0.15	0.87	2.09	5.70	7.51	-1.72	0.45	0.89	2.11	5.59	7.31	-1.54	0.00	1.09	2.12	5.32	6.86
rx^{FX}	Mean	-1.38	-0.20	1.36	1.46	2.72	4.10	-1.08	0.50	1.17	1.44	2.50	3.58	-0.66	0.04	1.04	1.41	2.37	3.04
	s.e.	[0.82]	[0.94]	[0.94]	[0.91]	[0.84]	[0.63]	[0.84]	[1.03]	[0.96]	[0.97]	[0.95]	[0.68]	[0.87]	[1.08]	[1.04]	[1.03]	[1.05]	[0.68]
	Std	6.44	7.40	7.43	7.14	6.68	4.98	6.66	11.04	7.38	7.60	7.26	5.46	7.47	8.20	8.71	8.08	8.14	5.71
	SR	-0.22	-0.03	0.18	0.20	0.41	0.82	-0.16	0.05	0.16	0.19	0.35	0.66	-0.09	0.00	0.12	0.18	0.29	0.53
	s.e.	[0.13]	[0.13]	[0.13]	[0.13]	[0.14]	[0.14]	[0.13]	[0.13]	[0.13]	[0.13]	[0.14]	[0.16]	[0.13]	[0.13]	[0.13]	[0.14]	[0.14]	[0.19]
$rx^{(10),*}$	Mean	3.02	1.86	1.45	1.16	0.44	-2.58	2.56	1.05	1.26	1.14	0.99	-1.58	1.95	1.18	1.04	1.04	1.64	-0.31
	s.e.	[0.46]	[0.52]	[0.48]	[0.54]	[0.54]	[0.64]	[0.47]	[0.65]	[0.55]	[0.55]	[0.61]	[0.65]	[0.63]	[0.82]	[0.59]	[0.64]	[0.71]	[0.65]
	Std	3.59	4.05	3.79	4.17	4.29	5.09	3.96	9.22	4.28	4.57	5.00	5.35	4.33	5.24	6.62	5.25	5.52	5.19
	SR	0.84	0.46	0.38	0.28	0.10	-0.51	0.65	0.11	0.29	0.25	0.20	-0.30	0.45	0.22	0.16	0.20	0.30	-0.06
	s.e.	[0.13]	[0.13]	[0.13]	[0.13]	[0.13]	[0.12]	[0.12]	[0.13]	[0.13]	[0.13]	[0.13]	[0.12]	[0.13]	[0.13]	[0.13]	[0.14]	[0.14]	[0.12]
$rx^{(10),\$}$	Mean	1.64	1.66	2.81	2.62	3.16	1.52	1.49	1.56	2.43	2.58	3.49	2.01	1.29	1.22	2.07	2.45	4.02	2.73
	s.e.	[0.99]	[1.12]	[1.08]	[1.09]	[1.05]	[0.97]	[1.00]	[1.26]	[1.11]	[1.08]	[1.16]	[1.01]	[1.03]	[1.35]	[1.24]	[1.21]	[1.25]	[0.95]
	Std	7.81	8.79	8.54	8.51	8.28	7.60	8.23	9.48	8.65	9.00	9.16	8.22	8.76	9.65	9.57	9.72	10.12	8.14
	SR	0.21	0.19	0.33	0.31	0.38	0.20	0.18	0.16	0.28	0.29	0.38	0.24	0.15	0.13	0.22	0.25	0.40	0.34
	s.e.	[0.13]	[0.13]	[0.13]	[0.13]	[0.13]	[0.13]	[0.13]	[0.13]	[0.13]	[0.13]	[0.13]	[0.13]	[0.13]	[0.13]	[0.13]	[0.14]	[0.13]	[0.15]
$rx^{(10),\$}-rx^{(10),US}$	Mean	0.12	0.15	1.30	1.11	1.65	1.52	-0.04	0.04	0.91	1.06	1.97	2.01	-0.26	-0.33	0.53	0.91	2.47	2.73
	s.e.	[1.18]	[1.23]	[1.14]	[1.21]	[1.30]	[0.97]	[1.12]	[1.31]	[1.20]	[1.22]	[1.45]	[1.01]	[1.11]	[1.45]	[1.39]	[1.31]	[1.45]	[0.95]
Panel B: 1971-2012																			
$-\Delta s$	Mean	0.74	0.09	0.46	-0.24	-3.78	-4.52	0.99	0.05	0.41	-0.67	-3.78	-4.77	1.17	0.15	-0.03	-0.74	-3.61	-4.78
$f-s$	Mean	-2.13	-0.16	1.03	2.62	6.91	9.04	-2.02	-0.12	1.07	2.64	6.74	8.76	-1.86	0.02	1.16	2.64	6.40	8.26
rx^{FX}	Mean	-1.39	-0.08	1.50	2.37	3.13	4.52	-1.03	-0.07	1.48	1.97	2.96	3.99	-0.69	0.17	1.13	1.90	2.79	3.48
	s.e.	[1.17]	[1.37]	[1.28]	[1.26]	[1.20]	[0.88]	[1.19]	[1.46]	[1.39]	[1.42]	[1.38]	[0.95]	[1.27]	[1.52]	[1.37]	[1.44]	[1.54]	[1.07]
	Std	7.47	8.75	8.27	8.10	7.76	5.57	7.76	8.97	8.64	8.71	8.47	6.29	8.76	9.44	8.97	9.33	9.80	6.62
	SR	-0.19	-0.01	0.18	0.29	0.40	0.81	-0.13	-0.01	0.17	0.23	0.35	0.63	-0.08	0.02	0.13	0.20	0.28	0.53
	s.e.	[0.16]	[0.16]	[0.16]	[0.17]	[0.16]	[0.17]	[0.16]	[0.16]	[0.16]	[0.16]	[0.17]	[0.19]	[0.16]	[0.16]	[0.16]	[0.17]	[0.18]	[0.27]
$rx^{(10),*}$	Mean	3.73	2.31	2.28	1.80	-0.04	-3.77	3.12	2.13	1.79	1.73	0.81	-2.31	2.52	1.60	1.56	1.19	1.73	-0.79
	s.e.	[0.67]	[0.74]	[0.68]	[0.70]	[0.71]	[0.85]	[0.73]	[0.88]	[0.78]	[0.82]	[0.79]	[0.89]	[0.85]	[1.15]	[0.82]	[0.98]	[1.00]	[0.82]
	Std	4.23	4.77	4.39	4.49	4.53	5.47	4.72	5.54	4.93	5.26	5.37	5.85	4.96	6.16	5.87	6.47	6.19	5.57
	SR	0.88	0.48	0.52	0.40	-0.01	-0.69	0.66	0.38	0.36	0.33	0.15	-0.40	0.51	0.26	0.27	0.18	0.28	-0.14
	s.e.	[0.15]	[0.17]	[0.16]	[0.16]	[0.16]	[0.15]	[0.15]	[0.17]	[0.17]	[0.17]	[0.16]	[0.15]	[0.17]	[0.16]	[0.17]	[0.17]	[0.18]	[0.16]
$rx^{(10),\$}$	Mean	2.34	2.24	3.77	4.17	3.09	0.75	2.09	2.06	3.27	3.70	3.77	1.68	1.83	1.77	2.69	3.08	4.52	2.69
	s.e.	[1.44]	[1.64]	[1.51]	[1.47]	[1.47]	[1.33]	[1.47]	[1.78]	[1.58]	[1.56]	[1.66]	[1.43]	[1.51]	[1.90]	[1.65]	[1.70]	[1.84]	[1.46]
	Std	9.13	10.43	9.74	9.37	9.45	8.48	9.71	10.96	10.01	9.97	10.49	9.43	10.29	11.13	10.84	11.14	12.07	9.49
	SR	0.26	0.21	0.39	0.45	0.33	0.09	0.22	0.19	0.33	0.37	0.36	0.18	0.18	0.16	0.25	0.28	0.37	0.28
	s.e.	[0.16]	[0.16]	[0.16]	[0.16]	[0.16]	[0.16]	[0.15]	[0.16]	[0.16]	[0.16]	[0.16]	[0.16]	[0.16]	[0.17]	[0.17]	[0.18]	[0.18]	[0.20]
$rx^{(10),\$}-rx^{(10),US}$	Mean	-0.17	-0.27	1.27	1.67	0.59	0.75	-0.44	-0.47	0.74	1.17	1.24	1.68	-0.73	-0.79	0.13	0.52	1.96	2.69
	s.e.	[1.61]	[1.66]	[1.53]	[1.58]	[1.83]	[1.33]	[1.54]	[1.82]	[1.58]	[1.67]	[2.07]	[1.43]	[1.51]	[2.01]	[1.77]	[1.87]	[2.11]	[1.46]

Annualized monthly log returns realized at $t+k$ on 10-year Bond Index and T-bills for k from 1 month to 12 months. Portfolios of 30 currencies sorted every month by T-bill rate at t. The unbalanced panel consists of Australia, Austria, Belgium, Canada, Denmark, Finland, France, Germany, Greece, India, Ireland, Italy, Japan Mexico, Malaysia, the Netherlands, New Zealand, Norway, Pakistan, the Philippines, Poland, Portugal, South Africa, Singapore, Spain, Sweden, Switzerland, Taiwan, Thailand, and the United Kingdom.

Table 6: Slope-Sorted Portfolios

Portfolio		Panel A: 12/1950–12/2012				Panel B: 12/1971–12/2012			
		1	2	3	$3-1$	1	2	3	$3-1$
$-\Delta s$	Mean	0.01	0.39	1.18	1.17	-0.08	0.61	1.51	1.60
$f-s$	Mean	2.96	0.42	-0.71	-3.68	3.31	0.54	-0.94	-4.25
$y^{(10),*}-r^{*,f}$	Mean	-0.25	0.33	0.74	0.99	-0.32	0.28	0.67	0.99
$y^{(10),*}-y^{(10)}-\overline{\Delta s}^{(10)}$	Mean	0.14	0.42	1.21	1.07	-0.44	0.26	0.82	1.25
rx^{FX}	Mean	2.97	0.81	0.47	-2.50	3.23	1.15	0.58	-2.65
	s.e.	[1.08]	[1.03]	[0.95]	[0.87]	[1.65]	[1.55]	[1.44]	[1.26]
	Std	8.25	7.75	7.60	6.84	10.09	9.38	9.16	8.15
	SR	0.36	0.10	0.06	-0.37	0.32	0.12	0.06	-0.32
	s.e.	[0.14]	[0.13]	[0.13]	[0.15]	[0.17]	[0.16]	[0.16]	[0.18]
$rx^{(10),*}$	Mean	-0.86	1.33	3.33	4.19	-0.52	1.70	3.64	4.16
	s.e.	[0.58]	[0.51]	[0.58]	[0.60]	[0.85]	[0.75]	[0.81]	[0.85]
	Std	4.60	4.24	4.65	4.67	5.45	5.03	5.20	5.26
	SR	-0.19	0.31	0.72	0.90	-0.10	0.34	0.70	0.79
	s.e.	[0.13]	[0.13]	[0.13]	[0.13]	[0.16]	[0.16]	[0.17]	[0.15]
$rx^{(10),\$}$	Mean	2.12	2.14	3.80	1.68	2.70	2.85	4.22	1.51
	s.e.	[1.19]	[1.17]	[1.18]	[1.08]	[1.81]	[1.74]	[1.73]	[1.56]
	Std	9.34	8.98	9.42	8.14	11.30	10.79	11.15	9.59
	SR	0.23	0.24	0.40	0.21	0.24	0.26	0.38	0.16
	s.e.	[0.13]	[0.13]	[0.13]	[0.12]	[0.16]	[0.16]	[0.16]	[0.15]
$rx^{(10),\$}-rx^{(10),US}$	Mean	0.60	0.62	2.28	1.68	0.17	0.32	1.69	1.51
	s.e.	[1.42]	[1.29]	[1.14]	[1.08]	[2.09]	[1.88]	[1.62]	[1.56]

Notes: The table reports the average change in exchange rates (Δs), the average interest rate difference ($f-s$), the average slope ($y^{(10),*}-r^{*,f}$), the average deviation from the long run U.I.P. condition ($y^{(10),*}-y^{(10)}-\overline{\Delta s}^{(10)}$, where $\overline{\Delta s}^{(10)}$ denotes the average change in exchange rate in the next 10 years), the average log currency excess return (rx^{FX}), the average log foreign bond excess return on 10-year government bond indices in foreign currency ($rx^{(10),*}$) and in U.S. dollars ($rx^{(10),\$}$), as well as the difference between the average foreign bond log excess return in U.S. dollars and the average U.S. bond log excess return ($rx^{(10),\$}-rx^{US}$). For the excess returns, the table also reports their annualized standard deviation (denoted Std) and their Sharpe ratios (denoted SR). The holding period of the returns is three months. Log returns are annualized. The balanced panel consists of Australia, Canada, Japan, Germany, Norway, New Zealand, Sweden, Switzerland, and the U.K. The countries are sorted by the slope of their yield curves into three portfolios. The slope of the yield curve is measured by the difference between the 10-year yield and the one-month interest rate at date t. The standard errors (denoted s.e. and reported between brackets) were generated by bootstrapping 10,000 samples of non-overlapping returns.

Table 7: Slope-Sorted Portfolios: Benchmark Sample

Portfolio		1	2	3	3 − 1	1	2	3	3 − 1	1	2	3	3 − 1
Horizon		1-Month				3-Month				12-Month			
						Panel A: 12/1950–12/2012							
$-\Delta s$	Mean	-0.01	0.77	0.83	0.84	0.01	0.39	1.18	1.17	-0.09	0.55	1.09	1.18
$f-s$	Mean	3.03	0.41	-0.77	-3.81	2.96	0.42	-0.71	-3.68	2.76	0.46	-0.55	-3.31
rx^{FX}	Mean	3.02	1.18	0.06	-2.97	2.97	0.81	0.47	-2.50	2.67	1.01	0.54	-2.13
	s.e.	[0.97]	[0.94]	[0.94]	[0.81]	[1.08]	[1.03]	[0.95]	[0.87]	[1.14]	[1.07]	[1.13]	[0.86]
	Std	7.59	7.37	7.36	6.30	8.25	7.75	7.60	6.84	9.05	8.31	8.39	6.65
	SR	0.40	0.16	0.01	-0.47	0.36	0.10	0.06	-0.37	0.30	0.12	0.06	-0.32
	s.e.	[0.13]	[0.13]	[0.13]	[0.13]	[0.14]	[0.13]	[0.13]	[0.15]	[0.14]	[0.13]	[0.13]	[0.14]
$rx^{(10),*}$	Mean	-1.82	1.61	4.00	5.82	-0.86	1.33	3.33	4.19	-0.22	1.20	2.79	3.01
	s.e.	[0.50]	[0.46]	[0.51]	[0.54]	[0.58]	[0.51]	[0.58]	[0.60]	[0.62]	[0.69]	[0.65]	[0.58]
	Std	3.97	3.67	4.09	4.29	4.60	4.24	4.65	4.67	5.10	4.88	5.29	4.90
	SR	-0.46	0.44	0.98	1.35	-0.19	0.31	0.72	0.90	-0.04	0.25	0.53	0.61
	s.e.	[0.12]	[0.12]	[0.13]	[0.13]	[0.13]	[0.13]	[0.13]	[0.13]	[0.13]	[0.13]	[0.15]	[0.11]
$rx^{(10),\$}$	Mean	1.21	2.79	4.06	2.85	2.12	2.14	3.80	1.68	2.45	2.21	3.33	0.88
	s.e.	[1.09]	[1.07]	[1.12]	[0.99]	[1.19]	[1.17]	[1.18]	[1.08]	[1.28]	[1.18]	[1.31]	[1.12]
	Std	8.61	8.36	8.84	7.76	9.34	8.98	9.42	8.14	10.45	9.57	10.29	8.67
	SR	0.14	0.33	0.46	0.37	0.23	0.24	0.40	0.21	0.23	0.23	0.32	0.10
	s.e.	[0.13]	[0.13]	[0.13]	[0.13]	[0.13]	[0.13]	[0.13]	[0.12]	[0.14]	[0.13]	[0.13]	[0.12]
$rx^{(10),\$}-rx^{(10),US}$	Mean	-0.30	1.28	2.55	2.85	0.60	0.62	2.28	1.68	0.91	0.66	1.79	0.88
	s.e.	[1.28]	[1.14]	[1.21]	[0.99]	[1.42]	[1.29]	[1.14]	[1.08]	[1.51]	[1.25]	[1.37]	[1.12]
						Panel B: 12/1971–12/2012							
$-\Delta s$	Mean	-0.08	1.21	1.03	1.11	-0.08	0.61	1.51	1.60	-0.30	0.76	1.47	1.77
$f-s$	Mean	3.40	0.54	-1.02	-4.42	3.31	0.54	-0.94	-4.25	3.08	0.57	-0.72	-3.80
rx^{FX}	Mean	3.32	1.75	0.01	-3.32	3.23	1.15	0.58	-2.65	2.78	1.33	0.75	-2.03
	s.e.	[1.45]	[1.41]	[1.37]	[1.16]	[1.65]	[1.55]	[1.44]	[1.26]	[1.71]	[1.62]	[1.66]	[1.22]
	Std	9.29	8.95	8.80	7.40	10.09	9.38	9.16	8.15	11.04	10.05	10.12	7.95
	SR	0.36	0.20	0.00	-0.45	0.32	0.12	0.06	-0.32	0.25	0.13	0.07	-0.26
	s.e.	[0.16]	[0.16]	[0.16]	[0.16]	[0.17]	[0.16]	[0.16]	[0.18]	[0.17]	[0.16]	[0.16]	[0.17]
$rx^{(10),*}$	Mean	-1.68	1.94	4.56	6.24	-0.52	1.70	3.64	4.16	0.10	1.71	2.98	2.87
	s.e.	[0.74]	[0.69]	[0.72]	[0.76]	[0.85]	[0.75]	[0.81]	[0.85]	[0.88]	[1.02]	[0.91]	[0.83]
	Std. Dev.	4.67	4.37	4.63	4.84	5.45	5.03	5.20	5.26	5.95	5.73	5.86	5.54
	SR	-0.36	0.44	0.98	1.29	-0.10	0.34	0.70	0.79	0.02	0.30	0.51	0.52
	s.e.	[0.15]	[0.15]	[0.17]	[0.15]	[0.16]	[0.16]	[0.17]	[0.15]	[0.16]	[0.17]	[0.20]	[0.12]
$rx^{(10),\$}$	Mean	1.64	3.69	4.56	2.93	2.70	2.85	4.22	1.51	2.88	3.04	3.73	0.84
	s.e.	[1.63]	[1.59]	[1.62]	[1.41]	[1.81]	[1.74]	[1.73]	[1.56]	[1.89]	[1.77]	[1.91]	[1.63]
	Std	10.45	10.13	10.46	8.97	11.30	10.79	11.15	9.59	12.54	11.33	12.15	10.23
	SR	0.16	0.36	0.44	0.33	0.24	0.26	0.38	0.16	0.23	0.27	0.31	0.08
	s.e.	[0.16]	[0.16]	[0.16]	[0.15]	[0.16]	[0.16]	[0.16]	[0.15]	[0.17]	[0.17]	[0.17]	[0.15]
$rx^{(10),\$}-rx^{(10),US}$	Mean	-0.87	1.18	2.06	2.93	0.17	0.32	1.69	1.51	0.32	0.47	1.16	0.84
	s.e.	[1.85]	[1.63]	[1.69]	[1.41]	[2.09]	[1.88]	[1.62]	[1.56]	[2.23]	[1.87]	[1.98]	[1.63]

Notes: The table reports the average change in exchange rates (Δs), the average interest rate difference ($f-s$), the average log currency excess return (rx^{FX}), the average log foreign bond excess return on 10-year government bond indices in foreign currency ($rx^{(10),*}$) and in U.S. dollars ($rx^{(10),\$}$), as well as the difference between the average foreign bond log excess return in U.S. dollars and the average U.S. bond log excess return ($rx^{(10),\$}-rx^{US}$). For the excess returns, the table also reports their annualized standard deviation (denoted Std) and their Sharpe ratios (denoted SR). The annualized monthly log returns are realized at date $t+k$, where the horizon k equals 1, 3, and 12 months. The balanced panel consists of Australia, Canada, Japan, Germany, Norway, New Zealand, Sweden, Switzerland, and the U.K. The countries are sorted by the slope of their yield curves into three portfolios. The slope of the yield curve is measured by the difference between the 10-year yield and the one-month interest rate at date t. The standard errors (denoted s.e. and reported between brackets) were generated by bootstrapping 10,000 samples of non-overlapping returns.

