

# An Experiment to Definitively Determine the Contributions of Multiple Photon Exchange in Elastic Lepton-Nucleon Scattering

K. Dow, W. Franklin, D. Hasell, E. Ihloff, J. Kelsey, M. Kohl,  
J. Matthews, R. Milner, R. Redwine, C. Tschalaer, E. Tsentalovich,  
B. Turchinets, J. van der Laan, and F. Wang  
MIT Laboratory for Nuclear Science and Bates Linear Accelerator Center

J. Arrington  
Argonne National Laboratory

R. Alarcon  
Arizona State University

J. Calarco  
University of New Hampshire

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## Abstract

Recent determinations of the proton electric to magnetic form factor ratio from polarization transfer measurements at Jefferson Lab indicate an unexpected and dramatic discrepancy with the form factor ratio obtained using the Rosenbluth separation technique in unpolarized cross section measurements. This discrepancy has been explained by the effects of multiple photon exchange beyond the usual one-photon exchange approximation in the calculation of the elastic electron-proton scattering cross section. Since most of our understanding on the structure of the proton and atomic nuclei is based upon lepton scattering analyzed in terms of the single photon approximation; it is essential to definitively verify the contribution of multiple photon exchange.

The most direct evidence for multiple photon exchange would be a deviation from unity in the ratio of positron-proton to electron-proton elastic scattering cross sections. The definitive experiment would utilize intense beams of electrons and positrons on a hydrogen target at incident energies from about 2 GeV to about 4 GeV and precisely measure elastic scattering at angles about  $60^\circ$  with high statistical and systematic precision. In this document the concept for such an experiment at the DORIS storage ring at DESY, Germany utilizing the Bates Large Acceptance Spectrometer Toroid (BLAST) detector is developed. Such an experiment could be installed and commissioned in parallel with existing light source operation at DORIS but would require dedicated data taking for about one month per year for several years.

# 1 Introduction

In the course of the more than 50 year long history of elastic electron-proton scattering since Hofstadter [1] the separation of the proton's electric and magnetic form factors,  $G_E(Q^2)$  and  $G_M(Q^2)$ , has been of particular interest. These two functions of  $Q^2$  describe the distribution of charge and magnetism of the proton and it is expected that precise *ab initio* calculations in terms of quarks and gluons will become available in the foreseeable future using lattice QCD techniques [2]. Until the 1990's the experimental method to separate  $G_E(Q^2)$  and  $G_M(Q^2)$  was based on the procedure by Rosenbluth [3] measuring the unpolarized elastic cross section at fixed four-momentum transfer,  $Q^2$ , but with different electron scattering angles and incident beam energies. It was found that the  $Q^2$  dependence of both  $G_E$  and  $G_M$ , to a good approximation, followed the form of the Fourier transform of an exponentially decaying distribution, namely the dipole form factor  $(1 + Q^2/0.71)^{-2}$ , implying a ratio of  $\mu G_E/G_M \approx 1$  as shown by the open symbols in Figure 1 (left panel).