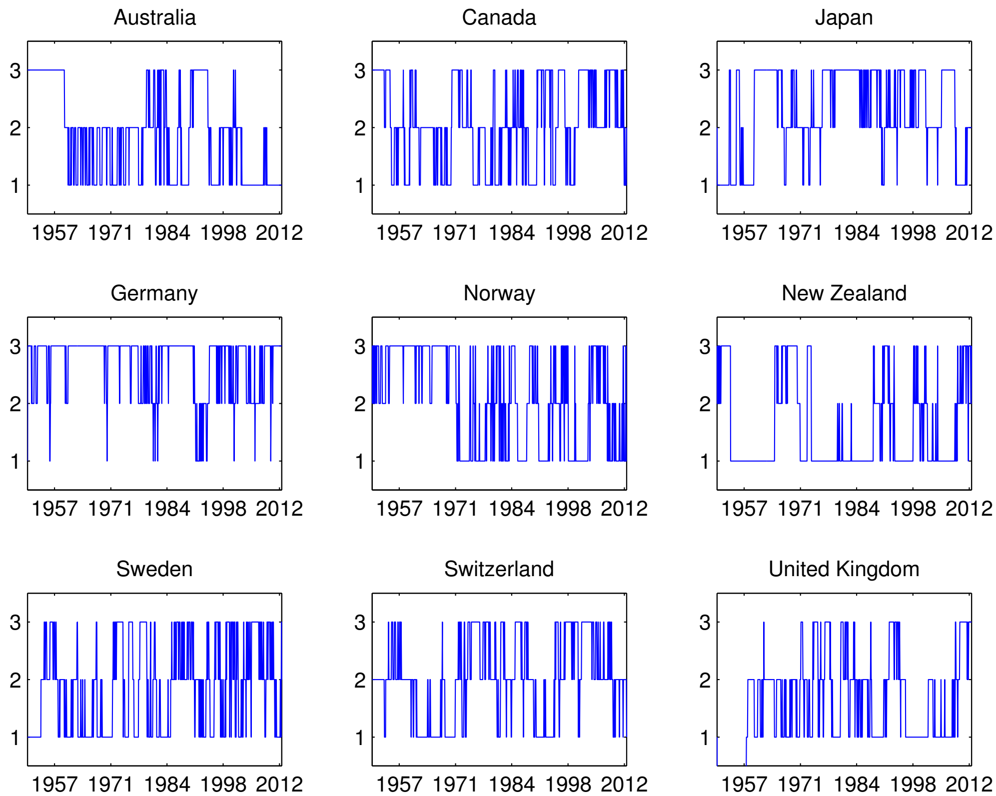

Figure 8: Composition of Slope-Sorted Portfolios — The figure presents the composition of portfolios of the currencies in the benchmark sample sorted by the slope of their yield curves. The portfolios are rebalanced monthly. The slope of the yield curve is measured as the 10-year interest rate minus the one-month Treasury bill rates. Data are monthly, from 12/1950 to 12/2012.

B.2 Developed Countries

Table 8 reports the results of sorting on the yield curve slope on the sample of developed countries. The results are commented in the main text.

B.3 Whole Sample

Table 9 reports the results obtained from using the entire cross-section of countries, including emerging countries. Here again, the results are commented in the main text.

C Foreign Bond Returns Across Maturities

This section reports additional results obtained with zero-coupon bonds. We start with the bond risk premia in our benchmark sample of G10 countries and then turn to a larger set of developed countries. We then show that holding period returns on zero-coupon bonds, once converted to a common currency (the U.S. dollar, in particular), become increasingly similar as bond maturities approach infinity.

Table 8: Slope Sorted Portfolios: Developed sample

Portfolio		1	2	3	4	4 − 1	1	2	3	4	4 − 1	1	2	3	4	4 − 1
Horizon		1-month					3-month					12-month				
Panel A: 1950-2012																
$-\Delta s$	Mean	-0.78	0.14	0.05	0.67	1.45	-0.94	0.25	-0.06	0.68	1.62	-0.79	0.13	-0.01	0.48	1.28
$f-s$	Mean	3.69	1.64	0.82	-0.18	-3.87	3.60	1.63	0.85	-0.12	-3.71	3.33	1.62	0.90	0.06	-3.27
rx^{FX}	Mean	2.91	1.78	0.87	0.49	-2.42	2.66	1.88	0.79	0.56	-2.10	2.54	1.74	0.89	0.54	-1.99
	s.e.	[0.96]	[1.01]	[1.05]	[1.03]	[0.63]	[1.09]	[1.07]	[1.11]	[1.05]	[0.64]	[1.22]	[1.07]	[1.23]	[1.15]	[0.65]
	Std	7.62	7.92	8.16	8.08	5.03	8.33	8.08	8.59	8.09	4.95	9.32	8.68	9.44	8.67	4.88
	SR	0.38	0.22	0.11	0.06	-0.48	0.32	0.23	0.09	0.07	-0.42	0.27	0.20	0.09	0.06	-0.41
	s.e.	[0.13]	[0.13]	[0.13]	[0.13]	[0.15]	[0.13]	[0.13]	[0.13]	[0.13]	[0.15]	[0.13]	[0.13]	[0.13]	[0.13]	[0.14]
$rx^{(10),*}$	Mean	-1.96	0.27	2.27	3.95	5.90	-1.29	0.95	1.88	3.15	4.44	-0.33	1.08	1.60	2.20	2.52
	s.e.	[0.51]	[0.52]	[0.51]	[0.74]	[0.84]	[0.61]	[0.58]	[0.61]	[0.78]	[0.86]	[0.68]	[0.86]	[0.67]	[1.08]	[1.01]
	Std	4.05	4.08	4.02	5.84	6.60	4.89	4.72	4.67	6.43	7.01	5.86	5.82	5.68	6.87	6.97
	SR	-0.48	0.07	0.56	0.68	0.89	-0.26	0.20	0.40	0.49	0.63	-0.06	0.19	0.28	0.32	0.36
	s.e.	[0.13]	[0.13]	[0.13]	[0.14]	[0.16]	[0.13]	[0.13]	[0.13]	[0.14]	[0.15]	[0.13]	[0.14]	[0.13]	[0.13]	[0.18]
$rx^{(10),\$}$	Mean	0.95	2.05	3.14	4.44	3.48	1.37	2.83	2.67	3.71	2.34	2.21	2.82	2.49	2.74	0.53
	s.e.	[1.09]	[1.15]	[1.15]	[1.39]	[1.10]	[1.22]	[1.18]	[1.28]	[1.40]	[1.12]	[1.36]	[1.35]	[1.37]	[1.59]	[1.30]
	Std	8.59	9.06	8.97	10.91	8.72	9.51	9.23	9.72	11.27	9.09	10.86	10.37	11.18	11.36	9.45
	SR	0.11	0.23	0.35	0.41	0.40	0.14	0.31	0.27	0.33	0.26	0.20	0.27	0.22	0.24	0.06
	s.e.	[0.13]	[0.13]	[0.13]	[0.13]	[0.13]	[0.13]	[0.13]	[0.13]	[0.13]	[0.12]	[0.13]	[0.13]	[0.13]	[0.13]	[0.14]
$rx^{(10),\$}-rx^{(10),US}$	Mean	-0.56	0.53	1.63	2.93	3.48	-0.15	1.31	1.15	2.19	2.34	0.67	1.28	0.95	1.20	0.53
	s.e.	[1.25]	[1.23]	[1.19]	[1.46]	[1.10]	[1.40]	[1.24]	[1.26]	[1.42]	[1.12]	[1.47]	[1.35]	[1.43]	[1.63]	[1.30]
Panel B: 1971-2012																
$-\Delta s$	Mean	-0.96	0.18	0.29	0.84	1.80	-1.20	0.37	0.00	0.83	2.03	-1.18	0.27	0.05	0.54	1.73
$f-s$	Mean	4.29	1.89	1.03	-0.20	-4.49	4.18	1.87	1.06	-0.11	-4.29	3.87	1.86	1.12	0.13	-3.74
rx^{FX}	Mean	3.33	2.07	1.32	0.64	-2.69	2.98	2.24	1.06	0.72	-2.26	2.69	2.13	1.18	0.68	-2.01
	s.e.	[1.44]	[1.51]	[1.52]	[1.55]	[0.94]	[1.61]	[1.60]	[1.62]	[1.58]	[0.95]	[1.82]	[1.62]	[1.82]	[1.75]	[0.95]
	Std	9.22	9.71	9.75	9.90	5.99	10.13	9.87	10.35	9.90	5.94	11.35	10.55	11.43	10.61	5.90
	SR	0.36	0.21	0.14	0.07	-0.45	0.29	0.23	0.10	0.07	-0.38	0.24	0.20	0.10	0.06	-0.34
	s.e.	[0.16]	[0.16]	[0.16]	[0.16]	[0.17]	[0.16]	[0.16]	[0.16]	[0.16]	[0.17]	[0.16]	[0.16]	[0.16]	[0.16]	[0.18]
$rx^{(10),*}$	Mean	-1.89	0.75	2.56	4.76	6.65	-1.10	1.72	2.32	3.50	4.60	0.07	1.81	2.03	2.17	2.10
	s.e.	[0.74]	[0.73]	[0.75]	[1.09]	[1.22]	[0.89]	[0.86]	[0.87]	[1.14]	[1.26]	[0.95]	[1.26]	[0.95]	[1.62]	[1.52]
	Std	4.77	4.75	4.77	6.92	7.79	5.77	5.58	5.50	7.61	8.33	6.85	6.77	6.69	8.12	8.33
	SR	-0.40	0.16	0.54	0.69	0.85	-0.19	0.31	0.42	0.46	0.55	0.01	0.27	0.30	0.27	0.25
	s.e.	[0.16]	[0.16]	[0.15]	[0.17]	[0.18]	[0.16]	[0.16]	[0.16]	[0.17]	[0.17]	[0.16]	[0.18]	[0.17]	[0.16]	[0.19]
$rx^{(10),\$}$	Mean	1.44	2.82	3.87	5.40	3.96	1.88	3.96	3.38	4.21	2.34	2.76	3.94	3.21	2.85	0.09
	s.e.	[1.60]	[1.70]	[1.66]	[2.09]	[1.63]	[1.77]	[1.75]	[1.85]	[2.07]	[1.66]	[1.97]	[1.97]	[1.97]	[2.40]	[1.95]
	Std	10.29	10.98	10.67	13.24	10.35	11.40	11.14	11.60	13.61	10.87	12.91	12.27	13.29	13.67	11.38
	SR	0.14	0.26	0.36	0.41	0.38	0.16	0.36	0.29	0.31	0.22	0.21	0.32	0.24	0.21	0.01
	s.e.	[0.16]	[0.16]	[0.16]	[0.16]	[0.16]	[0.16]	[0.16]	[0.16]	[0.16]	[0.16]	[0.16]	[0.17]	[0.16]	[0.16]	[0.16]
$rx^{(10),\$}-rx^{(10),US}$	Mean	-1.06	0.31	1.37	2.90	3.96	-0.66	1.43	0.85	1.68	2.34	0.19	1.37	0.64	0.28	0.09
	s.e.	[1.76]	[1.74]	[1.66]	[2.14]	[1.63]	[2.01]	[1.79]	[1.80]	[2.08]	[1.66]	[2.15]	[1.96]	[2.06]	[2.43]	[1.95]

Annualized monthly log returns realized at $t+k$ on 10-year Bond Index and T-bills for k from 1 month to 12 months. Portfolios of 21 currencies sorted every month by the slope of the yield curve (10-year yield minus T-bill rate) at t. The unbalanced panel consists of Australia, Austria, Belgium, Canada, Denmark, Finland, France, Germany, Greece, Ireland, Italy, Japan, the Netherlands, New Zealand, Norway, Portugal, Spain, Sweden, Switzerland, and the United Kingdom.

Table 9: Slope Sorted Portfolios: Whole sample

Portfolio		1	2	3	4	5	$5-1$	1	2	3	4	5	$5-1$	1	2	3	4	5	$5-1$
Horizon		1-month						3-month						12-month					
								Panel A: 1950-2012											
$-\Delta s$	Mean	-2.15	-0.71	-0.18	0.27	-0.47	1.67	-2.47	-0.53	0.02	-0.11	-0.32	2.14	-2.19	-0.59	-0.15	0.02	-0.55	1.64
$f-s$	Mean	4.63	2.06	1.28	0.52	-0.08	-4.71	4.45	2.04	1.30	0.54	0.50	-3.95	4.12	1.99	1.30	0.74	0.27	-3.85
rx^{FX}	Mean	2.49	1.35	1.10	0.79	-0.55	-3.04	1.99	1.51	1.32	0.43	0.18	-1.81	1.93	1.40	1.15	0.76	-0.28	-2.21
	s.e.	[0.94]	[0.91]	[0.98]	[1.03]	[0.82]	[0.72]	[1.04]	[0.97]	[1.05]	[1.08]	[0.84]	[0.73]	[1.16]	[1.02]	[1.11]	[1.18]	[0.89]	[0.81]
	Std	7.48	7.10	7.70	8.00	6.37	5.65	8.11	7.66	7.92	8.14	8.91	8.56	8.90	8.16	8.76	9.70	6.87	6.06
	SR	0.33	0.19	0.14	0.10	-0.09	-0.54	0.24	0.20	0.17	0.05	0.02	-0.21	0.22	0.17	0.13	0.08	-0.04	-0.36
	s.e.	[0.14]	[0.13]	[0.13]	[0.13]	[0.13]	[0.16]	[0.14]	[0.13]	[0.13]	[0.13]	[0.13]	[0.16]	[0.13]	[0.13]	[0.13]	[0.13]	[0.13]	[0.14]
$rx^{(10),*}$	Mean	-3.32	-0.82	1.46	2.56	5.05	8.37	-2.56	-0.03	1.43	2.11	3.83	6.38	-1.32	0.32	1.50	1.55	3.07	4.40
	s.e.	[0.53]	[0.49]	[0.46]	[0.49]	[0.66]	[0.81]	[0.62]	[0.54]	[0.56]	[0.59]	[0.68]	[0.82]	[0.58]	[0.69]	[0.75]	[0.66]	[0.96]	[0.89]
	Std	4.14	3.88	3.65	3.91	5.20	6.31	4.87	4.42	4.40	4.62	8.46	9.18	5.05	5.42	5.57	6.80	6.06	6.36
	SR	-0.80	-0.21	0.40	0.66	0.97	1.33	-0.53	-0.01	0.32	0.46	0.45	0.70	-0.26	0.06	0.27	0.23	0.51	0.69
	s.e.	[0.11]	[0.12]	[0.13]	[0.14]	[0.15]	[0.15]	[0.11]	[0.13]	[0.13]	[0.13]	[0.15]	[0.16]	[0.14]	[0.14]	[0.14]	[0.14]	[0.12]	[0.17]
$rx^{(10),\$}$	Mean	-0.83	0.53	2.56	3.35	4.50	5.33	-0.57	1.48	2.74	2.54	4.00	4.57	0.61	1.72	2.65	2.31	2.80	2.19
	s.e.	[1.09]	[1.06]	[1.08]	[1.17]	[1.16]	[1.14]	[1.24]	[1.07]	[1.16]	[1.28]	[1.18]	[1.18]	[1.28]	[1.16]	[1.38]	[1.34]	[1.33]	[1.20]
	Std	8.80	8.23	8.53	9.17	9.08	8.95	9.84	8.71	9.04	9.73	9.35	9.63	10.70	9.59	10.73	10.83	9.29	9.32
	SR	-0.09	0.06	0.30	0.37	0.50	0.60	-0.06	0.17	0.30	0.26	0.43	0.47	0.06	0.18	0.25	0.21	0.30	0.23
	s.e.	[0.13]	[0.13]	[0.13]	[0.13]	[0.14]	[0.12]	[0.13]	[0.13]	[0.13]	[0.13]	[0.13]	[0.12]	[0.13]	[0.13]	[0.13]	[0.13]	[0.13]	[0.13]
$rx^{(10),\$}-rx^{(10),US}$	Mean	-2.34	-0.99	1.04	1.84	2.99	5.33	-2.09	-0.04	1.22	1.02	2.48	4.57	-0.94	0.18	1.11	0.76	1.25	2.19
	s.e.	[1.32]	[1.19]	[1.19]	[1.19]	[1.33]	[1.14]	[1.50]	[1.21]	[1.22]	[1.27]	[1.33]	[1.18]	[1.53]	[1.21]	[1.51]	[1.30]	[1.45]	[1.20]
								Panel B: 1971-2012											
$-\Delta s$	Mean	-2.91	-0.62	-0.10	0.13	-0.96	1.95	-3.30	-0.43	-0.07	-0.08	-0.85	2.45	-2.73	-0.65	0.03	-0.08	-1.05	1.68
$f-s$	Mean	5.54	2.37	1.53	0.66	-0.11	-5.65	5.30	2.35	1.55	0.68	0.05	-5.25	4.94	2.29	1.55	0.74	0.33	-4.61
rx^{FX}	Mean	2.63	1.75	1.44	0.79	-1.06	-3.70	2.01	1.91	1.49	0.60	-0.80	-2.81	2.20	1.65	1.58	0.66	-0.72	-2.93
	s.e.	[1.39]	[1.32]	[1.41]	[1.37]	[1.13]	[1.06]	[1.53]	[1.41]	[1.45]	[1.48]	[1.16]	[1.13]	[1.60]	[1.50]	[1.55]	[1.67]	[1.24]	[1.11]
	Std	8.95	8.39	9.02	8.74	7.27	6.80	9.72	9.07	9.38	9.11	7.25	7.19	10.49	9.75	10.37	10.24	7.80	7.02
	SR	0.29	0.21	0.16	0.09	-0.15	-0.54	0.21	0.21	0.16	0.07	-0.11	-0.39	0.21	0.17	0.15	0.06	-0.09	-0.42
	s.e.	[0.17]	[0.16]	[0.16]	[0.16]	[0.16]	[0.20]	[0.16]	[0.16]	[0.16]	[0.16]	[0.16]	[0.18]	[0.17]	[0.16]	[0.16]	[0.16]	[0.16]	[0.17]
$rx^{(10),*}$	Mean	-3.73	-0.56	1.40	3.81	6.13	9.85	-2.73	0.47	1.54	2.93	5.15	7.87	-1.19	0.72	2.04	2.20	3.54	4.73
	s.e.	[0.77]	[0.72]	[0.66]	[0.70]	[0.90]	[1.11]	[0.89]	[0.78]	[0.84]	[0.80]	[0.93]	[1.15]	[0.82]	[1.00]	[1.12]	[0.89]	[1.32]	[1.23]
	Std	4.90	4.61	4.30	4.51	5.81	7.10	5.72	5.29	5.36	5.34	6.18	7.55	5.85	6.31	6.64	6.49	6.68	7.10
	SR	-0.76	-0.12	0.33	0.85	1.06	1.39	-0.48	0.09	0.29	0.55	0.83	1.04	-0.20	0.11	0.31	0.34	0.53	0.67
	s.e.	[0.14]	[0.15]	[0.16]	[0.16]	[0.18]	[0.18]	[0.14]	[0.16]	[0.16]	[0.16]	[0.18]	[0.18]	[0.16]	[0.18]	[0.19]	[0.18]	[0.16]	[0.20]
$rx^{(10),\$}$	Mean	-1.10	1.19	2.84	4.60	5.06	6.16	-0.72	2.39	3.03	3.53	4.35	5.07	1.02	2.37	3.62	2.86	2.82	1.80
	s.e.	[1.63]	[1.52]	[1.56]	[1.59]	[1.60]	[1.61]	[1.80]	[1.55]	[1.61]	[1.77]	[1.63]	[1.72]	[1.77]	[1.70]	[1.94]	[1.89]	[1.87]	[1.74]
	Std	10.49	9.74	10.03	10.17	10.31	10.25	11.69	10.33	10.66	11.09	10.49	11.16	12.56	11.25	12.53	12.62	10.38	10.66
	SR	-0.10	0.12	0.28	0.45	0.49	0.60	-0.06	0.23	0.28	0.32	0.41	0.45	0.08	0.21	0.29	0.23	0.27	0.17
	s.e.	[0.15]	[0.16]	[0.16]	[0.16]	[0.17]	[0.15]	[0.15]	[0.16]	[0.16]	[0.16]	[0.16]	[0.15]	[0.16]	[0.17]	[0.16]	[0.17]	[0.16]	[0.17]
$rx^{(10),\$}-rx^{(10),US}$	Mean	-3.60	-1.32	0.33	2.09	2.56	6.16	-3.25	-0.14	0.50	1.00	1.82	5.07	-1.55	-0.20	1.06	0.29	0.25	1.80
	s.e.	[1.89]	[1.63]	[1.65]	[1.56]	[1.78]	[1.61]	[2.17]	[1.68]	[1.64]	[1.74]	[1.77]	[1.72]	[2.17]	[1.72]	[2.12]	[1.78]	[2.00]	[1.74]