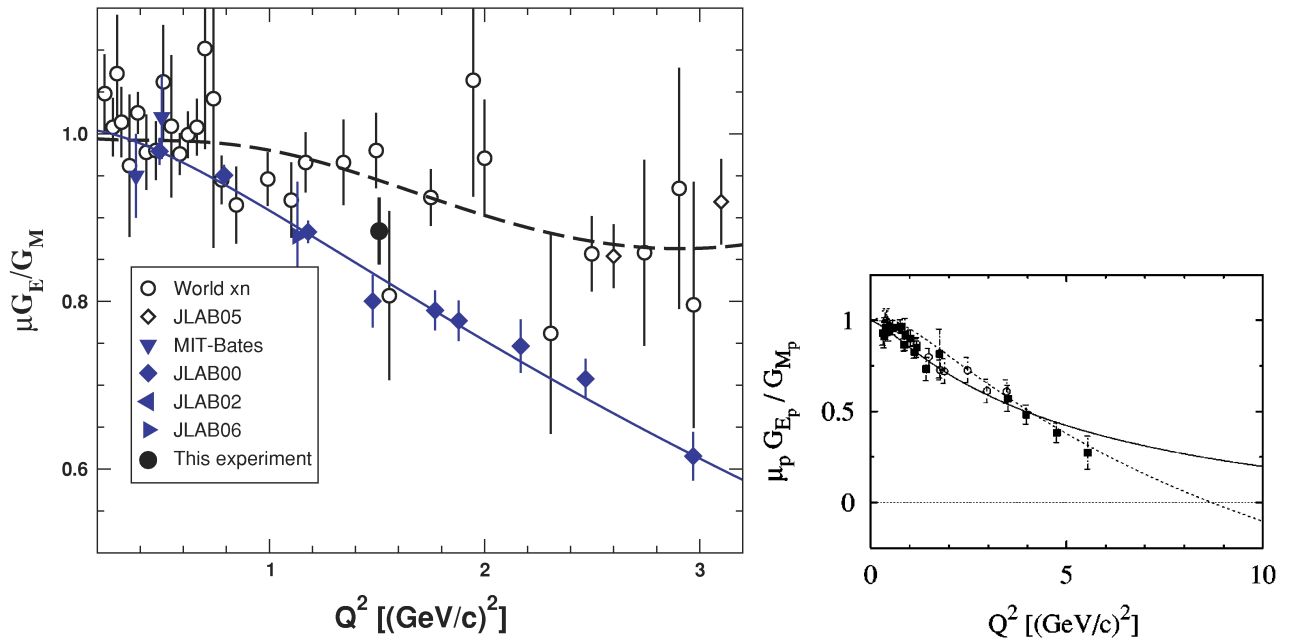


Figure 1: Left panel: Proton electric to magnetic form factor ratio  $\mu G_E/G_M$  from unpolarized (open symbols, "World xn" and JLAB05 [4, 6]) and polarized measurements (filled symbols, MIT-Bates [8], JLAB00 [12], JLAB02 [13], and JLAB06 [14]). The figure has been taken from [20] ("this experiment"). The curves are fits to unpolarized data [4] and to data from [12, 13] only for the high  $Q^2$  region. Right panel: form factor ratio  $\mu G_E/G_M$  from recoil polarization compared with calculations by Iachello from 2004 (solid) [16] and 1973 (dashed) [15].

Due to the nature of the Rosenbluth formula

$$\frac{d\sigma/d\Omega}{(d\sigma/d\Omega)_{Mott}} = \frac{G_E^2(Q^2) + \tau G_M^2(Q^2)}{1 + \tau} + 2\tau G_M^2(Q^2) \tan^2 \frac{\theta}{2} = \frac{\epsilon G_E^2(Q^2) + \tau G_M^2(Q^2)}{\epsilon(1 + \tau)}, \quad (1)$$

where  $\tau = Q^2/(4M_p^2)$ , the transverse virtual photon polarization  $\epsilon = [1+2(1+\tau)\tan^2(\theta/2)]^{-1}$ , and  $(d\sigma/d\Omega)_{Mott} = \alpha^2/(4E^2)(\cos^2\frac{\theta}{2}/\sin^4\frac{\theta}{2})(E'/E)$ ; the weight of  $G_E$  in the cross section becomes less at higher  $Q^2$  making the Rosenbluth separation of  $G_E(Q^2)$  and  $G_M(Q^2)$  at high momentum transfer rather difficult. While some experiments reported a scaling of the form factors; others occasionally observed significant deviations of the ratio  $\mu G_E/G_M$  from unity. The world data for elastic e-p scattering has recently been compiled by [4]. The most recent Rosenbluth-type measurements have again confirmed the scaling behavior of the proton form factor ratio [5, 6], and additional unpolarized precision measurements are planned [7].

In the late 1990's, development of polarized beams, targets and polarimeters permitted a new way to measure the form factor ratio more directly through the interference of  $G_E$  and  $G_M$  in the spin-dependent elastic cross section asymmetry [8, 9, 10, 11]. It came as a big surprise when the high precision polarization transfer measurements at Jefferson Laboratory at higher momentum transfers (up to  $5.5 \text{ (GeV/c)}^2$ ) gave striking evidence that the proton form factor ratio  $\mu G_E/G_M$  was monotonically falling with  $Q^2$  [12, 13, 14]. This  $Q^2$  dependence was dramatically different from that observed with the unpolarized Rosenbluth method. Linear extrapolation would even suggest a node of the electric form factor near  $8 \text{ (GeV/c)}^2$ . Note that this decline of the proton form factor ratio was predicted already in 1973 by calculations based on vector-meson dominance including the expected node around  $8 \text{ (GeV/c)}^2$  [15, 16] shown in Figure 1 (right panel). Future recoil polarization experiments at Jefferson Lab will extend the  $Q^2$  range up to  $9 \text{ (GeV/c)}^2$  with a new recoil polarimeter [17] and up to  $14 \text{ (GeV/c)}^2$  after the 12 GeV upgrade [18].