Annualized monthly log returns realized at $t+k$ on 10-year Bond Index and T-bills for k from 1 month to 12 months. Portfolios of 30 currencies sorted every month by the slope of the yield curve (10-year yield minus T-bill rate) at t. The unbalanced panel consists of Australia, Austria, Belgium, Canada, Denmark, Finland, France, Germany, Greece, India, Ireland, Italy, Japan Mexico, Malaysia, the Netherlands, New Zealand, Norway, Pakistan, the Philippines, Poland, Portugal, South Africa, Singapore, Spain, Sweden, Switzerland, Taiwan, Thailand, and the United Kingdom.

C.1 Benchmark Sample

Table 10 reports summary statistics on one-quarter holding period returns on zero-coupon bond positions with maturities from 4 (1 year) to 60 quarters (15 years).

At the short end of the maturity spectrum, it is profitable to invest in flat-yield-curve currencies and short the currencies of countries with steep yield curves: the annualized dollar excess return on that strategy using 1-year bonds is 4.10%. However, this excess return monotonically declines as the bond maturity increases: it is 2.33% using 5-year bonds and only 0.52% using 10-year bonds. At the long end of the maturity spectrum, this strategy delivers negative dollar excess returns: an investor who buys the 15-year bond of flat-yield-curve currencies and shorts the 15-year bond of steep-yield-curve currencies loses 0.42% per year on average. Foreign bond risk premia decrease with the bond maturity.

Figure 9 reports results for all maturities. The figure shows the local currency excess returns (in logs) in the top panel, and the dollar excess returns (in logs) in the bottom panel. The top panel in Figure 9 shows that countries with the steepest local yield curves (Portfolio 3, center) exhibit local bond excess returns that are higher, and increase faster with the maturity than the flat yield curve countries (Portfolio 1, on the left-hand side). Thus, ignoring the effect of exchange rates, investors should invest in the short-term and long-term bonds of steep yield curve currencies.

Considering the effect of currency fluctuations by focusing on dollar returns radically alters the results. Figure 9 shows that the dollar excess returns of Portfolio 1 are higher than those of Portfolio 3 at the short end of the yield curve, consistent with the carry trade results of Ang and Chen (2010). Yet, an investor who would attempt to replicate the short-maturity carry trade strategy at the long end of the maturity curve would incur losses on average: the long-maturity excess returns of flat yield curve currencies are lower than those of steep yield curve currencies, as currency risk premia more than offset term premia. This result is apparent in the lower panel on the right, which is the same as Figure 1 in the main text.

C.2 Developed Countries

Table 11 is the equivalent of Table 10 but for a larger set of developed countries. Results are very similar to those of our benchmark sample.

An investor who buys the one-year bonds of flat-yield curve currencies and shorts the one-year bonds of steep-yield-curve currencies realizes a dollar excess return of 4.1% per year on average. However, at the long end of the maturity structure this strategy generates negative and insignificant excess returns: the average annualized dollar excess return of an investor who pursues this strategy using 15-year bonds is −0.4%. Foreign bond risk premia decrease with the bonds' maturity.

Table 10: The Maturity Structure of Returns in Slope-Sorted Portfolios

Maturity		One Year				Five Years				Ten Years				Fifteen Years			
Portfolio		1	2	3	3 − 1	1	2	3	3 − 1	1	2	3	3 − 1	1	2	3	3 − 1
$-\Delta s$	Mean	2.80	2.51	2.96	0.16	2.80	2.51	2.96	0.16	2.80	2.51	2.96	0.16	2.75	2.55	2.87	0.13
$f - s$	Mean	3.15	0.79	-0.31	-3.46	3.15	0.79	-0.31	-3.46	3.15	0.79	-0.31	-3.46	3.12	0.73	-0.36	-3.48
rx^{FX}	Mean	5.95	3.30	2.64	-3.31	5.95	3.30	2.64	-3.31	5.95	3.30	2.64	-3.31	5.87	3.29	2.52	-3.35
	s.e.	[1.94]	[1.75]	[1.62]	[1.62]	[1.98]	[1.75]	[1.65]	[1.63]	[1.95]	[1.76]	[1.62]	[1.62]	[1.96]	[1.78]	[1.69]	[1.61]
	Std	10.98	9.61	9.00	8.88	10.98	9.61	9.00	8.88	10.98	9.61	9.00	8.88	11.00	9.68	9.09	8.94
	SR	0.54	0.34	0.29	-0.37	0.54	0.34	0.29	-0.37	0.54	0.34	0.29	-0.37	0.53	0.34	0.28	-0.38
	s.e.	[0.23]	[0.21]	[0.19]	[0.21]	[0.23]	[0.21]	[0.19]	[0.21]	[0.23]	[0.21]	[0.19]	[0.21]	[0.23]	[0.21]	[0.19]	[0.21]
$rx^{(k),*}$	Mean	-0.16	0.36	0.45	0.61	1.39	2.62	3.31	1.92	2.27	4.34	5.69	3.42	2.65	5.52	7.78	5.13
	s.e.	[0.18]	[0.16]	[0.14]	[0.18]	[1.05]	[0.88]	[0.92]	[0.94]	[1.86]	[1.54]	[1.57]	[1.47]	[2.54]	[2.20]	[2.20]	[2.00]
	Std	1.01	0.80	0.73	0.97	5.43	4.54	4.67	4.57	9.61	7.96	8.23	7.19	12.95	11.06	11.67	10.39
	SR	-0.16	0.45	0.62	0.63	0.26	0.58	0.71	0.42	0.24	0.55	0.69	0.48	0.20	0.50	0.67	0.49
	s.e.	[0.19]	[0.20]	[0.22]	[0.21]	[0.19]	[0.20]	[0.21]	[0.19]	[0.20]	[0.21]	[0.22]	[0.20]	[0.20]	[0.21]	[0.21]	[0.19]
$rx^{(k),\$}$	Mean	5.79	3.66	3.10	-2.69	7.34	5.92	5.95	-1.39	8.22	7.64	8.34	0.12	8.52	8.81	10.30	1.77
	s.e.	[1.95]	[1.74]	[1.67]	[1.62]	[2.36]	[1.90]	[2.03]	[1.83]	[2.86]	[2.29]	[2.40]	[2.16]	[3.32]	[2.83]	[2.93]	[2.48]
	Std	11.00	9.53	9.19	8.82	12.33	9.98	10.70	9.44	14.72	11.56	12.68	10.99	16.87	13.64	15.00	13.03
	SR	0.53	0.38	0.34	-0.31	0.60	0.59	0.56	-0.15	0.56	0.66	0.66	0.01	0.51	0.65	0.69	0.14
	s.e.	[0.22]	[0.21]	[0.19]	[0.21]	[0.20]	[0.20]	[0.19]	[0.20]	[0.20]	[0.20]	[0.20]	[0.19]	[0.19]	[0.20]	[0.20]	[0.19]
$rx^{(k),\$} - rx^{(k),US}$	Mean	8.77	6.64	6.08	-2.69	7.22	5.81	5.84	-1.39	5.71	5.13	5.83	0.12	4.43	4.72	6.20	1.77
	s.e.	[1.95]	[1.74]	[1.62]	[1.62]	[2.24]	[1.74]	[1.69]	[1.83]	[2.46]	[1.91]	[1.90]	[2.16]	[2.80]	[2.26]	[2.39]	[2.48]

Notes: The table reports summary statistics on annualized log returns realized on zero coupon bonds with maturity varying from $k = 4$ to $k = 60$ quarters. The holding period is one quarter. The table reports the average change in exchange rates ($-\Delta s$), the average interest rate difference ($f - s$), the average currency excess return (rx^{FX}), the average foreign bond excess return in foreign currency ($rx^{(k),*}$) and in U.S. dollars ($rx^{(k),\$}$), as well as the difference between the average foreign bond excess return in U.S. dollars and the average U.S. bond excess return ($rx^{(k),\$} - rx^{(k),US}$). For the excess returns, the table also reports their annualized standard deviation (denoted Std) and their Sharpe ratios (denoted SR). The balanced panel consists of Australia, Canada, Japan, Germany, Norway, New Zealand, Sweden, Switzerland, and the U.K. The countries are sorted by the slope of their yield curves into three portfolios. The slope of the yield curve is measured by the difference between the 10-year yield and the 3-month interest rate at date t. The standard errors (denoted s.e. and reported between brackets) were generated by bootstrapping 10,000 samples of non-overlapping returns. Data are monthly, from the zero-coupon dataset, and the sample window is 4/1985–12/2012.

Table 11: The Maturity Structure of Returns in Slope-Sorted Portfolios: Extended Sample

Maturity		One Year						Five Years						Ten Years						Fifteen Years					
Portfolio		1	2	3	4	5	5 − 1	1	2	3	4	5	5 − 1	1	2	3	4	5	5 − 1	1	2	3	4	5	5 − 1
$-\Delta s$	Mean	0.90	0.59	0.29	2.34	-0.30	-1.19	0.90	0.59	0.29	2.34	-0.30	-1.19	0.90	0.59	0.29	2.34	-0.30	-1.19	0.94	0.43	0.35	2.38	-0.39	-1.33
$f-s$	Mean	3.99	1.99	0.99	0.39	0.36	-3.63	3.99	1.99	0.99	0.39	0.36	-3.63	3.99	1.99	0.99	0.39	0.36	-3.63	3.93	2.00	0.95	0.32	0.32	-3.62
rx^{FX}	Mean	4.89	2.58	1.28	2.73	0.07	-4.82	4.89	2.58	1.28	2.73	0.07	-4.82	4.89	2.58	1.28	2.73	0.07	-4.82	4.87	2.43	1.30	2.70	-0.07	-4.94
	s.e.	[2.35]	[2.14]	[2.11]	[2.10]	[1.79]	[1.56]	[2.33]	[2.15]	[2.10]	[2.06]	[1.80]	[1.56]	[2.33]	[2.15]	[2.11]	[2.06]	[1.79]	[1.57]	[2.32]	[2.13]	[2.12]	[2.13]	[1.78]	[1.63]
	Std	11.00	10.36	10.44	10.24	8.56	7.95	11.00	10.36	10.44	10.24	8.56	7.95	11.00	10.36	10.44	10.24	8.56	7.95	11.01	10.34	10.44	10.44	8.65	7.99
	SR	0.44	0.25	0.12	0.27	0.01	-0.61	0.44	0.25	0.12	0.27	0.01	-0.61	0.44	0.25	0.12	0.27	0.01	-0.61	0.44	0.24	0.12	0.26	-0.01	-0.62
	s.e.	[0.21]	[0.21]	[0.21]	[0.20]	[0.20]	[0.21]	[0.22]	[0.21]	[0.21]	[0.21]	[0.20]	[0.21]	[0.21]	[0.21]	[0.21]	[0.20]	[0.20]	[0.22]	[0.22]	[0.21]	[0.21]	[0.21]	[0.20]	[0.22]
$rx^{(k),*}$	Mean	-0.09	0.11	0.32	0.33	0.63	0.72	1.11	2.12	2.56	2.73	3.60	2.50	1.59	2.69	4.16	4.32	5.90	4.31	2.42	2.84	6.36	4.78	7.78	5.36
	s.e.	[0.21]	[0.20]	[0.17]	[0.17]	[0.17]	[0.23]	[1.08]	[0.95]	[1.06]	[0.95]	[1.10]	[1.21]	[1.90]	[1.66]	[1.86]	[1.61]	[1.77]	[1.69]	[2.63]	[2.56]	[2.42]	[2.12]	[2.54]	[2.40]
	Std	1.04	0.88	0.88	0.84	0.88	1.15	5.13	4.70	5.22	4.96	5.42	5.53	9.34	8.56	9.19	8.78	9.01	8.44	12.67	12.82	12.55	11.92	12.70	11.76
	SR	-0.09	0.12	0.37	0.39	0.72	0.63	0.22	0.45	0.49	0.55	0.67	0.45	0.17	0.31	0.45	0.49	0.65	0.51	0.19	0.22	0.51	0.40	0.61	0.46
	s.e.	[0.20]	[0.20]	[0.20]	[0.21]	[0.21]	[0.20]	[0.21]	[0.20]	[0.23]	[0.21]	[0.21]	[0.20]	[0.20]	[0.20]	[0.23]	[0.21]	[0.22]	[0.20]	[0.20]	[0.20]	[0.21]	[0.22]	[0.21]	[0.20]
$rx^{(k),\$}$	Mean	4.80	2.69	1.60	3.06	0.70	-4.10	6.00	4.70	3.84	5.46	3.67	-2.33	6.48	5.27	5.45	7.05	5.97	-0.52	7.29	5.27	7.66	7.47	7.71	0.42
	s.e.	[2.35]	[2.10]	[2.09]	[2.10]	[1.83]	[1.54]	[2.50]	[2.17]	[2.25]	[2.26]	[2.30]	[1.83]	[2.93]	[2.54]	[2.72]	[2.62]	[2.71]	[2.08]	[3.40]	[3.20]	[3.11]	[2.97]	[3.29]	[2.67]
	Std	11.05	10.29	10.32	10.27	8.72	7.99	11.87	11.07	11.02	11.50	10.71	9.17	14.10	13.13	13.25	13.84	13.14	11.24	16.45	16.10	15.74	16.04	15.96	13.97
	SR	0.43	0.26	0.16	0.30	0.08	-0.51	0.51	0.42	0.35	0.47	0.34	-0.25	0.46	0.40	0.41	0.51	0.45	-0.05	0.44	0.33	0.49	0.47	0.48	0.03
	s.e.	[0.21]	[0.21]	[0.21]	[0.20]	[0.20]	[0.21]	[0.20]	[0.21]	[0.21]	[0.21]	[0.21]	[0.20]	[0.20]	[0.21]	[0.21]	[0.21]	[0.20]	[0.20]	[0.21]	[0.21]	[0.21]	[0.21]	[0.20]	[0.20]
$rx^{(k),\$}-rx^{(k),US}$	Mean	7.66	5.56	4.47	5.92	3.57	-4.10	6.28	4.99	4.13	5.75	3.96	-2.33	4.89	3.68	3.86	5.46	4.38	-0.52	4.67	2.65	5.04	4.85	5.09	0.42
	s.e.	[2.35]	[2.08]	[2.07]	[2.08]	[1.79]	[1.54]	[2.57]	[2.13]	[2.05]	[2.21]	[2.13]	[1.83]	[2.95]	[2.41]	[2.38]	[2.59]	[2.42]	[2.08]	[3.37]	[3.10]	[2.78]	[2.97]	[3.00]	[2.67]

Notes: The table reports summary statistics on annualized log returns realized on zero coupon bonds with maturity varying from $k=4$ to $k=60$ quarters. The holding period is one quarter. The table reports the average change in exchange rates ($-\Delta s$), the average interest rate difference ($f-s$), the average currency excess return (rx^{FX}), the average foreign bond excess return in foreign currency ($rx^{(k),*}$) and in U.S. dollars ($rx^{(k),\$}$), as well as the difference between the average foreign bond excess return in U.S. dollars and the average U.S. bond excess return ($rx^{(k),\$}-rx^{(k),US}$). For the excess returns, the table also reports their annualized standard deviation (denoted Std) and their Sharpe ratios (denoted SR). The unbalanced panel consists of Australia, Austria, Belgium, Canada, the Czech Republic, Denmark, Finland, France, Germany, Hungary, Indonesia, Ireland, Italy, Japan, Malaysia, Mexico, the Netherlands, New Zealand, Norway, Poland, Portugal, Singapore, South Africa, Spain, Sweden, Switzerland, and the U.K. The countries are sorted by the slope of their yield curves into five portfolios. The slope of the yield curve is measured by the difference between the 10-year yield and the 3-month interest rate at date t. The standard errors (denoted s.e. and reported between brackets) were generated by bootstrapping 10,000 samples of non-overlapping returns. Data are quarterly and the sample window is 5/1987–12/2012.

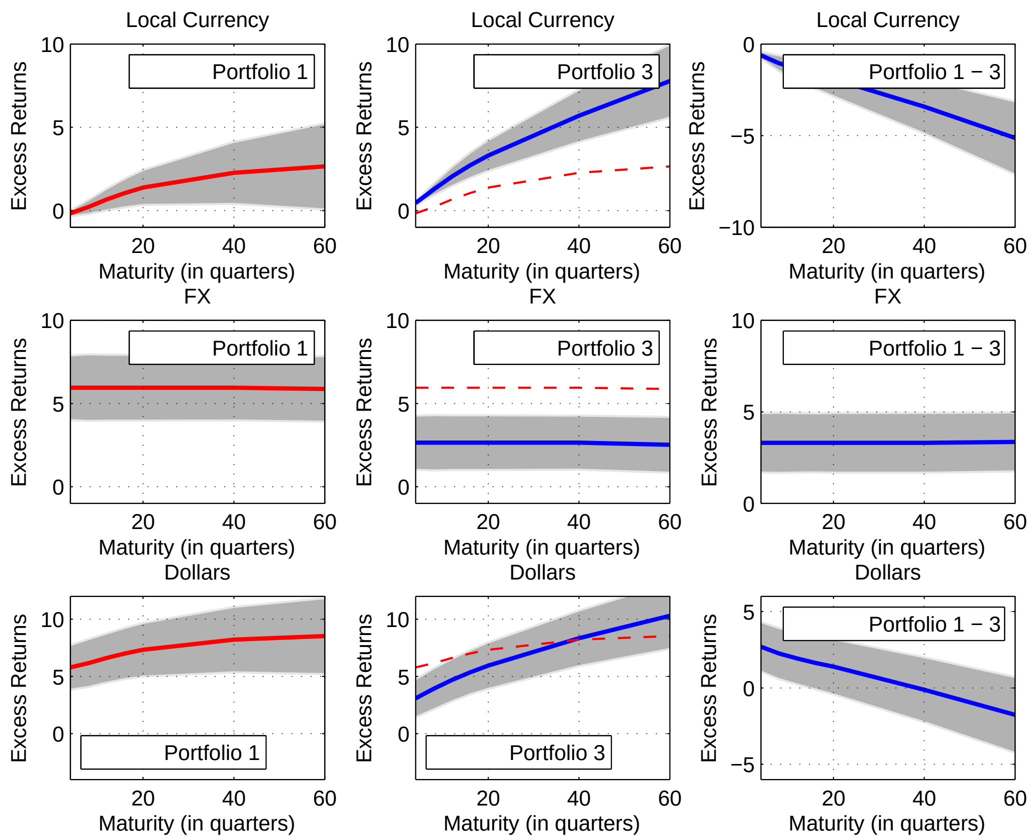

Figure 9: Dollar Bond Risk Premia Across Maturities— The figure shows the log excess returns on foreign bonds in local currency in the top panel, the currency excess return in the middle panel, and the log excess returns on foreign bonds in U.S. dollars in the bottom panel as a function of the bond maturities. The left panel focuses on Portfolio 1 (flat yield curve currencies) excess returns, while the middle panel reports Portfolio 3 (steep yield curve currencies) excess returns. The middle panels also report the Portfolio 1 excess returns in dashed lines for comparison. The right panel reports the difference. Data are monthly, from the zero-coupon dataset, and the sample window is 4/1985–12/2012. The unbalanced panel consists of Australia, Canada, Japan, Germany, Norway, New Zealand, Sweden, Switzerland, and the U.K. The countries are sorted by the slope of their yield curves into three portfolios. The slope of the yield curve is measured by the difference between the 10-year yield and the 3-month interest rate at date t. The holding period is one quarter. The returns are annualized. The shaded areas correspond to one standard deviation above and below each point estimate. Standard deviations are obtained by bootstrapping 10,000 samples of non-overlapping returns.

Figure 10 shows the local currency log excess returns in the top panel, and the dollar log excess returns in the bottom panel as a function of the bond maturities for zero-coupon bonds of our extended sample of developed countries. The results are also commented in the main text.

C.3 The Correlation and Volatility of Dollar Bond Returns

If the permanent components of the SDFs are the same across countries, holding period returns on zero-coupon bonds, once converted to a common currency (the U.S. dollar, in particular), should become increasingly similar as bond maturities approach infinity. To determine whether this hypothesis has merit, Figure 11 reports the correlation coefficient between three-month returns on foreign zero-coupon bonds (either in local currency or in U.S. dollars) and corresponding returns on U.S. bonds for bonds of maturity ranging from 1 year to 15 years. All foreign currency yield curves exhibit the same pattern: correlation coefficients for U.S. dollar returns start from very low (often negative) values and increase monotonically with bond maturity, tending towards one for

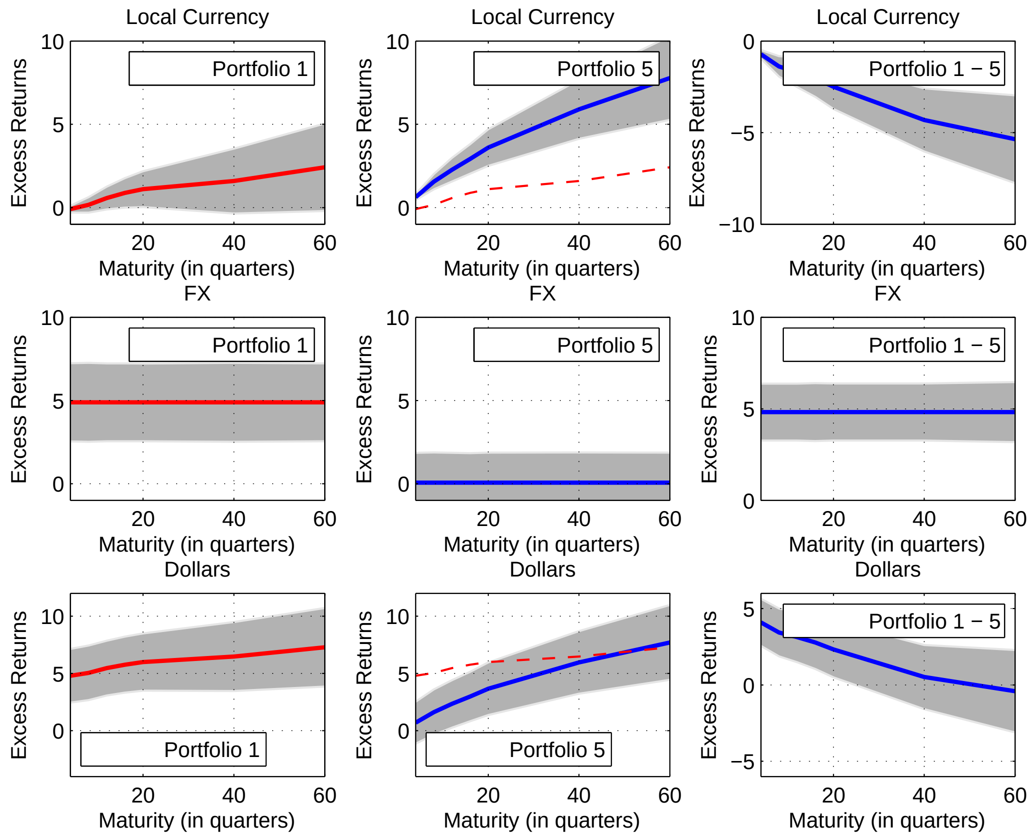

Figure 10: Dollar Bond Risk Premia Across Maturities-: Extended Sample — The figure shows the local currency log excess returns in the top panel, and the dollar log excess returns in the bottom panel as a function of the bond maturities. The left panel focuses on Portfolio 1 (flat yield curve currencies) excess returns, while the middle panel reports Portfolio 5 (steep yield curve currencies) excess returns. The middle panels also report the Portfolio 1 excess returns in dashed lines for comparison. The right panel reports the difference. Data are monthly, from the zero-coupon dataset, and the sample window is 5/1987–12/2012. The unbalanced sample includes Australia, Austria, Belgium, Canada, the Czech Republic, Denmark, Finland, France, Germany, Hungary, Indonesia, Ireland, Italy, Japan, Malaysia, Mexico, the Netherlands, New Zealand, Norway, Poland, Portugal, Singapore, South Africa, Spain, Sweden, Switzerland, and the U.K. The countries are sorted by the slope of their yield curves into five portfolios. The slope of the yield curve is measured by the difference between the 10-year yield and the 3-month interest rate at date t. The holding period is one quarter. The returns are annualized. The shaded areas correspond to one standard deviation above and below each point estimate. Standard deviations are obtained by bootstrapping 10,000 samples of non-overlapping returns.

long-term bonds. The clear monotonicity is not observed on local currency returns. The local currency three-month return correlations do not exhibit any discernible pattern with maturity, implying that the convergence of U.S. dollar return correlations towards the value of one results from exchange rate changes that partially offset differences in local currency bond returns. Similar results hold true for volatility ratios (instead of correlations); we report those in the Online Appendix.

In sum, the behavior of U.S. dollar bond returns and local currency bond returns differs markedly as bond maturity changes. While U.S. dollar bond returns become more correlated and roughly equally volatile across countries as the maturity increases, the behavior of local currency returns does not appear to change when bond maturity changes.

To check the robustness of our time-varying dollar bond betas, we run rolling window regressions on a longer sample. We consider an equally-weighted portfolio of all the currencies in the developed country sample and regress its dollar return and its components on the U.S. bond return from 12/1950 to 12/2012.

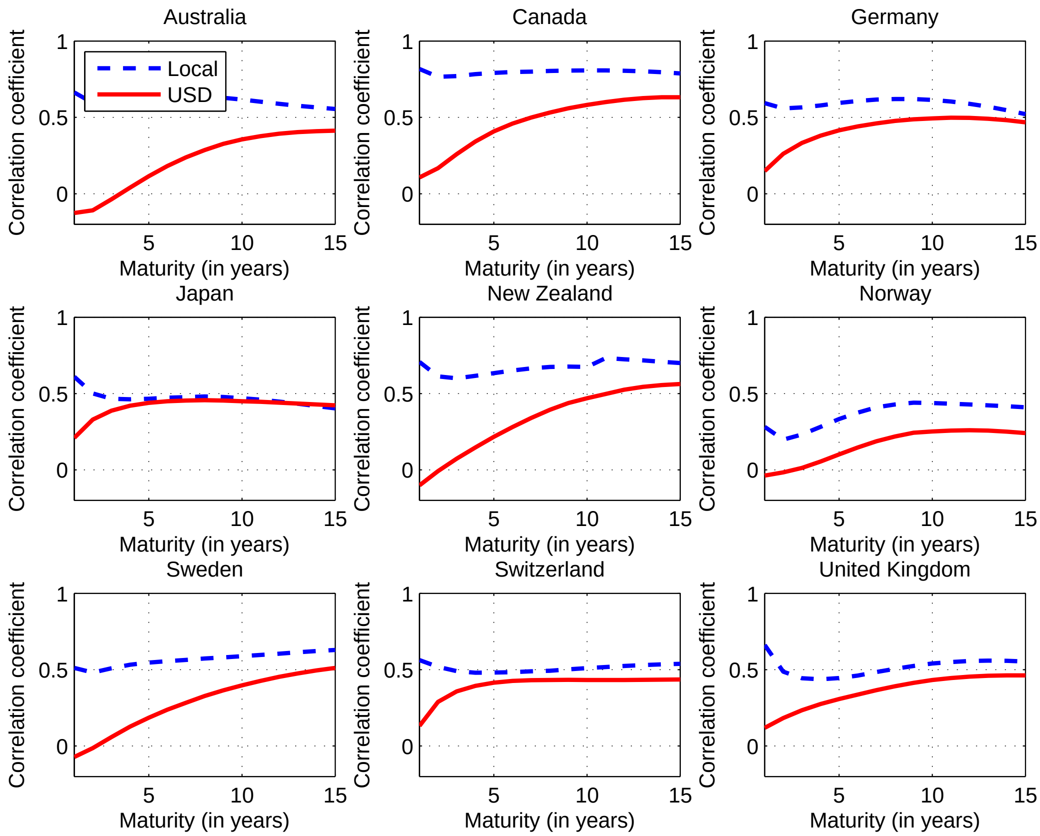

Figure 11: The Maturity Structure of Bond Return Correlations — The figure presents the correlation of foreign bond returns with U.S. bond returns. The time-window is country-dependent. Data are monthly. The holding period is three-months.

Figure 12 plots the 60-month rolling window of the regression coefficients. We note large increases in the dollar beta after the demise of the Bretton-Woods regime, mostly driven by increases in the exchange rate betas. The same is true around the early 1990s. Furthermore, there is a secular increase in the local return beta over the entire sample.

The exchange rate coefficient is positive during most of our sample period, providing evidence that the currency exposure hedges the interest rate exposure of the foreign bond position. There are two main exceptions: the Long Term Capital Management (LTCM) crisis in 1998 and the recent financial crisis. During these episodes, the dollar appreciated, despite the strong performance of the U.S. bond market, weakening the comovement between foreign and local bond returns.

D Finite vs. Infinite Maturity Bond Returns

We estimate a version of the Joslin, Singleton, and Zhu (2011) term structure model with three factors. The three factors are the three first principal components of the yield covariance matrix. We thank the authors for making their code available on their web pages.

This Gaussian dynamic term structure model is estimated on zero-coupon rates over the period from April 1985 to December 2012, the same period used in our empirical work, for each country in our benchmark sample. Each country-specific model is estimated independently, without using any exchange rate data. The maturities considered are 6 months, and 1, 2, 3, 5, 7, and 10 years. Using the parameter estimates, we derive the implied bond

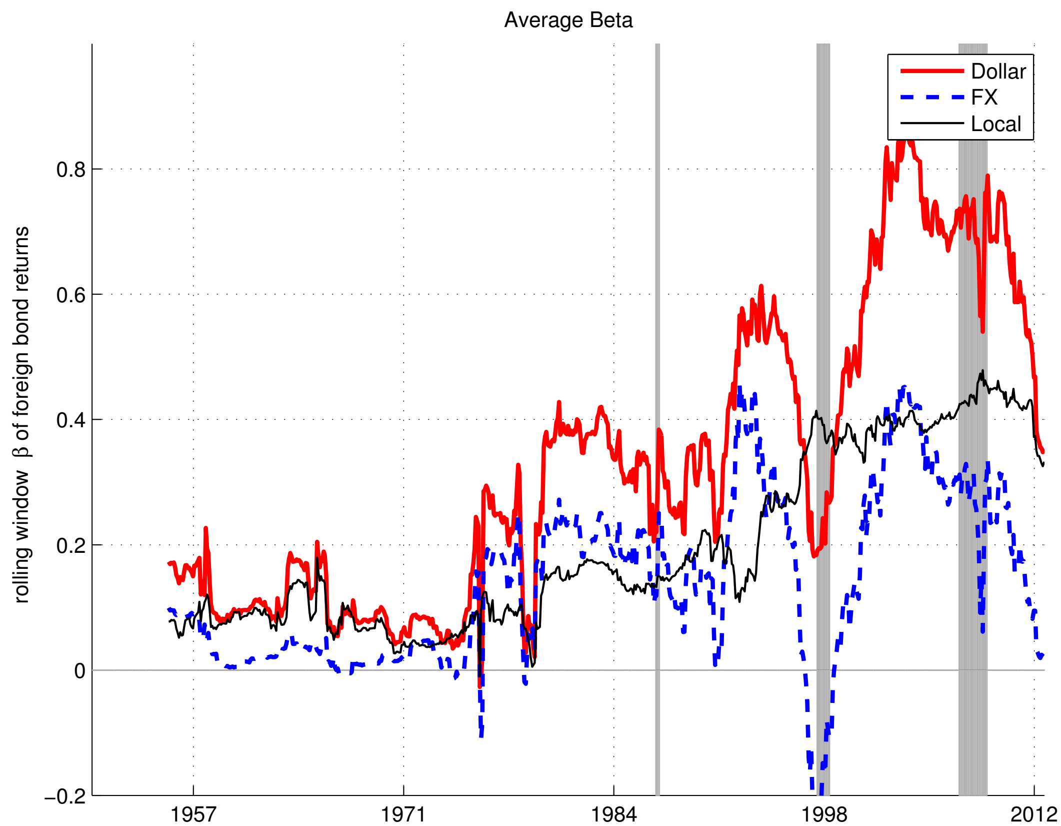

Figure 12: Foreign Bond Return Betas — This figure presents the 60-month rolling window estimation of beta with respect to US bond returns for the equal-weighted average of log bond returns in local currency, the log change in the exchange rate and the log dollar bond returns for the benchmark sample of countries. The panel consists of Australia, Austria, Belgium, Canada, Denmark, Finland, France, Germany, Greece, Ireland, Italy, Japan, the Netherlands, New Zealand, Norway, Portugal, Spain, Sweden, Switzerland, and the United Kingdom. The sample is 12/1950–12/2012. The dark shaded areas represent the 1987 crash, the 1998 LTCM crisis and the 2007-2008 U.S. financial crisis.

returns for different maturities. We report simulated data for Germany, Japan, Norway, Switzerland, U.K., and U.S. and ignore the simulated data for Australia, Canada, New Zealand and Sweden as the parameter estimates imply there that the yield curves turn negative on long maturities. Table 12 reports the simulated moments.