Alternative measurements of  $G_E/G_M$  are based on the spin-dependent asymmetries with polarized beam and target. Experiments of this kind are considered equivalent to polarization transfer and constitute important independent tests to verify the recoil polarization results. Such measurements have recently been performed with the Bates Large Acceptance Spectrometer Toroid (BLAST) at low  $Q^2$  using an internal polarized hydrogen target [19]. The result is consistent with scaling of the form factor ratio, albeit at low  $Q^2$  where no discrepancy between polarized and unpolarized measurements was expected. Another experiment used a frozen-spin ammonia target [20] to extract the form factor ratio at somewhat higher  $Q^2 \approx 1.51 \text{ (GeV/c)}^2$  with a result for  $\mu G_E/G_M$  between the unpolarized and polarization transfer data (solid circle in left plot of Figure 1). Clearly, further measurements are needed to resolve this discrepancy.

## 2 Beyond the Single Photon Approximation

The generally accepted explanation for the discrepancy between the recoil polarization and Rosenbluth determinations of the elastic proton form factor ratio is the exchange of multiple ( $>1$ ) photons during the electron-proton elastic scattering process [21, 23]. This implies that certain lepton-nucleon scattering observables will differ significantly from their one-photon exchange (or first-order Born approximation) expectation value.

Multiple-photon exchange processes will exhibit a characteristic dependence of the elastic lepton-proton scattering cross section on the value of the virtual photon polarization,  $\epsilon$ . As  $\epsilon$  decreases, the effects of multiple-photon exchange on the elastic cross section tend to increase in magnitude.

The discrepancy between the recoil polarization and Rosenbluth determinations of the elastic proton form factor ratio grows with increasing  $Q^2$ . At high  $Q^2$ , the cross section is dominated by magnetic (i.e. transverse) scattering. This explains why the effect on the extraction of  $G_E$  from Rosenbluth separations can be sizable, while the effect on the cross section at all values of  $Q^2$  is rather modest. At the same time, the form factor ratio from polarization experiments is less affected.

The effect of multiple-photon exchange on the electromagnetic elastic form factors involves the real part of the multiple-photon exchange amplitude. The observable most sensitive to this amplitude is the ratio of the elastic cross section for electron-proton to positron-proton scattering. In the presence of multiple-photon exchange, the cross section for unpolarized lepton-proton scattering contains an interference term between the one- and two-photon amplitudes. This interference is odd under time reversal, and hence has the opposite sign for elastic positron-proton and electron-proton scattering. Therefore, a non-zero two-photon amplitude would result in different cross sections for unpolarized electron-proton and positron-proton scattering.

Figure 2 shows the ratio of the two cross sections as a function of the virtual photon polarization  $\epsilon$ . The ratio would be unity in the case of pure single photon exchange, i.e. the Born approximation. The sensitivity is enhanced at low  $\epsilon$ , exceeding 4% for  $\epsilon \leq 0.4$ , provided  $Q^2 \geq 2$  (GeV/c)<sup>2</sup>. Beyond  $Q^2 = 2$  (GeV/c)<sup>2</sup> the  $Q^2$  dependence of the two-photon effect is small, and since the cross section decreases rapidly with  $Q^2$ , one would want to keep  $Q^2$  as low as possible for optimized statistics. This is clear in Figure 3 which displays the  $e^+p/e^-p$  cross section ratio as a function of the scattering angle for three beam energies. Up to scattering angles of about  $80^\circ$ , the cross section ratio is almost independent of the beam energy, and hence of  $Q^2$  for a given scattering angle.

The effects of radiative corrections on the  $e^+p/e^-p$  cross section ratio are expected to be negligible. Bethe-Heitler corrections will be identical for positrons and electrons and interference effects which change sign for  $e^+$  and  $e^-$ , e.g. between the lepton bremsstrahlung process and the proton bremsstrahlung process (a very small effect at these energies) are also expected to be negligible [22].

Figure 4 shows the elastic proton electric to magnetic form factor ratio under various conditions: The red diamonds correspond to the form factor ratio as determined from recoil polarization, which has only little sensitivity to multi-photon effects. The magenta crosses correspond to the form factor ratio from existing  $e^-p$  Rosenbluth separation data (Bosted fit [24]). The green open circles represent the effect of two-photon exchange on the

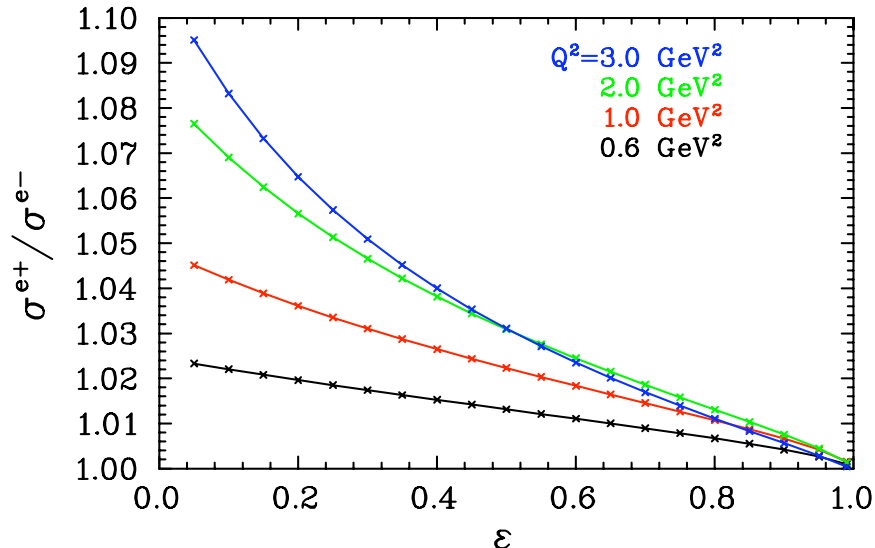


Figure 2: Ratio of elastic positron-proton to electron-proton cross section versus virtual photon polarization for given  $Q^2$  [23].

Rosenbluth measurements, using a simple fit to the two-photon correction that explains that discrepancy between polarization and Rosenbluth measurements. The blue solid circles are the result of applying this two-photon correction to Rosenbluth measurements using  $e^+p$  scattering. The expected node at  $\approx 2.6$  (GeV/c) $^2$  is remarkable. Above 2.6 (GeV/c) $^2$ , one would expect to find negative values for  $G_E^{p,2}$  from  $e^+p$  Rosenbluth separations.

Previous experiments from the 1960's at SLAC [25] have measured the  $e^+p/e^-p$  cross section ratio. However, high-precision measurements with uncertainties of 1% were done only at low  $Q^2$  or very large  $\epsilon$ , where the multiple-photon exchange effects appear to be small. Measurements at low  $\epsilon$  had uncertainties of  $\approx 5\%$ , too large to see conclusive deviations from unity. Recent reanalysis of the (limited) low- $\epsilon$  data give an indication of multiple-photon exchange effects, consistent with recent calculations, but only at the three-sigma level [4].

Recently, two new experiments have been proposed to study the  $e^+p$  and  $e^-p$  cross section ratio: one at Jefferson Lab [26] using a secondary electron/positron beam from a pair production target, and another at Novosibirsk [27] based on stored electron and positron beams incident on an internal unpolarized hydrogen target.

The effect of two-photon exchange on the real part of the lepton-nucleon scattering amplitude can also be investigated by studying the  $\epsilon$ -dependence of the proton form factor ratio from polarization experiments. Such an experiment has been proposed at Jefferson Lab [28]. Precise mapping of Rosenbluth cross sections in unpolarized  $e^-p$  scattering will also reveal any nonlinearities in the  $\epsilon$  dependence of the cross section [7].

The imaginary part of the two-photon amplitude would give rise to non-zero transverse single-spin asymmetries, of either the beam ( $A_n$ ), the target ( $A_y$ ) or the induced polarization ( $P_y$ ). These single-spin asymmetries will be studied at Jefferson Lab as well [28, 29].

In this document, we argue that the use of the intense, multi-GeV stored electron and positron beams at the storage ring DORIS at DESY, Hamburg, Germany in combination with the BLAST detector can produce the most definitive data to determine the effect of