We first consider the unconditional holding period bond returns across countries. The average (annualized) log return on the 10-year bond is lower than the log return on the infinite-maturity bond for all countries except the U.K., but the differences are not statistically significant. The unconditional correlation between the two log returns ranges from 0.88 to 0.96 across countries; for example, it is 0.93 for the U.S. Furthermore, the estimations imply very volatile log SDFs that exhibit little correlation across countries. As a result, the implied exchange rate changes are much more volatile than in the data. We then turn to conditional bond returns, obtained by sorting countries into two portfolios, either by the level of their short-term interest rate or by the slope of their yield curve. The portfolio sorts recover the results highlighted in the previous section: low (high) short-term interest rates correspond to high (low) average local bond returns. Likewise, low (high) slopes correspond to low (high) average local bond returns. The infinite maturity bonds tend to offer larger conditional returns than the 10-year bonds but the differences are not significant. The correlation between the conditional returns of the 10-year and infinite maturity bond portfolios ranges from 0.93 to 0.94 across portfolios.

Table 12: Simulated Bond Returns

	Panel A: Country Returns					
	US	Germany	UK	Japan	Switzerland	Norway
$y^{(10)}$ (data)	5.92	5.38	6.51	3.01	3.49	4.67
$y^{(10)}$	5.93	5.38	6.50	3.01	3.50	4.68
$rx^{(10)}$	6.06	4.21	3.63	4.18	2.84	3.07
s.e.	[1.52]	[1.25]	[1.63]	[1.26]	[1.22]	[1.91]
$rx^{(\infty)}$	12.54	5.76	3.36	7.20	5.67	5.90
s.e.	[5.44]	[2.58]	[4.45]	[2.75]	[3.01]	[4.34]
Corr ($rx^{(10)}$, $rx^{(\infty)}$)	0.93	0.92	0.88	0.94	0.94	0.96
$rx^{(\infty)} - rx^{(10)}$	6.47	1.55	-0.27	3.02	2.83	2.84
s.e.	[4.05]	[1.51]	[3.12]	[1.61]	[1.93]	[2.57]
$\sigma_{m^\star}$	245.55	112.44	156.56	206.23	226.16	128.55
$corr(m, m^\star)$	1.00	0.19	0.03	0.03	0.13	0.03
$\sigma_{\Delta s}$		249.80	286.73	315.23	279.01	207.43
	Panel B: Portfolio Returns					
		Sorted by Level			Sorted by Slope	
Sorting variable (level/slope)		2.42	5.44		0.07	1.87
$rx^{(10)}$		4.47	3.96		2.72	5.34
s.e.		[1.17]	[1.27]		[1.26]	[1.18]
$rx^{(\infty)}$		7.62	6.27		3.97	9.40
s.e.		[2.93]	[3.37]		[3.22]	[3.35]
Corr ($rx^{(10)}$, $rx^{(\infty)}$)		0.93	0.93		0.94	0.93
$rx^{(\infty)} - rx^{(10)}$		3.16	2.31		1.26	4.06
s.e.		[1.87]	[2.24]		[2.04]	[2.27]

Notes: Panel A reports moments on simulated data at the country level. For each country, the table first compares the 10-year yield in the data and in the model, and then reports the annualized average simulated log excess return (in percentage terms) of bonds with maturities of 10 years and infinity, as well as the correlation between the two bond returns. The table also reports the annualized volatility of the log SDF, the correlation between the foreign log SDF and the U.S. log SDF, and the annualized volatility of the implied exchange rate changes. Panel B reports conditional moments obtained by sorting countries by either the level of their short-term interest rates or the slope of their yield curves into two portfolios. The table reports the average value of the sorting variable, and then the average returns on the 10-year and infinite-maturity bonds, along with their correlation. The simulated data come from the benchmark 3-factor model (denoted RPC) in Joslin, Singleton, and Zhu (2011) that sets the first 3 principal components of bond yields as the pricing factors. The model is estimated on zero-coupon rates for Germany, Japan, Norway, Switzerland, U.K., and U.S. The sample estimation period is 4/1985–12/2012. The standard errors (denoted s.e. and reported between brackets) were generated by block-bootstrapping 10,000 samples of 333 monthly observations.

E Dynamic Term Structure Models

This section reports additional results on dynamic term structure models, starting with the simple Vasicek one-factor model, before turning to essentially affine k-factor models and the model studied in Lustig, Roussanov, and Verdelhan (2011).

For the reader's convenience, we repeat the three main equations that will be key to analyze the currency and bond risk premia:

$$E_t\left[rx_{t+1}^{FX}\right] = (f_t - s_t) - E_t(\Delta s_{t+1}) = L_t\left(\frac{\Lambda_{t+1}}{\Lambda_t}\right) - L_t\left(\frac{\Lambda^*_{t+1}}{\Lambda^*_t}\right), \quad (26)$$

$$E_t\left[rx_{t+1}^{(\infty),*}\right] = \lim_{k\to\infty} E_t\left[rx_{t+1}^{(k),*}\right] = L_t\left(\frac{\Lambda^*_{t+1}}{\Lambda^*_t}\right) - L_t\left(\frac{\Lambda^{*,\mathbb{P}}_{t+1}}{\Lambda^{*,\mathbb{P}}_t}\right), \quad (27)$$

$$E_t\left[rx_{t+1}^{(\infty),*}\right] + E_t\left[rx_{t+1}^{FX}\right] = E_t\left[rx_{t+1}^{(\infty)}\right] + L_t\left(\frac{\Lambda^{\mathbb{P}}_{t+1}}{\Lambda^{\mathbb{P}}_t}\right) - L_t\left(\frac{\Lambda^{\mathbb{P},*}_{t+1}}{\Lambda^{\mathbb{P},*}_t}\right). \quad (28)$$

As already noted, Equation (26) shows that the currency risk premium is equal to the difference between the entropy of the domestic and foreign SDFs (Backus, Foresi, and Telmer, 2001). Equation (27) shows that the term premium is equal to the difference between the total entropy of the SDF and the entropy of its permanent component (Alvarez and Jermann, 2005). Equation (28) shows that the foreign term premium in dollars is equal to the domestic term premium plus the difference in the entropy of the foreign and domestic permanent component of the SDFs.

E.1 Vasicek (1977)

Model In the Vasicek model, the log SDF evolves as:

$$-m_{t+1} = y_{1,t} + \frac{1}{2}\lambda^2\sigma^2 + \lambda\varepsilon_{t+1},$$

where $y_{1,t}$ denotes the short-term interest rate. It is affine in a single factor:

$$\begin{aligned} x_{t+1} &= \rho x_t + \varepsilon_{t+1}, \quad \varepsilon_{t+1} \sim \mathcal{N}\left(0, \sigma^2\right) \\ y_{1,t} &= \delta + x_t. \end{aligned}$$

In this model, x_t is the level factor and ε_{t+1} are shocks to the level of the term structure. The Jensen term is there to ensure that $E_t\left(M_{t+1}\right) = \exp\left(-y_{1,t}\right)$. Bond prices are exponentially affine. For any maturity n, bond prices are equal to $P_t^{(n)} = \exp\left(-B_0^n - B_1^n x_t\right)$. The price of the one-period risk-free note ($n = 1$) is naturally:

$$P_t^{(1)} = \exp\left(-y_{1,t}\right) = \exp\left(-B_0^1 - B_1^1 x_t\right),$$

with $B_0^1 = \delta, B_1^1 = 1$. Bond prices are defined recursively by the Euler equation: $P_t^{(n)} = E_t\left(M_{t+1}P_{t+1}^{(n-1)}\right)$, which implies:

$$-B_0^n - B_1^n x_t = -\delta - x_t - B_0^{n-1} - B_1^{n-1}\rho x_t + \frac{1}{2}\left(B_1^{n-1}\right)^2\sigma^2 x_t + \lambda B_1^{n-1}\sigma^2.$$

The coefficients B_0^n and B_1^n satisfy the following recursions:

$$\begin{aligned} B_0^n &= \delta + B_0^{n-1} - \frac{1}{2}\sigma^2(B_1^{n-1})^2 - \lambda B_1^{n-1}\sigma^2, \\ B_1^n &= 1 + B_1^{n-1}\rho. \end{aligned}$$

Decomposition (Alvarez and Jermann, 2005) We first implement the Alvarez and Jermann (2005) approach. The temporary pricing component of the pricing kernel is:

$$\Lambda_t^{\mathbb{T}} = \lim_{n\to\infty}\frac{\beta^{t+n}}{P_t^n} = \lim_{n\to\infty}\beta^{t+n}e^{B_0^n + B_1^n x_t},$$

where the constant β is chosen in order to satisfy Assumption 1 in Alvarez and Jermann (2005):

$$0 < \lim_{n\to\infty} \frac{P_t^n}{\beta^n} < \infty.$$

The limit of $B_0^n - B_0^{n-1}$ is finite: $\lim_{n\to\infty} B_0^n - B_0^{n-1} = \delta - \frac{1}{2}\sigma^2(B_1^\infty)^2 - \lambda B_1^\infty \sigma^2$, where B_1^∞ is $1/(1-\rho)$. As a result, B_0^n grows at a linear rate in the limit. We choose the constant β to offset the growth in B_0^n as n becomes very large. Setting $\beta = e^{-\delta + \frac{1}{2}\sigma^2(B_1^\infty)^2 + \lambda B_1^\infty \sigma^2}$ guarantees that Assumption 1 in Alvarez and Jermann (2005) is satisfied. The temporary pricing component of the pricing kernel is thus equal to:

$$\frac{\Lambda_{t+1}^{\mathbb{T}}}{\Lambda_t^{\mathbb{T}}} = \beta e^{B_1^\infty(x_{t+1}-x_t)} = \beta e^{\frac{1}{1-\rho}(\rho-1)x_t + \frac{1}{1-\rho}\varepsilon_{t+1}} = \beta e^{-x_t + \frac{1}{1-\rho}\varepsilon_{t+1}}.$$

The martingale component of the pricing kernel is then:

$$\frac{\Lambda_{t+1}^{\mathbb{P}}}{\Lambda_t^{\mathbb{P}}} = \frac{\Lambda_{t+1}}{\Lambda_t}\left(\frac{\Lambda_{t+1}^{\mathbb{T}}}{\Lambda_t^{\mathbb{T}}}\right)^{-1} = \beta^{-1} e^{x_t - \frac{1}{1-\rho}\varepsilon_{t+1} - \delta - x_t - \frac{1}{2}\lambda^2\sigma^2 - \lambda\varepsilon_{t+1}} = \beta^{-1} e^{-\delta - \frac{1}{2}\lambda^2\sigma^2 - (\frac{1}{1-\rho}+\lambda)\varepsilon_{t+1}}.$$

In the case of $\lambda = -B_1^\infty = -\frac{1}{1-\rho}$, the martingale component of the pricing kernel is constant and all the shocks that affect the pricing kernel are transitory.

Decomposition (Hansen and Scheinkman, 2009) We now show that the Hansen and Scheinkman (2009) methodology leads to similar results. Guess an eigenfunction ϕ of the form

$$\phi(x) = e^{cx}$$

where c is a constant. Then, the (one-period) eigenfunction problem can be written as

$$E_t\left[\exp(-\delta - x_t - \frac{1}{2}\lambda^2\sigma^2 - \lambda\varepsilon_{t+1} + cx_{t+1})\right] = \exp(\beta + cx_t).$$

Expanding and matching coefficients, we solve for the constants c and β:

$$c = -\frac{1}{1-\rho}$$

$$\beta = -\delta + \frac{1}{2}\sigma^2\left(\frac{1}{1-\rho}\right)^2 + \lambda\sigma^2\left(\frac{1}{1-\rho}\right)$$

As shown above, the recursive definition of the bond price coefficients B_0^n and B_1^n implies that:

$$c = -B_1^\infty.$$

The transitory component of the pricing kernel is by definition:

$$\Lambda_t^{\mathbb{T}} = e^{\beta t - cx_t}$$

The transitory and permanent SDF component are thus:

$$\frac{\Lambda_{t+1}^{\mathbb{T}}}{\Lambda_t^{\mathbb{T}}} = e^{\beta - c(x_{t+1}-x_t)} = e^{\beta - x_t + \frac{1}{1-\rho}\varepsilon_{t+1}}$$

$$\frac{\Lambda_{t+1}^{\mathbb{P}}}{\Lambda_t^{\mathbb{P}}} = \frac{\Lambda_{t+1}}{\Lambda_t}\left(\frac{\Lambda_{t+1}^{\mathbb{T}}}{\Lambda_t^{\mathbb{T}}}\right)^{-1} = e^{-\delta - x_t - \frac{1}{2}\lambda^2\sigma^2 - \lambda\varepsilon_{t+1}} e^{-\beta + x_t - \frac{1}{1-\rho}\varepsilon_{t+1}} = e^{-\left[\frac{1}{2}\left(\frac{1}{1-\rho}\right)^2 + \lambda\frac{1}{1-\rho} + \frac{1}{2}\lambda^2\right]\sigma^2 - \left(\frac{1}{1-\rho}+\lambda\right)\varepsilon_{t+1}}$$

If $\lambda = -\frac{1}{1-\rho}$, then the martingale SDF component becomes

$$\frac{\Lambda_{t+1}^{\mathbb{P}}}{\Lambda_t^{\mathbb{P}}} = 1$$

so the entirety of the SDF is its transitory component.

Term and Risk Premium The expected log excess return of an infinite maturity bond is then:

$$E_t[rx_{t+1}^{(\infty)}] = -\frac{1}{2}\sigma^2(B_1^{\infty})^2 - \lambda B_1^{\infty}\sigma^2.$$

The first term is a Jensen term. The risk premium is constant and positive if λ is negative. The SDF is homoskedastic. The expected log currency excess return is therefore constant:

$$E_t[-\Delta s_{t+1}] + y_t^* - y_t = \frac{1}{2}Var_t(m_{t+1}) - \frac{1}{2}Var_t(m_{t+1}^*) = \frac{1}{2}\lambda\sigma^2 - \frac{1}{2}\lambda^*\sigma^{*2}.$$

When $\lambda = -B_1^{\infty} = -\frac{1}{1-\rho}$, the martingale component of the pricing kernel is constant and all the shocks that affect the pricing kernel are transitory. By using the expression for the bond risk premium in Equation (27), it is straightforward to verify that the expected log excess return of an infinite maturity bond is in this case:

$$E_t[rx_{t+1}^{(\infty)}] = \frac{1}{2}\sigma^2\lambda^2.$$

Model with Country-Specific Factor We start by examining the case in which each country has its own factor. We assume the foreign pricing kernel has the same structure, but it is driven by a different factor with different shocks:

$$\begin{aligned} -\log M_{t+1}^* &= y_{1,t}^* + \frac{1}{2}\lambda^{*2}\sigma^{*2} + \lambda^*\varepsilon_{t+1}^*, \\ x_{t+1}^* &= \rho x_t^* + \varepsilon_{t+1}^*, \quad \varepsilon_{t+1}^* \sim \mathcal{N}\left(0, \sigma^{*2}\right) \\ y_{1,t} &= \delta^* + x_t^*. \end{aligned}$$

Equation (26) shows that the expected log currency excess return is constant: $E_t[rx_{t+1}^{FX}] = \frac{1}{2}Var_t(m_{t+1}) - \frac{1}{2}Var_t(m_{t+1}^*) = \frac{1}{2}\lambda^2\sigma^2 - \frac{1}{2}\lambda^{2*}\sigma^{*2}$.

Result 3. *In a Vasicek model with country-specific factors, the long bond uncovered return parity holds only if the model parameters satisfy the following restriction:* $\lambda = -\frac{1}{1-\rho}$.

Under these conditions, there is no martingale component in the pricing kernel and the foreign term premium on the long bond expressed in home currency is simply $E_t[rx_{t+1}^{(*,\infty)}] = \frac{1}{2}\lambda^2\sigma^2$. This expression equals the domestic term premium. The nominal exchange rate is stationary.

Symmetric Model with Global Factor Next, we examine the case in which the single state variable x_t is global. The foreign SDF is thus:

$$\begin{aligned} -\log M_{t+1}^* &= y_{1,t}^* + \frac{1}{2}\lambda^{*2}\sigma^2 + \lambda^*\varepsilon_{t+1}, \\ x_{t+1} &= \rho x_t + \varepsilon_{t+1}, \quad \varepsilon_{t+1} \sim \mathcal{N}\left(0, \sigma^2\right) \\ y_{1,t} &= \delta^* + x_t. \end{aligned}$$

This case is key for our understanding of carry risk. Since carry trade returns are base-currency-invariant and obtained on portfolios of countries that average out country-specific shocks, heterogeneity in the exposure of the pricing kernel to *global* shocks is required to explain the carry trade premium (Lustig, Roussanov, and Verdelhan, 2011). Note that here $B_1^{\infty} = 1/(1-\rho)$ is the same for all countries, since it only depends on the persistence of the global state variable. Likewise, $\sigma = \sigma^{*2}$ in this case.

Result 4. *In a Vasicek model with a single global factor and permanent shocks, the long bond uncovered return parity condition holds only if the countries' SDFs share the same exposure (λ) to the global shocks.*

If countries SDFs share the same parameter λ, then the permanent components of their SDFs are perfectly correlated. In this case, the result is trivial, because the currency risk premium is zero, and the local term premia are identical across countries. we now turn to a model where the currency risk premium is potentially time-varying.