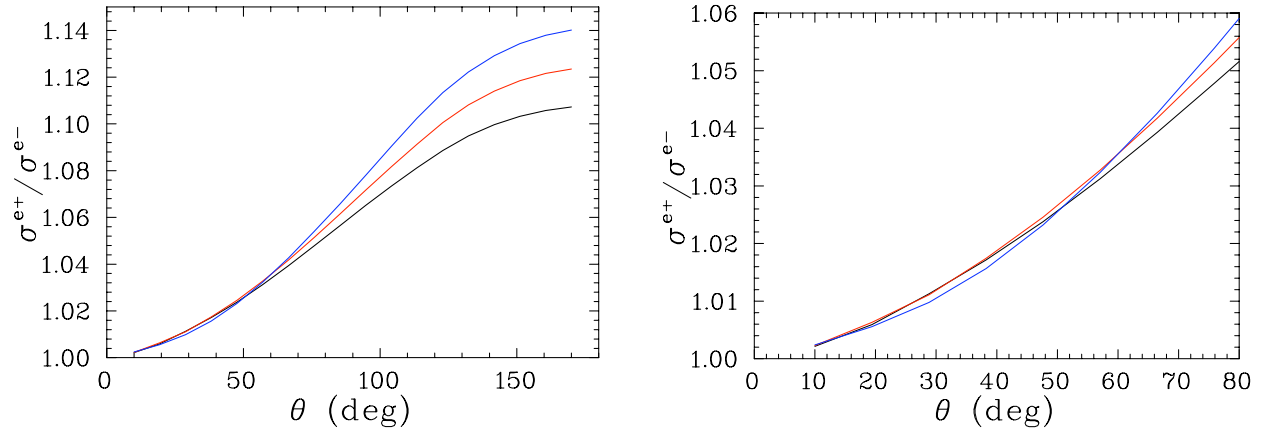


Figure 3:  $e^+p/e^-p$  cross section ratio as a function of scattering angle, for three beam energies (black=2.0 GeV, red=2.5 GeV, blue=3.0 GeV) [23]. The figure on the right shows the region up to  $80^\circ$  where only little energy dependence of the two-photon effect is evident.

multiple photon exchange in elastic lepton-proton scattering and verify the recent theoretical predictions.

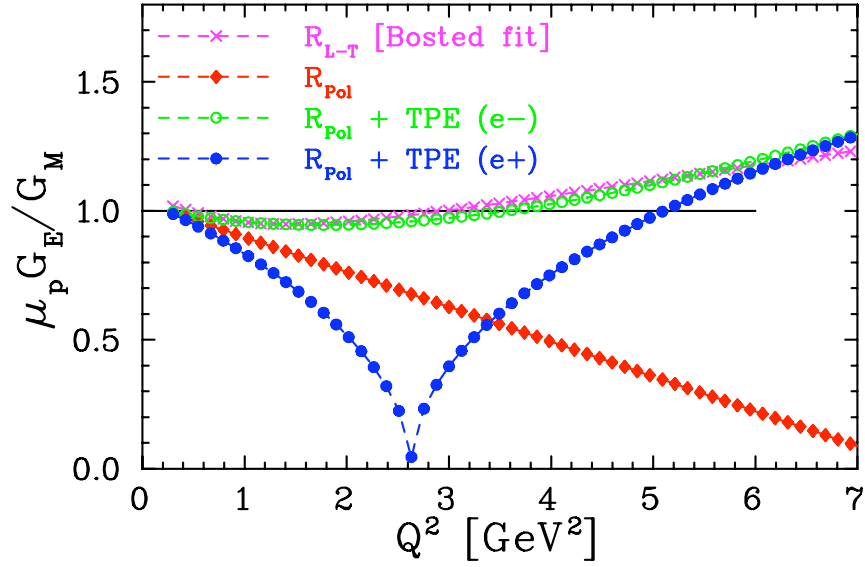


Figure 4: Proton electric to magnetic form factor ratio  $\mu_p G_E^p / G_M^p$  without (red diamonds) and with two-photon effects calculated for  $e^-p$  (green open circles) and  $e^+p$  (blue solid circles) Rosenbluth separations [23]. The magenta crosses represent a fit to existing Rosenbluth-separated  $e^-p$  data. The electric form factor  $G_E^p$  from unpolarized  $e^+p$  scattering has a node expected at  $Q^2 \approx 2.6$  (GeV/c) $^2$ , with  $G_E^p < 0$  for  $Q^2 > 2.6$  (GeV/c) $^2$ .

### 3 Proposed Experiment

We propose to measure the ratio of elastic electron-proton to positron-proton cross sections over a range of  $\epsilon$  with the BLAST detector using an internal, unpolarized hydrogen target and intense stored beams of unpolarized positrons and electrons at energies between 2.3 and 4.5 GeV at the site of the ARGUS experiment on the storage ring DORIS at DESY in Hamburg, Germany. To carry out this experiment it will be required:

- to operate the DORIS storage ring at energies from  $\sim 2$  GeV to  $\sim 4.5$  GeV,
- to switch between beams of electrons and positrons at a frequency on the order of once per day,
- to relocate the BLAST detector from MIT-Bates to DESY/DORIS, and
- to install an unpolarized hydrogen internal gas target in the DORIS storage ring.