E.2 Cox, Ingersoll, and Ross (1985) Model

Model The Cox, Ingersoll, and Ross (1985) model (denoted CIR) is defined by the following two equations:

$$\begin{aligned} -\log M_{t+1} &= \alpha + \chi z_t + \sqrt{\gamma z_t} u_{t+1}, \\ z_{t+1} &= (1-\phi)\theta + \phi z_t - \sigma \sqrt{z_t} u_{t+1}, \end{aligned} \tag{29}$$

where M denotes the stochastic discount factor. In this model, log bond prices are affine in the state variable z: $p_t^{(n)} = -B_0^n - B_1^n z_t$. The price of a one period-bond is: $P^{(1)} = E_t(M_{t+1}) = e^{-\alpha-(\chi-\frac{1}{2}\gamma)z_t}$. Bond prices are defined recursively by the Euler equation: $P_t^{(n)} = E_t(M_{t+1}P_{t+1}^{(n-1)})$. Thus the bond price coefficients evolve according to the following second-order difference equations:

$$\begin{aligned} B_0^n &= \alpha + B_0^{n-1} + B_1^{n-1}(1-\phi)\theta, \\ B_1^n &= \chi - \frac{1}{2}\gamma + B_1^{n-1}\phi - \frac{1}{2}\left(B_1^{n-1}\right)^2\sigma^2 + \sigma\sqrt{\gamma}B_1^{n-1}. \end{aligned} \tag{30}$$

Decomposition (Alvarez and Jermann, 2005) We first implement the Alvarez and Jermann (2005) approach. The temporary pricing component of the pricing kernel is:

$$\Lambda_t^{\mathbb{T}} = \lim_{n\to\infty} \frac{\beta^{t+n}}{P_t^{(n)}} = \lim_{n\to\infty} \beta^{t+n} e^{B_0^n + B_1^n z_t},$$

where the constant β is chosen in order to satisfy Assumption 1 in Alvarez and Jermann (2005):

$$0 < \lim_{n\to\infty} \frac{P_t^{(n)}}{\beta^n} < \infty.$$

The limit of $B_0^n - B_0^{n-1}$ is finite: $\lim_{n\to\infty} B_0^n - B_0^{n-1} = \alpha + B_1^\infty(1-\phi)\theta$, where B_1^∞ is defined implicitly in a second-order equation above. As a result, B_0^n grows at a linear rate in the limit. We choose the constant β to offset the growth in B_0^n as n becomes very large. Setting $\beta = e^{-\alpha - B_1^\infty(1-\phi)\theta}$ guarantees that Assumption 1 in Alvarez and Jermann (2005) is satisfied. The temporary pricing component of the SDF is thus equal to:

$$\frac{\Lambda_{t+1}^{\mathbb{T}}}{\Lambda_t^{\mathbb{T}}} = \beta e^{B_1^\infty(z_{t+1}-z_t)} = \beta e^{B_1^\infty\left[(\phi-1)(z_t-\theta)-\sigma\sqrt{z_t}u_{t+1}\right]}.$$

As a result, the martingale component of the SDF is then:

$$\frac{\Lambda_{t+1}^{\mathbb{P}}}{\Lambda_t^{\mathbb{P}}} = \frac{\Lambda_{t+1}}{\Lambda_t}\left(\frac{\Lambda_{t+1}^{\mathbb{T}}}{\Lambda_t^{\mathbb{T}}}\right)^{-1} = \beta^{-1} e^{-\alpha-\chi z_t - \sqrt{\gamma z_t}u_{t+1}} e^{-B_1^\infty\left[(\phi-1)(z_t-\theta)-\sigma\sqrt{z_t}u_{t+1}\right]}. \tag{31}$$

Decomposition (Hansen and Scheinkman, 2009) We now show that the Hansen and Scheinkman (2009) methodology leads to similar results. We guess an eigenfunction ϕ of the form

$$\phi(x) = e^{cz}$$

where c is a constant. Then, the (one-period) eigenfunction problem can be written as

$$E_t\left[\exp(\alpha + \chi z_t + \sqrt{\gamma z_t}u_{t+1} + cz_{t+1})\right] = \exp(\beta + cz_t).$$

Expanding and matching coefficients, we get:

$$\beta = -\alpha + c(1-\phi)\theta$$

$$\left[\frac{1}{2}\sigma^2\right]c^2 + \left[\sigma\sqrt{\gamma} + \phi - 1\right]c + \left[\frac{1}{2}\gamma - \chi\right] = 0$$

so c solves a quadratic equation. The transitory component of the pricing kernel is by definition:

$$\Lambda_t^{\mathbb{T}} = e^{\beta t - cz_t}$$

The transitory and permanent SDF component are thus:

$$\begin{aligned}\frac{\Lambda_{t+1}^{\mathbb{T}}}{\Lambda_t^{\mathbb{T}}} &= e^{\beta-c(z_{t+1}-z_t)} = e^{\beta-c\left[(1-\phi)(\theta-z_t)-\sigma\sqrt{z_t}u_{t+1}\right]} = e^{-\alpha+c\left[(1-\phi)z_t+\sigma\sqrt{z_t}u_{t+1}\right]} \\ \frac{\Lambda_{t+1}^{\mathbb{P}}}{\Lambda_t^{\mathbb{P}}} &= \frac{\Lambda_{t+1}}{\Lambda_t}\left(\frac{\Lambda_{t+1}^{\mathbb{T}}}{\Lambda_t^{\mathbb{T}}}\right)^{-1} = e^{-\alpha-\chi z_t-\sqrt{\gamma z_t}u_{t+1}}e^{\alpha-c\left[(1-\phi)z_t+\sigma\sqrt{z_t}u_{t+1}\right]} = e^{-[\chi+c(1-\phi)]z_t-\left[\sqrt{\gamma}+c\sigma\right]\sqrt{z_t}u_{t+1}}\end{aligned}$$

The law of motion of bond prices implies that $c=-B_1^{\infty}$. If $\chi=-c(1-\phi)$, then the quadratic equation for c becomes

$$\sigma^2c^2+2\sigma\sqrt{\gamma}c+\gamma=0$$

with unique solution $c=-\frac{\sqrt{\gamma}}{\sigma}$. Then, the martingale component of the SDF becomes

$$\frac{\Lambda_{t+1}^{\mathbb{P}}}{\Lambda_t^{\mathbb{P}}}=1$$

so the entirety of the SDF is its transitory component.

Term Premium The expected log excess return is thus given by:

$$E_t[rx_{t+1}^{(n)}]=[-\frac{1}{2}\left(B_1^{n-1}\right)^2\sigma^2+\sigma\sqrt{\gamma}B_1^{n-1}]z_t.$$

The expected log excess return of an infinite maturity bond is then:

$$\begin{aligned}E_t[rx_{t+1}^{(\infty)}] &= [-\frac{1}{2}\left(B_1^{\infty}\right)^2\sigma^2+\sigma\sqrt{\gamma}B_1^{\infty}]z_t, \\ &= [B_1^{\infty}(1-\phi)-\chi+\frac{1}{2}\gamma]z_t.\end{aligned}$$

The $-\frac{1}{2}\left(B_1^{\infty}\right)^2\sigma^2$ is a Jensen term. The term premium is driven by $\sigma\sqrt{\gamma}B_1^{\infty}z_t$, where B_1^{∞} is defined implicitly in the second order equation $B_1^{\infty}=\chi-\frac{1}{2}\gamma+B_1^{\infty}\phi-\frac{1}{2}\left(B_1^{\infty}\right)^2\sigma^2+\sigma\sqrt{\gamma}B_1^{\infty}$.

Model with Country-specific Factors Consider the special case of $B_1^{\infty}(1-\phi)=\chi$. In this case, the expected term premium is simply $E_t[rx_{t+1}^{(\infty)}]=\frac{1}{2}\gamma z_t$, which is equal to one-half of the variance of the log stochastic discount factor.

Suppose that the foreign pricing kernel is specified as in Equation (29) with the same parameters. However, the foreign country has its own factor z^*. As a result, the difference between the domestic and foreign log term premia is equal to the log currency risk premium, which is given by $E_t[rx_{t+1}^{FX}]=\frac{1}{2}\gamma(z_t-z_t^*)$. In other words, the expected foreign log holding period return on a foreign long bond converted into U.S. dollars is equal to the U.S. term premium: $E_t[rx_{t+1}^{(\infty),*}]+E_t[rx_{t+1}^{FX}]=\frac{1}{2}\gamma z_t$.

This special case corresponds to the absence of permanent shocks to the SDF: when $B_1^{\infty}(1-\phi)=\chi$, the permanent component of the stochastic discount factor is constant. To see this result, let us go back to the implicit definition of B_1^{∞} in Equation (31):

$$\begin{aligned}0 &= \frac{1}{2}\left(B_1^{\infty}\right)^2\sigma^2+(1-\phi-\sigma\sqrt{\gamma})B_1^{\infty}-\chi+\frac{1}{2}\gamma, \\ 0 &= \frac{1}{2}\left(B_1^{\infty}\right)^2\sigma^2-\sigma\sqrt{\gamma}B_1^{\infty}+\frac{1}{2}\gamma, \\ 0 &= \left(\sigma B_1^{\infty}-\sqrt{\gamma}\right)^2.\end{aligned}$$

In this special case, $B_1^{\infty}=\sqrt{\gamma}/\sigma$. Using this result in Equation (31), the permanent component of the SDF reduces to:

$$\frac{M_{t+1}^{\mathbb{P}}}{M_t^{\mathbb{P}}} = \frac{M_{t+1}}{M_t}\left(\frac{M_{t+1}^{\mathbb{T}}}{M_t^{\mathbb{T}}}\right)^{-1} = \beta^{-1}e^{-\alpha-\chi z_t-\sqrt{\gamma z_t}u_{t+1}}e^{-B_1^{\infty}\left[(\phi-1)(z_t-\theta)-\sigma\sqrt{z_t}u_{t+1}\right]} = \beta^{-1}e^{-\alpha-\chi\theta},$$

which is a constant.

Model with Global Factors We assume that all the shocks are global and that z_t is a global state variable (and thus $\sigma = \sigma^*$, $\phi = \phi^*$, $\theta = \theta^*$). The state variable is referred as "permanent" if it has some impact on the permanent component of the SDF. The difference in term premia between the domestic and foreign bond (once expressed in the same currency) is pinned down by the difference in conditional variances of the permanent components of the SDFs. Therefore the two bonds have the same risk premia when:

$$\sqrt{\gamma} + B_1^{\infty}\sigma = \sqrt{\gamma^*} + B_1^{\infty*}\sigma$$

Note that B_1^{∞} depends on χ and γ, as well as on the global parameters ϕ and σ. The domestic and foreign infinite-maturity bonds have the same risk premia (once expressed in the same currency) when $\gamma = \gamma^*$ and $\chi = \chi^*$, i.e. when the domestic and foreign SDFs react similarly to changes in the global "permanent " state variable and its shocks

E.3 Multi-Factor Vasicek Models

Model Under some conditions, the previous results can be extended to a more k-factor model. The standard $k-$factor essentially affine model in discrete time generalizes the Vasicek (1977) model to multiple risk factors. The log SDF is given by:

$$-\log M_{t+1} = y_{1,t} + \frac{1}{2}\Lambda_t'\Sigma\Lambda_t + \Lambda_t'\varepsilon_{t+1}$$

To keep the model affine, the law of motion of the risk-free rate and of the market price of risk are:

$$\begin{aligned} y_{1,t} &= \delta_0 + \delta_1' x_t, \\ \Lambda_t &= \Lambda_0 + \Lambda_1 x_t, \end{aligned}$$

where the state vector ($x_t \in R^k$) is:

$$x_{t+1} = \Gamma x_t + \varepsilon_{t+1}, \quad \varepsilon_{t+1} \sim \mathcal{N}(0, \Sigma).$$

x_t is a $k \times 1$ vector, and so are ε_{t+1}, δ_1, Λ_t, and Λ_0, while Γ, Λ_1, and Σ are $k \times k$ matrices.[16]

We assume that the market price of risk is constant ($\Lambda_1 = \mathbf{0}$), so that we can define orthogonal temporary shocks. We decompose the shocks into two groups: the first $h < k$ shocks affect both the temporary and the permanent SDF components and the last $k-h$ shocks are temporary.[17] The parameters of the temporary shocks satisfy $B_{1k-h}^{\infty\prime} = (I_{k-h} - \Gamma_{k-h})^{-1}\delta_{1k-h}' = -\Lambda_{0k-h}'$. This ensures that these shocks do not affect the permanent component of the SDF.

Symmetric Model with Global Factor Now we assume that x_t is a global state variable:

$$\begin{aligned} -\log M_{t+1}^* &= y_{1,t}^* + \frac{1}{2}\Lambda_t^{*\prime}\Sigma\Lambda_t^* + \Lambda_t^{*\prime}\varepsilon_{t+1}, \\ y_{1,t} &= \delta_0^* + \delta_1^{*\prime} x_t, \\ \Lambda_t^* &= \Lambda_0^*, \\ x_{t+1} &= \Gamma x_t + \varepsilon_{t+1}, \quad \varepsilon_{t+1} \sim \mathcal{N}(0, \Sigma). \end{aligned}$$

In a multi-factor Vasicek model with global factors and constant risk prices, long bond uncovered return parity obtains only if countries share the same Λ_h and δ_{1h}, which govern exposure to the permanent, global shocks.

This condition eliminates any differences in permanent risk exposure across countries.[18] The nominal exchange rate has no permanent component $\left(\frac{S_t^{\mathbb{P}}}{S_{t+1}^{\mathbb{P}}} = 1\right)$. From equation (26) , the expected log currency excess return is

[16] Note that if $k = 1$ and $\Lambda_1 = 0$, we are back to the Vasicek (1977) model with one factor and a constant market price of risk. The Vasicek (1977) model presented in the first section is a special case where $\Lambda_0 = \lambda$, $\delta_0' = \delta$, $\delta_0' = 1$ and $\Gamma = \rho$.

[17] A block-diagonal matrix whose blocks are invertible is invertible, and its inverse is a block diagonal matrix (with the inverse of each block on the diagonal). Therefore, if Γ is block-diagonal and $(I - \Gamma)$ is invertible, we can decompose the shocks as described

[18] The terms δ_1' and $\delta_{1h}^{*\prime}$ do not appear in the single-factor Vasicek (1977) model of the first section because that single-factor model assumes $\delta_1 = \delta_{1h}^* = 1$.

equal to:

$$E_t[rx_{t+1}^{FX}] = \frac{1}{2}Var_t(m_{t+1}) - \frac{1}{2}Var_t(m_{t+1}^*) = \frac{1}{2}\Lambda_0'\Sigma\Lambda_0 - \frac{1}{2}\Lambda_0^{*\prime}\Sigma\Lambda_0^*.$$

Non-zero currency risk premia will be only due to variation in the exposure to transitory shocks (Λ_{0k-h}^*).

E.4 Gaussian Dynamic Term Structure Models

Model The $k-$factor heteroskedastic Gaussian Dynamic Term Structure Model (DTSM) generalizes the CIR model. When market prices of risk are constant, the log SDF is given by:

$$\begin{aligned} -m_{t+1} &= y_{1,t} + \frac{1}{2}\Lambda' V(x_t)\Lambda + \Lambda' V(x_t)^{1/2}\varepsilon_{t+1}, \\ x_{t+1} &= \Gamma x_t + V(x_t)^{1/2}\varepsilon_{t+1}, \quad \varepsilon_{t+1} \sim \mathcal{N}(0, I), \\ y_{1,t} &= \delta_0 + \delta_1' x_t, \end{aligned}$$

where $V(x)$ is a diagonal matrix with entries $V_{ii}(x_t) = \alpha_i + \beta_i' x_t$. To be clear, x_t is a $k \times 1$ vector, and so are ε_{t+1}, Λ, δ_1, and β_i. But Γ and V are $k \times k$ matrices. A restricted version of the model would impose that β_i is a scalar and $V_{ii}(x_t) = \alpha_i + \beta_i x_{it}$— this is equivalent to assuming that the price of shock i only depends on the state variable i.

Bond Prices The price of a one period-bond is:

$$P_t^{(1)} = E_t(M_{t+1}) = e^{-\delta_0 - \delta_1' x_t}.$$

For any maturity n, bond prices are exponentially affine, $P_t^{(n)} = \exp(-B_0^n - B_1^{n\prime} x_t)$. Note that B_0^n is a scalar, while B_1^n is a $k \times 1$ vector. The one period-bond corresponds to $B_0^1 = \delta_0$, $B_1' = \delta_1'$. Bond prices are defined recursively by the Euler equation: $P_t^{(n)} = E_t(M_{t+1}P_{t+1}^{n-1})$, which implies:

$$\begin{aligned} P_t^{(n)} &= E_t\left(\exp\left(-y_{1,t} - \frac{1}{2}\Lambda' V(x_t)\Lambda - \Lambda' V(x_t)^{1/2}\varepsilon_{t+1} - B_0^{n-1} - B_1^{n-1\prime} x_{t+1}\right)\right) \\ &= \exp\left(-B_0^n - B_1^{n\prime} x_t\right). \end{aligned}$$

This delivers the following difference equations:

$$\begin{aligned} B_0^n &= \delta_0 + B_0^{n-1} - \frac{1}{2}B_1^{n-1\prime} V(0) B_1^{n-1} - \Lambda' V(0) B_1^{n-1}, \\ B_1^{n\prime} &= \delta_1' + B_1^{n-1\prime}\Gamma - \frac{1}{2}B_1^{n-1\prime} V_x B_1^{n-1} - \Lambda' V_x B_1^{n-1}, \end{aligned}$$

where V_x denotes all the diagonal slope coefficients β_i of the V matrix.
The CIR model studied in the previous pages is a special case of this model. It imposes that $k = 1$, $\sigma = -\sqrt{\beta}$, and $\Lambda = -\frac{1}{\sigma}\sqrt{\gamma}$. Note that the CIR model has no constant term in the square root component of the log SDF, but that does not imply $V(0) = 0$ here as the CIR model assumes that the state variable has a non-zero mean (while it is zero here).

Decomposition (Alvarez and Jermann, 2005) From there, we can define the Alvarez and Jermann (2005) pricing kernel components as for the Vasicek model. The limit of $B_0^n - B_0^{n-1}$ is finite: $\lim_{n\to\infty} B_0^n - B_0^{n-1} = \delta_0 - \frac{1}{2}B_1^{\infty\prime} V(0) B_1^{\infty} - \Lambda_0' V(0) B_1^{\infty}$, where $B_1^{\infty\prime}$ is the solution to the second-order equation above. As a result, B_0^n grows at a linear rate in the limit. We choose the constant β to offset the growth in B_0^n as n becomes very large. Setting $\beta = e^{-\delta_0 + \frac{1}{2}B_1^{\infty\prime} V(0) B_1^{\infty} + \Lambda' V(0) B_1^{\infty}}$ guarantees that Assumption 1 in Alvarez and Jermann (2005) is satisfied. The temporary pricing component of the pricing kernel is thus equal to:

$$\frac{\Lambda_{t+1}^{\mathbb{T}}}{\Lambda_t^{\mathbb{T}}} = \beta e^{B_1^{\infty\prime}(x_{t+1} - x_t)} = \beta e^{B_1^{\infty\prime}(\Gamma - 1)x_t + B_1^{\infty\prime} V(x_t)^{1/2}\varepsilon_{t+1}}.$$

The martingale component of the pricing kernel is then:

$$\begin{aligned}\frac{\Lambda^{\mathbb{P}}_{t+1}}{\Lambda^{\mathbb{P}}_t} = \frac{\Lambda_{t+1}}{\Lambda_t}\left(\frac{\Lambda^{\mathbb{T}}_{t+1}}{\Lambda^{\mathbb{T}}_t}\right)^{-1} &= \beta^{-1} e^{-B_1^{\infty}(\Gamma-1)x_t - B_1^{\infty\prime} V(x_t)^{1/2}\varepsilon_{t+1} - y_{1,t} - \frac{1}{2}\Lambda' V(x_t)\Lambda_t - \Lambda' V(x_t)^{1/2}\varepsilon_{t+1}} \\ &= \beta^{-1} e^{-B_1^{\infty}(\Gamma-1)x_t - \delta_0 - \delta_1' x_t - \frac{1}{2}\Lambda' V(x_t)\Lambda - (\Lambda' + B_1^{\infty\prime})V(x_t)^{1/2}\varepsilon_{t+1}}.\end{aligned}$$

For the martingale component to be constant, we need that $\Lambda' = -B_1^{\infty\prime}$ and $B_1^{\infty}(\Gamma-1) + \delta_1' + \frac{1}{2}\Lambda' V_x \Lambda = 0$. Note that the second condition is automatically satisfied if the first one holds: this result comes from the implicit value of $B_1^{\infty\prime}$ implied by the law of motion of B_1. As a result, the martingale component is constant as soon as $\Lambda = -B_1^{\infty}$.

Decomposition (Hansen and Scheinkman, 2009) We guess an eigenfunction ϕ of the form

$$\phi(x) = e^{c'x}$$

where c is a $k \times 1$ vector of constants. Then, the (one-period) eigenfunction problem can be written as

$$E_t\left[\exp(-\delta_0 - \delta_1' x_t - \frac{1}{2}\Lambda' V(x_t)\Lambda - \Lambda' V^{1/2}(x_t)\varepsilon_{t+1} + cx_{t+1})\right] = \exp(\beta + cx_t)$$

Expanding and matching coefficients, we get:

$$\begin{aligned}\beta &= -\delta_0 - \frac{1}{2}\Lambda' V(0)\Lambda + \frac{1}{2}(c-\Lambda)' V(0)(c-\Lambda) \\ 0 &= c'(\Gamma - I) - \delta_1' + \sum_{i=1}^{k} c_i(c_i - 2\Lambda_i)\mu_i'.\end{aligned}$$

The transitory component of the pricing kernel is by definition:

$$\Lambda^{\mathbb{T}}_t = e^{\beta t - c' x_t}$$

The transitory and permanent SDF component are thus:

$$\begin{aligned}\frac{\Lambda^{\mathbb{T}}_{t+1}}{\Lambda^{\mathbb{T}}_t} &= e^{\beta - c'(x_{t+1} - x_t)} = e^{\beta - c'(\Gamma - I)x_t - c' V^{1/2}(x_t)\varepsilon_{t+1}} \\ \frac{\Lambda^{\mathbb{P}}_{t+1}}{\Lambda^{\mathbb{P}}_t} &= \frac{\Lambda_{t+1}}{\Lambda_t}\left(\frac{\Lambda^{\mathbb{T}}_{t+1}}{\Lambda^{\mathbb{T}}_t}\right)^{-1} \\ &= e^{-\delta_0 - \delta_1' x_t - \frac{1}{2}\Lambda' V(x_t)\Lambda - \Lambda' V^{1/2}(x_t)\varepsilon_{t+1}} e^{-\beta + c'(\Gamma - I)x_t + c' V^{1/2}(x_t)\varepsilon_{t+1}} \\ &= e^{-\delta_0 - \beta + \left[c'(\Gamma - I) - \delta_1'\right]x_t - \frac{1}{2}\Lambda' V(x_t)\Lambda + (c-\Lambda)' V^{1/2}(x_t)\varepsilon_{t+1}}\end{aligned}$$

If $\Lambda = c$, then the equations for β and c become:

$$\begin{aligned}\beta &= -\delta_0 - \frac{1}{2}c' V(0) c \\ 0 &= c'(\Gamma - I) - \delta_1' - \sum_{i=1}^{k} c_i^2 \mu_i'\end{aligned}$$

The martingale component of the SDF is then

$$\frac{\Lambda^{\mathbb{P}}_{t+1}}{\Lambda^{\mathbb{P}}_t} = e^{-\delta_0 - \beta + \left[c'(\Gamma - I) - \delta_1'\right]x_t - \frac{1}{2}c' V(x_t) c} = 1$$

The entirety of the SDF is described by its transitory component in this case.

Term Premium The expected log holding period excess return is:

$$E_t[rx^{(n)}_{t+1}] = -\delta_0 + \left(-B_1^{n-1\prime}\Gamma + B_1^{n\prime} - \delta_1'\right) x_t.$$

The term premium on an infinite-maturity bond is therefore:

$$E_t[rx_{t+1}^{(\infty)}] = -\delta_0 + \left((1-\Gamma)B_1^{\infty\prime} - \delta_1'\right) x_t.$$

The expected log currency excess return is equal to:

$$E_t[-\Delta s_{t+1}] + y_t^* - y_t = \frac{1}{2}Var_t(m_{t+1}) - \frac{1}{2}Var_t(m_{t+1}^*) = \frac{1}{2}\Lambda' V(x_t)\Lambda - \frac{1}{2}\Lambda^{*\prime} V(x_t^*)\Lambda^*.$$

We assume that all the shocks are global and that x_t is a global state variable ($\Gamma = \Gamma^*$ and $V = V^*$, no country-specific parameters in the V matrix— cross-country differences will appear in the vectors Λ). Let us decompose the shocks into two groups: the first $h < k$ shocks affect both the temporary and the permanent SDF components and the last $k - h$ shocks are temporary. Temporary shocks are such that $\Lambda_{k-h} = -B_{1,k-h}^{\infty}$ (i.e., they do not affect the value of the permanent component of the SDF).
The risk premia on the domestic and foreign infinite-maturity bonds (once expressed in the same currency) will be the same provided that the entropy of the domestic and foreign permanent components is the same:

$$\begin{aligned}
(\Lambda_h' + B_{1h}^{\infty\prime})V(0)(\Lambda_h + B_{1h}^{\infty}) &= (\Lambda_h^{*\prime} + B_{1h}^{*\infty\prime})V(0)(\Lambda_h^* + B_{1h}^{\infty *}), \\
(\Lambda_h' + B_{1h}^{\infty\prime})V_x(\Lambda_h + B_{1h}^{\infty}) &= (\Lambda_h^{*\prime} + B_{1h}^{*\infty\prime})V_x(\Lambda_h^* + B_{1h}^{\infty *}).
\end{aligned}$$

To compare these conditions to the results obtained in the one-factor CIR model, recall that $\sigma^{CIR} = -\sqrt{\beta}$, and $\Lambda = -\frac{1}{\sigma^{CIR}}\sqrt{\gamma^{CIR}}$. Differences in Λ_h in the k-factor model are equivalent to differences in γ in the CIR model: in both cases, they correspond to different loadings of the log SDF on the "permanent" shocks. As in the CIT model, differences in term premia can also come form differences in the sensitivity of infinite-maturity bond prices to the global "permanent" state variable ($B_{1h}^{\infty\prime}$), which can be traced back to differences in the sensitivity of the risk-free rate to the "permanent" state variable (i.e., different δ_1 parameters).

Special case Let us start with the special case of no permanent innovations: $h = 0$, the martingale component is constant. Two conditions need to be satisfied for the martingale component to be constant: $\Lambda' = -B_1^{\infty\prime}$ and $B_1^{\infty}(\Gamma - 1) + \delta_1' + \frac{1}{2}\Lambda' V_x \Lambda = 0$. The second condition imposes that the cumulative impact on the pricing kernel of an innovation today given by $\left(\delta_1' + \frac{1}{2}\Lambda' V_x\Lambda\right)(1-\Gamma)^{-1}$ equals the instantaneous impact of the innovation on the long bond price. The second condition is automatically satisfied if the first one holds, as can be verified from the implicit value of $B_1^{\infty\prime}$ implied by the law of motion of B_1. As a result, the martingale component is constant as soon as $\Lambda = -B_1^{\infty}$.

As implied by Equation (27), the term premium on an infinite-maturity zero coupon bond is:

$$E_t[rx_{t+1}^{(\infty)}] = -\delta_0 + \left((1-\Gamma)B_1^{\infty\prime} - \delta_1'\right) x_t. \tag{32}$$

In the absence of permanent shocks, when $\Lambda = -B_1^{\infty}$, this log bond risk premium equals half of the stochastic discount factor variance $E_t[rx_{t+1}^{(\infty)}] = \frac{1}{2}\Lambda' V(x_t)\Lambda$; it attains the upper bound on log risk premia. Consistent with the result in Equation (26), the expected log currency excess return is equal to:

$$E_t\left[rx_{t+1}^{FX}\right] = \frac{1}{2}\Lambda' V(x_t)\Lambda - \frac{1}{2}\Lambda^{*\prime} V(x_t)\Lambda^*. \tag{33}$$

Differences in the market prices of risk Λ imply non-zero currency risk premia. Adding the previous two expressions in Equations (32) and (33), we obtain the foreign bond risk premium in dollars. The foreign bond risk premium in dollars equals the domestic bond premium in the absence of permanent shocks: $E_t\left[rx_{t+1}^{(\infty),*}\right] + E_t\left[rx_{t+1}^{FX}\right] = \frac{1}{2}\Lambda' V(x_t)\Lambda$.

General case In general, there is a spread between dollar returns on domestic and foreign bonds. We describe a general condition for long-run uncovered return parity in the presence of permanent shocks.

Result 5. *In a GDTSM with global factors, the long bond uncovered return parity condition holds only if the countries' SDFs share the parameters $\Lambda_h = \Lambda_h^*$ and $\delta_{1h} = \delta_{1h}^*$, which govern exposure to the permanent global shocks.*

The log risk premia on the domestic and foreign infinite-maturity bonds (once expressed in the same currency) are identical provided that the entropies of the domestic and foreign permanent components are the same:

$$
\begin{aligned}
(\Lambda_h' + B_{1h}^{\infty\prime})V(0)(\Lambda_h + B_{1h}^{\infty}) &= (\Lambda_h^{*\prime} + B_{1h}^{*\infty\prime})V(0)(\Lambda_h^* + B_{1h}^{\infty *}), \\
(\Lambda_h' + B_{1h}^{\infty\prime})V_x(\Lambda_h + B_{1h}^{\infty}) &= (\Lambda_h^{*\prime} + B_{1h}^{*\infty\prime})V_x(\Lambda_h^* + B_{1h}^{\infty *}).
\end{aligned}
$$

These conditions are satisfied if that these countries share $\Lambda_h = \Lambda_h^*$ and $\delta_{1h} = \delta_{1h}^*$ which govern exposure to the global shocks. In this case, the expected log currency excess return is driven entirely by differences between the exposures to transitory shocks: Λ_{k-h} and Λ_{k-h}^*. If there are only permanent shocks ($h = k$), then the currency risk premium is zero.[19]

E.5 Lustig, Roussanov, and Verdelhan (2014)

Model Following Lustig, Roussanov, and Verdelhan (2014), we consider a world with N countries and currencies. Hodrick and Vassalou (2002) have argued that multi-country models can help to better explain interest rates, bond returns and exchange rates (also see recent work by Graveline and Joslin (2011) and Jotikasthira, Le, and Lundblad (2015) in the same spirit).

In the model, the risk prices associated with country-specific shocks depend only on the country-specific factors, but the risk prices of world shocks depend on world and country-specific factors. To describe these risk prices, the authors introduce a common state variable z_t^w shared by all countries and a country-specific state variable z_t^i. The country-specific and world state variables follow autoregressive square-root processes:

$$
\begin{aligned}
z_{t+1}^i &= (1-\phi)\theta + \phi z_t^i - \sigma\sqrt{z_t^i}u_{t+1}^i, \\
z_{t+1}^w &= (1-\phi^w)\theta^w + \phi^w z_t^w - \sigma^w\sqrt{z_t^w}u_{t+1}^w.
\end{aligned}
$$

Lustig, Roussanov, and Verdelhan (2014) assume that in each country i, the logarithm of the real SDF m^i follows a three-factor conditionally Gaussian process:

$$
-m_{t+1}^i = \alpha + \chi z_t^i + \sqrt{\gamma z_t^i}u_{t+1}^i + \tau z_t^w + \sqrt{\delta^i z_t^w}u_{t+1}^w + \sqrt{\kappa z_t^i}u_{t+1}^g,
$$

where u_{t+1}^i is a country-specific SDF shock while u_{t+1}^w and u_{t+1}^g are common to all countries SDFs. All of these three innovations are Gaussian, with zero mean and unit variance, independent of one another and over time. There are two types of common shocks. The first type u_{t+1}^w is priced identically in all countries that have the same exposure δ, and all differences in exposure are *permanent*. The second type of common shock, u_{t+1}^g, is not, as heterogeneity with respect to this innovation is *transitory*: all countries are equally exposed to this shock *on average*, but conditional exposures vary over time and depend on country-specific economic conditions

To be parsimonious, Lustig, Roussanov, and Verdelhan (2014) limit the heterogeneity in the SDF parameters to the different loadings δ^i on the world shock u_{t+1}^w; all the other parameters are identical for all countries. The model is therefore a restricted version of the GDTSM; bond yields and risk premia are available in close forms.

Inflation is composed of a country-specific component and a global component. We simply assume that the same factors driving the real pricing kernel also drive expected inflation. In addition, inflation innovations in our model are not priced. Thus, country i's inflation process is given by $\pi_{t+1}^i = \pi_0 + \eta^w z_t^w + \sigma_\pi \epsilon_{t+1}^i$, where the inflation innovations ϵ_{t+1}^i are independent and identically distributed gaussian. It follows that the nominal risk-free interest rates (in logarithms) are given by

$$
r_t^{i,\$} = \pi_0 + \alpha + \left(\chi - \frac{1}{2}(\gamma + \kappa)\right) z_t^i + \left(\tau + \eta^w - \frac{1}{2}\delta^i\right) z_t^w - \frac{1}{2}\sigma_\pi^2.
$$

[19] To compare these conditions to the results obtained in the CIR model, recall that we have constrained the parameters in the CIR model such that: $\sigma^{CIR} = -\sqrt{\beta}$, and $\Lambda = -\frac{1}{\sigma^{CIR}}\sqrt{\gamma^{CIR}}$. Differences in Λ_h in the k-factor model are equivalent to differences in γ in the CIR model: in both cases, they correspond to different loadings of the log SDF on the "permanent" shocks. Differences in term premia can also come form differences in the sensitivity of the risk-free rate to the permanent state variable (i.e., different δ_1 parameters). These correspond to differences in χ in the CIR model.

Decomposition The log nominal bond prices are affine in the state variable z and z^w: $p_t^{i,(n)} = -C_0^{i,n} - C_1^n z_t - C_2^{i,n} z_t^w$. The price of a one-period nominal bond is:

$$P^{i,(0)} = E_t(M_{t+1}^{i,\$}) = E_t\left(e^{-\alpha-\chi z_t-\tau z_t^w-\sqrt{\gamma z_t^i}u_{t+1}^i-\sqrt{\delta^i z_t^w}u_{t+1}^w-\sqrt{\kappa z_t^i}u_{t+1}^g-\pi_0-\eta^w z_t^w-\sigma_\pi\epsilon_{t+1}^i}\right).$$

As a result, $C_0^1 = \alpha+\pi_0-\frac{1}{2}\sigma_\pi^2$, $C_1^1 = \chi - \frac{1}{2}(\gamma+\kappa)$, and $C_2^{i,1} = \tau - \frac{1}{2}\delta^i + \eta^w$. Bond prices are defined recursively by the Euler equation: $P_t^{i,(n)} = E_t(M_{t+1}^{i,\$}P_{t+1}^{i,(n-1)})$. This leads to the following difference equations:

$$\begin{aligned}
-C_0^{i,n} - C_1^n z_t - C_2^{i,n} z_t^w &= -\alpha - \chi z_t - \tau z_t^w - C_0^{n-1} - C_1^{n-1}\left[(1-\phi)\theta + \phi z_t\right] - C_2^{i,n-1}\left[(1-\phi^w)\theta^w + \phi^w z_t^w\right] \\
&+ \frac{1}{2}(\gamma+\kappa)z_t + \frac{1}{2}\left(C_1^{n-1}\right)^2\sigma^2 z_t - \sigma\sqrt{\gamma}C_1^{n-1}z_t \\
&+ \frac{1}{2}\delta^i z_t^w + \frac{1}{2}\left(C_2^{i,n-1}\right)^2(\sigma^w)^2 z_t^w - \sigma^w\sqrt{\delta^i}C_2^{i,n-1}z_t^w \\
&- \pi_0 - \eta^w z_t^w + \frac{1}{2}\sigma_\pi^2
\end{aligned}$$

Thus bond parameters evolve as:

$$\begin{aligned}
C_0^{i,n} &= \alpha + \pi_0 - \frac{1}{2}\sigma_\pi^2 + C_0^{n-1} + C_1^{n-1}(1-\phi)\theta + C_2^{i,n-1}(1-\phi^w)\theta^w, \\
C_1^n &= \chi - \frac{1}{2}(\gamma+\kappa) + C_1^{n-1}\phi - \frac{1}{2}\left(C_1^{n-1}\right)^2\sigma^2 + \sigma\sqrt{\gamma}C_1^{n-1} \\
C_2^{i,n} &= \tau - \frac{1}{2}\delta^i + \eta^w + C_2^{i,n-1}\phi^w - \frac{1}{2}\left(C_2^{i,n-1}\right)^2(\sigma^w)^2 + \sigma^w\sqrt{\delta^i}C_2^{i,n-1}.
\end{aligned}$$

The temporary pricing component of the pricing kernel is:

$$\Lambda_t^{\mathbb{T}} = \lim_{n\to\infty}\frac{\beta^{t+n}}{P_t^n} = \lim_{n\to\infty}\beta^{t+n}e^{C_0^{i,n}+C_1^n z_t + C_2^{i,n}z_t^w},$$

where the constant β is chosen in order to satisfy Assumption 1 in Alvarez and Jermann (2000): $0 < \lim_{n\to\infty}\frac{P_t^n}{\beta^n} < \infty$. The temporary pricing component of the SDF is thus equal to:

$$\frac{\Lambda_{t+1}^T}{\Lambda_t^T} = \beta e^{C_1^\infty(z_{t+1}-z_t)+C_2^{i,\infty}(z_{t+1}^w - z_t^w)} = \beta e^{C_1^\infty\left[(\phi-1)(z_t^i-\theta)-\sigma\sqrt{z_t^i}u_{t+1}^i\right]+C_2^{i,\infty}\left[(\phi^w-1)(z_t^w-\theta^w)-\sigma\sqrt{z_t^w}u_{t+1}^w\right]}.$$

The martingale component of the SDF is then:

$$\begin{aligned}
\frac{\Lambda_{t+1}^P}{\Lambda_t^P} &= \frac{\Lambda_{t+1}}{\Lambda_t}\left(\frac{\Lambda_{t+1}^T}{\Lambda_t^T}\right)^{-1} = \beta^{-1}e^{-\alpha-\chi z_t^i-\sqrt{\gamma z_t^i}u_{t+1}^i-\tau z_t^w-\sqrt{\delta^i z_t^w}u_{t+1}^w-\sqrt{\kappa z_t^i}u_{t+1}^g} \\
&\quad e^{C_1^\infty\left[(\phi-1)(z_t^i-\theta)-\sigma\sqrt{z_t^i}u_{t+1}^i\right]+C_2^{i,\infty}\left[(\phi^w-1)(z_t^w-\theta^w)-\sigma\sqrt{z_t^w}u_{t+1}^w\right]}.
\end{aligned}$$