At DORIS, both electron and positron beams can be stored with high intensity and energies up to 4.5 GeV. The DORIS storage ring was operated as a  $e^+e^-$  collider until 1993, and is currently used as a source for synchrotron radiation using  $\approx 150$  mA positrons with a lifetime of about 20 hours. Comparable beam intensities for electrons as for positrons are anticipated. With modification of the ring magnet power supplies, it is expected that switching between electron and positron beams in DORIS could be accomplished in about one hour.

With sufficient luminosity and appropriate control of systematic uncertainties, a storage ring experiment with both electrons and positrons incident on an internal hydrogen gas target is the best way to measure the  $e^+p/e^-p$  cross section ratio. Simultaneous measurement both at low and at high  $\epsilon$  with a large-acceptance detector configuration (BLAST) will allow a determination of the  $\epsilon$ -dependence of the cross section ratio, and hence the size of the multiple photon contribution. Measurement at different beam energies will also enable a Rosenbluth separation for the positron cross sections for a wide range of four-momentum transfer when the measured  $e^+p/e^-p$  ratios are combined with existing Rosenbluth data for elastic electron-proton scattering.

#### 3.1 Detector

We propose to utilize the existing Bates Large Acceptance Spectrometer Toroid (BLAST) detector system from MIT-Bates. BLAST is a toroidal spectrometer with eight sectors. The two in-plane sectors are instrumented with wire chambers for charged-particle tracking, plastic scintillators for trigger and particle identification, and aerogel-Cerenkov counters for pion rejection. The detector is symmetric about the beam direction and allows for complete reconstruction of coincident elastic events with both electron and proton four-vectors being determined. The symmetry of the detector doubles the solid angle for elastic scattering. The angle acceptance covers approximately  $20^\circ$  to  $80^\circ$  of the polar and  $\pm 15^\circ$  for the azimuthal angle.

The kinematic coverage of the BLAST detector is shown in Figure 5 for virtual photon polarization  $\epsilon$  versus  $Q^2$  for different incident beam energies (colors) and scattering angles



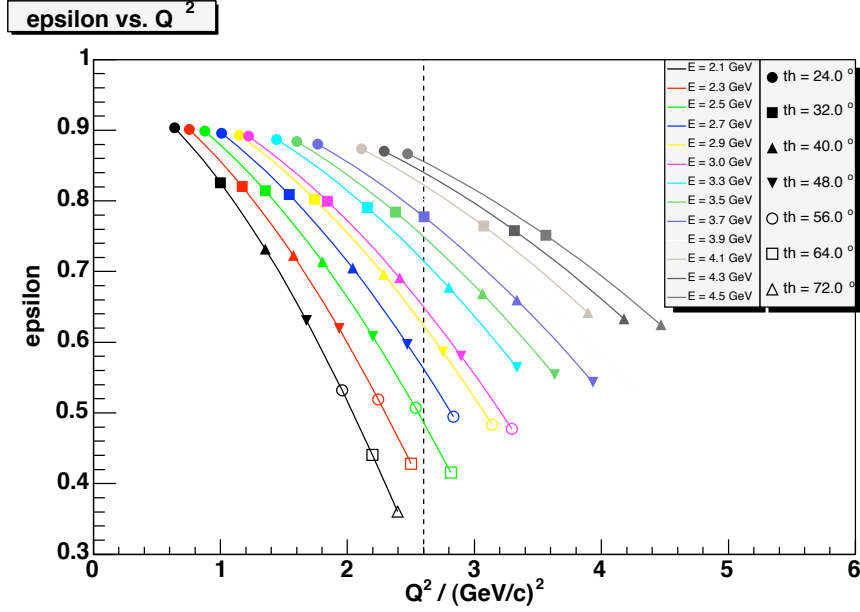


Figure 5: Kinematic coverage of  $\epsilon$  versus  $Q^2$  for the BLAST detector for various beam energies.

(symbols) corresponding to the BLAST acceptance. For any given beam energy, the parameters  $\epsilon$  and  $Q^2$  are kinematically correlated within the large angle acceptance. For the BLAST detector geometry, the acceptance becomes smaller at higher beam energies, thereby setting a lower limit for the reachable value of  $\epsilon$  since for backward lepton scattering angles the scattered proton is recoiling at decreasing angles and eventually misses the detector system. For the acceptance limitation by BLAST a proton angle  $\theta_p > 23^\circ$  was assumed.

As a consequence, the lowest reachable values of  $\epsilon$  are about 0.4 and are only established at a beam energy of less than 2.3 GeV. At the same time, the beam energy should also not be smaller than 2 GeV in order to maintain a high enough  $Q^2 > 2$  (GeV/c) $^2$ .