As a result, we need $\chi = C_1^\infty(1-\phi)$ to make sure that the country-specific factor does not contribute a martingale component. This special case corresponds to the absence of permanent shocks to the SDF: when $C_1^\infty(1-\phi) = \chi$ and $\kappa = 0$, the permanent component of the stochastic discount factor is constant. To see this result, let us go back to the implicit definition of B_1^∞ in Equation (31):

$$\begin{aligned}
0 &= -\frac{1}{2}(\gamma+\kappa) - \frac{1}{2}\left(C_1^\infty\right)^2\sigma^2 + \sigma\sqrt{\gamma}C_1^\infty \\
0 &= \left(\sigma C_1^\infty - \sqrt{\gamma}\right)^2,
\end{aligned}$$

where we have imposed $\kappa = 0$. In this special case, $C_1^\infty = \sqrt{\gamma}/\sigma$. Using this result in Equation (31), the permanent component of the SDF reduces to:

$$\frac{\Lambda_{t+1}^P}{\Lambda_t^P} = \frac{\Lambda_{t+1}}{\Lambda_t}\left(\frac{\Lambda_{t+1}^T}{\Lambda_t^T}\right)^{-1} = \beta^{-1}e^{-\tau z_t^w-\sqrt{\delta^i z_t^w}u_{t+1}^w}e^{C_2^{i,\infty}\left[(\phi^w-1)(z_t^w-\theta^w)-\sigma\sqrt{z_t^w}u_{t+1}^w\right]}.$$

Term Premium The expected log excess return on a zero coupon bond is thus given by:

$$E_t[rx_{t+1}^{(n)}] = [-\frac{1}{2}\left(C_1^{n-1}\right)^2\sigma^2 + \sigma\sqrt{\gamma}C_1^{n-1}]z_t + [-\frac{1}{2}\left(C_2^{i,n-1}\right)^2\sigma^2 + \sigma\sqrt{\delta}^i C_2^{i,n-1}]z_t^w.$$

The expected log excess return of an infinite maturity bond is then:

$$E_t[rx_{t+1}^{(\infty)}] = [-\frac{1}{2}\left(C_1^{\infty}\right)^2\sigma^2 + \sigma\sqrt{\gamma}C_1^{\infty}]z_t + [-\frac{1}{2}\left(C_2^{i,\infty}\right)^2\sigma^2 + \sigma\sqrt{\delta}^i C_2^{i,\infty}]z_t^w.$$

The $-\frac{1}{2}\left(C_1^{\infty}\right)^2\sigma^2$ is a Jensen term. The term premium is driven by $\sigma\sqrt{\gamma}C_1^{\infty}z_t$, where C_1^{∞} is defined implicitly in the second order equation $B_1^{\infty} = \chi - \frac{1}{2}(\gamma+\kappa) + C_1^{\infty}\phi - \frac{1}{2}\left(C_1^{\infty}\right)^2\sigma^2 + \sigma\sqrt{\gamma}C_1^{\infty}$. Consider the special case of $C_1^{\infty}(1-\phi) = \chi$ and $\kappa = 0$ and $C_2^{i,\infty}(1-\phi) = \tau$. In this case, the expected term premium is simply $E_t[rx_{t+1}^{(\infty)}] = \frac{1}{2}(\gamma z_t + \delta z_t^w)$, which is equal to one-half of the variance of the log stochastic discount factor.

Country-specific Factors Suppose that the foreign pricing kernel is specified as in Equation (29) with the same parameters. However, the foreign country has its own factor z^*. As a result, the difference between the domestic and foreign log term premia is equal to the log currency risk premium, which is given by $E_t[rx_{t+1}^{FX}] = \frac{1}{2}\gamma(z_t - z_t^*)$. In other words, the expected foreign log holding period return on a foreign long bond converted into U.S. dollars is equal to the U.S. term premium: $E_t[rx_{t+1}^{(\infty),*}] + E_t[rx_{t+1}^{FX}] = \frac{1}{2}\gamma z_t$.

Uncovered Bond Return Parity In this model, the expected log excess return of an infinite maturity bond is then:

$$E_t[rx_{t+1}^{(i,\infty)}] = \left[C_1^{\infty}(1-\phi) - \chi + \frac{1}{2}(\gamma+\kappa)\right]z_t^i + \left[C_2^{i,\infty}(1-\phi^w) - \tau + \frac{1}{2}\delta^i - \eta^w\right]z_t^w.$$

The foreign currency risk premium is given by:

$$E_t[rx_{t+1}^{FX,i}] = -\frac{1}{2}(\gamma+\kappa)(z_t^i - z_t) + \frac{1}{2}(\delta - \delta^i)(z_t^w).$$

Investors obtain high foreign currency risk premia when investing in currencies whose exposure to the global shocks is smaller. That is the source of short-term carry trade risk premia. The foreign bond risk premium in dollars is simply given by the sum of the two expressions above:

$$\begin{aligned} E_t[rx_{t+1}^{(i,\infty)}] + E_t[rx_{t+1}^{FX,i}] &= \left[\frac{1}{2}(\gamma+\kappa)z_t + (C_1^{\infty}(1-\phi) - \chi)z_t^i\right] \\ &+ \left[\frac{1}{2}\delta + C_2^{i,\infty}(1-\phi^w) - \tau - \eta^w\right]z_t^w. \end{aligned}$$

Next, we examine the conditions that are necessary for long-run uncovered bond return parity in this model.

Result 6. *The long-run uncovered bond return parity holds if $C_1^{\infty}(1-\phi) = \chi$, $\kappa = 0$, and $C_2^{i,\infty}(1-\phi^w) = \tau + \eta^w$.*

The first two restrictions rule out permanent effects of country-specific shocks. The last restriction rules out permanent effects of global shocks (u^w). When these restrictions are satisfied, the pricing kernel is not subject to permanent shocks. The U.S. term premium is simply $E_t[rx_{t+1}^{(\infty)}] = \frac{1}{2}(\gamma z_t + \delta z_t^w)$, which is equal to one-half of the variance of the log stochastic discount factor. As can easily be verified, the expected foreign log holding period return on a foreign long bond converted into U.S. dollars is equal to the U.S. term premium: $E_t[rx_{t+1}^{i,(\infty)}] + E_t[rx_{t+1}^{FX,i}] = \frac{1}{2}(\gamma z_t + \delta z_t^w)$. The higher foreign currency risk premium for investing in high δ countries is exactly offset by the lower bond risk premium.

Calibration The calibration is reported in Table 13.

Simulation Results We simulate the Lustig, Roussanov, and Verdelhan (2014) model, obtaining a panel of $T = 33600$ monthly observations and $N = 30$ countries. The simulation results are reported in Table 14. Each month, the 30 countries are ranked by their interest rates into six portfolios. Low interest rate currencies on average have higher exposure δ to the world factor. As a result, these currencies appreciate in case of an adverse world shocks. Long positions in these currencies earn negative excess returns (rx^{fx}) of -2.91% per annum. On the

Table 13: Parameter estimates.

This table reports the parameter values for the estimated version of the model. The model is defined by the following set of equations:

$$
\begin{aligned}
-m^i_{t+1} &= \alpha + \chi z^i_t + \sqrt{\gamma z^i_t} u^i_{t+1} + \tau z^w_t + \sqrt{\delta^i z^w_t} u^w_{t+1} + \sqrt{\kappa z^i_t} u^g_{t+1}, \\
z^i_{t+1} &= (1-\phi)\theta + \phi z^i_t - \sigma \sqrt{z^i_t} u^i_{t+1}, \\
z^w_{t+1} &= (1-\phi^w)\theta^w + \phi^w z^w_t - \sigma^w \sqrt{z^w_t} u^w_{t+1}, \\
\pi^i_{t+1} &= \pi_0 + \eta^w z^w_t + \sigma_\pi \epsilon^i_{t+1}.
\end{aligned}
$$

The 17 parameters were obtained to match the moments in Table 13 under the assumption that all countries share the same parameter values except for δ^i, which is distributed uniformly on $[\delta_h, \delta_l]$. The home country exhibits the average δ, which is equal to 0.36. The standard errors obtained by bootstrapping are reported between brackets.

Stochastic discount factor					
α (%)	χ	τ	γ	κ	δ
0.76	0.89	0.06	0.04	2.78	0.36
State variable dynamics					
ϕ	θ (%)	σ (%)	ϕ^w	θ^w (%)	σ^w (%)
0.91	0.77	0.68	0.99	2.09	0.28
Inflation dynamics				Heterogeneity	
η^w	π_0 (%)	σ^π (%)		δ_h	δ_l
0.25	-0.31	0.37		0.22	0.49
Implied SDF dynamics					
$E(Std_t(m))$	$Std(Std_t(m))$ (%)	$E(Corr(m_{t+1}, m^i_{t+1}))$		$Std(z)$ (%)	$Std(z^w)$ (%)
0.59	4.21	0.98		0.50	1.32

other hand, high interest rate currencies typically have high δ. Long positions in these currencies earn positive excess returns (rx^{FX}) of 2.61% per annum. At the short end, the carry trade strategy, which goes long in the sixth portfolio and short in the first one, delivers an excess return of 5.51% and a Sharpe ratio of 0.47.

This spread is only partly offset by higher local currency bond risk premia in the low interest rate countries with higher δ. The log excess return on the 30-year zero coupon bond is 1.28% in the first portfolio compared to 0.01 % in the last portfolio. While the low interest rate currencies do have steeper yield curves and higher local bond risk premia, this effect is too weak to offset the effect of the currency risk premia. At the 30-year maturity, the high-minus-low carry trade strategy still delivers a profitable excess return of 4.24% and a Sharpe ratio of 0.36. This large currency risk premium at the long end of the curve stands in stark contrast to the data.

Our theoretical results help explain the shortcomings of this simulation. In Lustig, Roussanov, and Verdelhan (2014) calibration, the conditions for long run bond parity are not satisfied. First, global shocks have permanent effects in all countries, because $C_2^{i,\infty}(1-\phi^w) < \tau + \eta^w$ for all $i = 1, \ldots, 30$. Second, the global shocks are not symmetric, because δ varies across countries. The heterogeneity in δ's across countries generates substantial dispersion in exposure to the permanent component. As a result, our long-run uncovered bond parity condition is violated.

Table 14: Simulated Excess Returns on Carry Strategies

	Low	2	3	4	5	High
	Panel A: Interest rates, Bond Returns and Exchange Rates					
Δs	0.57	−0.38	−0.14	−0.70	−0.84	−1.25
$f-s$	−2.33	−1.33	−0.71	−0.15	0.40	1.35
$rx^{*,15}$	1.03	0.52	0.32	0.23	0.17	−0.16
$rx^{*,30}$	1.28	0.72	0.51	0.41	0.35	0.01
	Panel B: Carry Returns with Short-Term Bills					
rx^{fx}	−2.91	−0.95	−0.57	0.54	1.24	2.61
	Panel C: Carry Returns with Long-Term Bonds					
$rx^{\$,15}$	−1.88	−0.43	−0.25	0.77	1.41	2.45
$rx^{\$,30}$	−1.62	−0.23	−0.06	0.95	1.59	2.62

Notes: The table reports summary statistics on simulated data from the Lustig, Roussanov, and Verdelhan (2014) model. Data are obtained from a simulated panel with $T = 33600$ monthly observations and $N = 30$ countries. Countries are sorted by interest rates into six portfolios. Panel A reports the average change in exchange rate (Δs), the average interest rate difference or forward discount ($f-s$), the average foreign bond excess returns for bonds of 15- and 30-year maturities in local currency ($rx^{*,15}$, $rx^{*,30}$). Panel B reports the average log currency excess returns (rx^{fx}). Panel C reports the average foreign bond excess returns for bonds of 15- and 30-year maturities in home currency ($rx^{\$,15}$, $rx^{\$,30}$). The moments are annualized (i.e., means are multiplied by 12).

E.6 Lustig, Roussanov, and Verdelhan (2014) with Temporary and Permanent Shocks

The Lustig, Roussanov, and Verdelhan (2014) calibration fails to satisfy the long term bond return parity condition. We turn to a model that explicitly features global permanent and transitory shocks. We show that the heterogeneity in the SDFs' loadings on the *permanent* global shocks needs to be ruled out in order to match the empirical evidence on the term structure of carry risk.

Model We assume that in each country i, the logarithm of the real SDF m^i follows a three-factor conditionally Gaussian process:

$$-m^i_{t+1} = \alpha + \chi z^i_t + \sqrt{\gamma z^i_t} u^i_{t+1} + \tau^i z^w_t + \sqrt{\delta^i z^w_t} u^w_{t+1} + \tau^{\mathbb{P},i} z^{\mathbb{P},w}_t + \sqrt{\delta^{\mathbb{P}} z^{\mathbb{P},w}_t} u^w_{t+1} + \sqrt{\kappa z^i_t} u^g_{t+1}.$$

The inflation process is the same as before. Note that the model now features two global state variables, z^w_t and $z^{\mathbb{P},w}_t$. The state variables follow similar square root processes as in the previous model:

$$\begin{aligned} z^i_{t+1} &= (1-\phi)\theta + \phi z^i_t - \sigma\sqrt{z^i_t} u^i_{t+1}, \\ z^w_{t+1} &= (1-\phi^w)\theta^w + \phi^w z^w_t - \sigma^w \sqrt{z^w_t} u^w_{t+1}. \\ z^{\mathbb{P},w}_{t+1} &= (1-\phi^{\mathbb{P},w})\theta^{\mathbb{P},w} + \phi^{\mathbb{P},w} z^w_t - \sigma^{\mathbb{P},w} \sqrt{z^{\mathbb{P},w}_t} u^{\mathbb{P},w}_{t+1}. \end{aligned}$$

But one of the common factors, z^w_t, is rendered transitory by imposing that $C^{i,\infty}_2(1-\phi^w) = \tau^i$. To make sure that the global shocks have no permanent effect for each value of δ^i, we need to introduce another source of heterogeneity across countries. Countries must differ in τ, ϕ^w, σ^w, or η^w (or a combination of those). Without this additional source of heterogeneity, there are at most two values of δ^i that are possible (for each set of parameters).[20] Here we simply choose to let the parameters τ differ across countries.

Bond Prices Our model only allows for heterogeneity in the exposure to the transitory common shocks (δ^i), but not in the exposure to the permanent common shock ($\delta^{\mathbb{P}}$). The nominal log zero-coupon n-month yield of maturity in local currency is given by the standard affine expression $y_t^{(n)} = \frac{1}{n}\left(C_0^n + C_1^n z_t + C_2^n z_t^w + C_3^n z_t^{\mathbb{P},w}\right)$, where the coefficients satisfy second-order difference equations. Given this restriction, the bond risk premium is equal to:

$$\begin{aligned} E_t[rx_{t+1}^{(i,\infty)}] &= \left[C_1^\infty(1-\phi) - \chi + \frac{1}{2}(\gamma+\kappa)\right] z_t + \frac{1}{2}\delta^i z_t^w \\ &+ \left[C_3^\infty(1-\phi^{\mathbb{P},w}) - \tau^{\mathbb{P}} + \frac{1}{2}\delta^{\mathbb{P}} - \eta^w\right] z_t^{\mathbb{P},w}. \end{aligned}$$

The log currency risk premium is equal to $E_t[rx_{t+1}^{FX,i}] = (\gamma+\kappa)(z_t - z_t^i)/2 + (\delta - \delta^i)z_t^w/2$. The permanent factor $z_t^{w,\mathbb{P}}$ drops out. This also implies that the expected foreign log holding period return on a foreign long bond converted into U.S. dollars is equal to:

$$\begin{aligned} E_t[rx_{t+1}^{(i,\infty)}] + E_t[rx_{t+1}^{FX,i}] &= \left[(C_1^\infty(1-\phi) - \chi)z_t^i + \frac{1}{2}(\gamma+\kappa)z_t\right] + \frac{1}{2}\delta z_t^w \\ &+ \left[C_3^\infty(1-\phi^{\mathbb{P},w}) - \tau^{\mathbb{P}} + \frac{1}{2}\delta^{\mathbb{P}} - \eta^w\right] z_t^{\mathbb{P},w}. \end{aligned}$$

Given the symmetry that we have imposed, the difference between the foreign term premium in dollars and the domestic term premium is then given by: $[C_1^\infty(1-\phi) - \chi]\,(z_t^i - z_t)$. There is no difference in long bond returns that can be traced back to the common factor; only the idiosyncratic factor. The spread due to the common factor is the only part that matters for the long-term carry trade, which approximately produces zero returns here.

Foreign Bond Risk Premia Across Maturities To match short-term carry trade returns, we need asymmetric exposure to the transitory shocks, governed by (δ), but not to permanent shocks, governed by ($\delta^{\mathbb{P}}$). If the foreign kernel is less exposed to the transitory shocks than the domestic kernel ($\delta > \delta^i$), there is a large positive foreign currency risk premium (equal here to $(\delta - \delta^i)z_t^w/2$), but that premium is exactly offset by a smaller foreign term premium and hence does not affect the foreign bond risk premium in dollars. The countries with higher exposure will also tend to have lower interest rates when the transitory volatility z_t increases, provided that $\left(\tau - \frac{1}{2}\delta\right) < 0$. Hence, in this model, the high δ^i funding currencies in the lowest interest rate portfolios will tend to earn negative currency risk premia, but positive term premia. The reverse would be true for the low δ^i investment currencies in the high interest rate portfolios. This model thus illustrates our main theoretical findings: chasing high interest rates does not necessarily work at the long end of the maturity spectrum. If there is no heterogeneity in the loadings on the permanent global component of the SDF, then the foreign term premium on the longest bonds, once converted to U.S. dollars is identical to the U.S. term premium.

Challenge The simulation of the LRV (2014) model highlights a key tension: without the heterogeneity in δs, the model cannot produce short-term carry trade risk premia; the heterogeneity in δs, however, leads to counterfactual long-term carry risk premia. The decomposition of the pricing kernel suggests a potential route to address this tension: two global state variables, one describing transitory shocks and one describing permanent shocks. As noted in the study of the term structure models, the heterogeneity in the SDFs' loadings on the *permanent* global shocks needs to be ruled out in order to match the long term bond parity. The heterogeneity in the SDFs' loadings on the *transitory* global shocks accounts for the carry trade excess returns at the short end of the yield curve. We sketch such a model in the Online Appendix, and show that the heterogeneity in the SDFs' loadings on pure *transitory* global shocks can only be obtained if countries differ in more than one dimension (δ, but also τ, ϕ^w, σ^w, or η^w, or a combination of those parameters). A potential solution to the tension entails a very rich model that is beyond the scope of this paper. We leave the empirical estimation of a N-country model that replicates the bond risk premia at different maturities as a challenge for the literature.

[20] This result appears when plugging the no-permanent-component condition in the differential equation that governs the loading of the bond price on the global state variable.