For a fixed  $Q^2$  of 2.6 (GeV/c) $^2$  (where the  $G_E(e^+)$  node is expected), only beam energies of 2.3-4.5 GeV are appropriate for use with BLAST. At this value of  $Q^2$  the BLAST acceptances for these beam energies are overlapping, suitable to map out the  $\epsilon$  dependence of the cross section ratio at fixed  $Q^2$  (similar to a Rosenbluth separation). The lowest beam energy corresponds to the lowest  $\epsilon$  value for that respective  $Q^2$  value. In combination with existing electron-proton cross sections, a Rosenbluth separation of the positron-proton elastic cross section can be carried out.

### 3.2 Target

The target will be an unpolarized hydrogen gas target confined by a thin walled, cryogenically cooled aluminum tube, similar to that used in the HERMES/DESY and BLAST/MIT experiments. To carry out measurements of the elastic electron-proton cross section at the lowest value of  $\epsilon \approx 0.4$  with  $\approx 1\%$  statistical uncertainty in about 1 month, a luminosity of  $2 \cdot 10^{33}/(\text{cm}^2\text{s})$  will be required for this experiment. Assuming 100 mA circulating electron

and positron currents, this implies a target thickness of about  $3 \cdot 10^{15}$  atoms/cm<sup>2</sup>. Large vacuum pumps will be required to pump away the hydrogen gas so that the lifetime of the stored beam can be on the order of several hours. The Argonne and MIT groups have considerable experience in designing, installing and operating such internal gas targets in storage rings [30].

We can estimate the beam lifetime in DORIS based on a simple model for losses accounting for bremsstrahlung, Moller and Rutherford scattering [31]. The current aperture at DORIS is limited by an undulator with an 11 mm gap, allowing only for a vertical emittance of about 7 mm-mrad. The momentum acceptance of DORIS (or bucket size) is estimated with 0.8%. The lifetime without any target in the current operation mode as a light source is on the order of 20 h. Figure 6 shows the expected partial lifetimes due to the various above

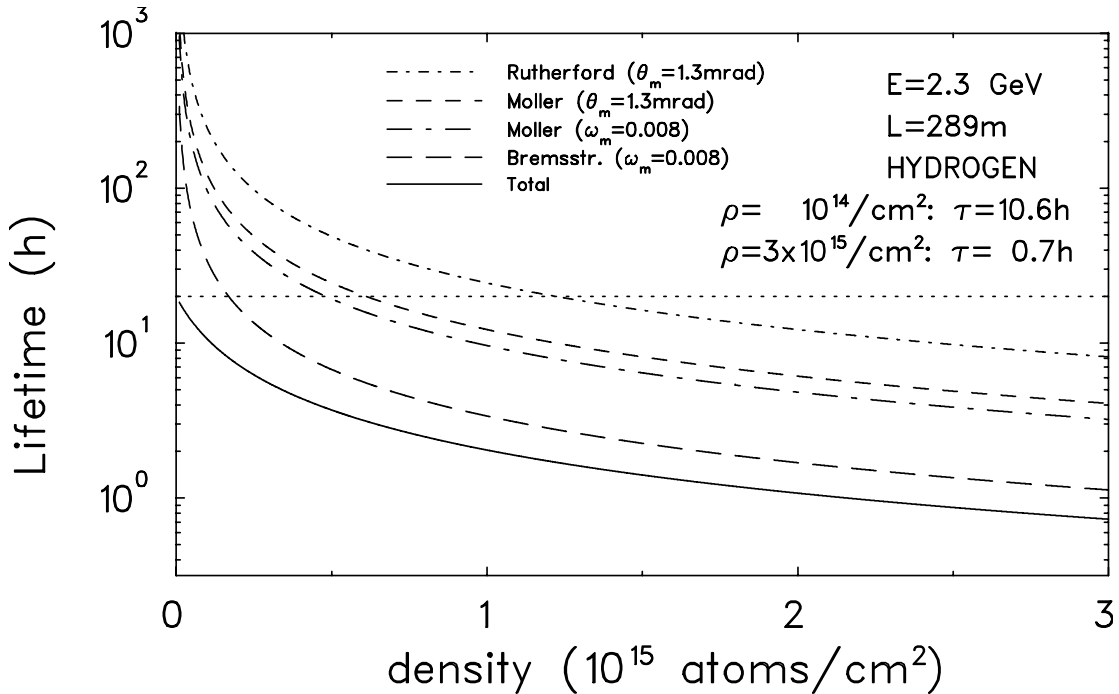


Figure 6: Expected beam lifetime in DORIS as a function of the target thickness. Based on a lifetime of 20 h without target (dotted line), the lifetime is reduced by Rutherford and Moller scattering and bremsstrahlung due to the given aperture limits (angle acceptance  $\theta_m$  and momentum acceptance  $\omega_m$ ).

mentioned processes that are causing losses, along with the resulting lifetime. It is assumed that the insertion of a target cell does not further limit the aperture. With a beta function sufficiently small at the location of the target, which can be achieved with a set of quadrupole magnets upstream and downstream of the internal target, this is a realistic assumption. The expected lifetime at a beam energy of 2.3 GeV amounts to 10.6 h for a target thickness of  $10^{14}$  atoms/cm<sup>2</sup> and 44 min for the required thickness of  $3 \cdot 10^{15}$  atoms/cm<sup>2</sup>. The momentum acceptance is still the dominant limitation. In comparison, the lifetime at MIT-Bates with a target thickness of  $5 \cdot 10^{13}$  atoms/cm<sup>2</sup> was about 30 minutes.

### 3.3 Luminosity Monitor

The target thickness will be monitored over time by continuously measuring the pressure and temperature of the reservoir and by an additional flow meter to measure the flux from the buffer. The stored current of positrons and electrons in the ring will be measured with an absolute precision of 1% with a parametric current transformer as was done for BLAST running at MIT-Bates, providing a precise monitor of the luminosity when combined with the gas flow information from the buffer system.

Besides measuring target thickness and beam current separately, we also propose to measure and monitor the luminosity with elastic scattering at low momentum transfer. At low  $Q^2 < 1$  (GeV/c)<sup>2</sup>, the proton form factors  $G_E$  and  $G_M$  are known to the 1% level. Moreover, at  $\epsilon$  close to 1, two-photon effects are expected to be negligible, hence the rate for both  $e^+p$  and  $e^-p$  elastic scattering is proportional to the luminosity.

To this extent, we will use a position-sensitive counter at a forward angle of about 10° to detect electrons or positrons in coincidence with the recoiling proton at large angle covered by the acceptance of BLAST. At such a forward angle, the field integral of the BLAST toroid is quite small, resulting in almost straight tracks for elastically scattered electrons or positrons. The forward-angle detector will have to be radiation-hard, capable of handling high rates in the MHz region and has to provide good angular ( $< 0.5^\circ$ ) and vertex resolution ( $< 1$  cm) for the forward tracks.

One possibility for the forward detector would be a package of two  $30 \times 30$  cm<sup>2</sup> planar triple-GEM detectors, identical to the COMPASS-GEM [32], allowing the lepton tracks to be measured with high resolution. An alternative would be several crossed layers of thin scintillator hodoscopes read out on both ends with fast photomultipliers for good position resolution. We will also use a Cerenkov counter or electromagnetic shower calorimeter behind the position-sensitive element to identify the electrons and positrons. At a distance of 200 cm from the target, a solid angle of 22.5 msr is covered by the area of  $30 \times 30$  cm<sup>2</sup>. The angular resolution of the track should be better than  $0.5^\circ$ , which corresponds to a spatial resolution requirement of 1.7 cm. While this modest requirement can already be achieved with the hodoscope, a higher resolution may be required if also the vertex needs to be resolved.

Another option for a luminosity monitor would be to measure symmetric Moller scattering at small angles. The final decision will be based on a more detailed study.

For beam energies between 2.3 and 4.5 GeV, the four-momentum transfer at  $\theta_e = 10^\circ$  varies between 0.15 and 0.57 (GeV/c)<sup>2</sup>, and the virtual photon polarization parameter  $\epsilon$  is above 0.98. Here the single photon approximation is good to better than 1%. The proton is recoiling with momenta of 400-800 MeV/c at angles of  $63^\circ$ – $73^\circ$ , well within the rear-angle acceptance of the BLAST detector.

The coincidence requirement between the forward detector and BLAST, as well as further kinematic correlations between the lepton and proton track, will suppress backgrounds from any source including random coincidences.

The cross section at low  $Q^2$  and  $\epsilon > 0.98$  is large enough to provide  $< 1\%$  statistical error for the above configuration in less than one hour, indicating the suitability of this setup as a luminosity monitor. The expected count rate for this luminosity monitor is listed for three beam energies in Table 1.

$E_0$ [GeV]	$Q^2$ [(GeV/c) <sup>2</sup> ]	$p_{e'}$ [GeV/c]	$\epsilon$	$\theta_p$	$p_p$ [MeV/c]	Rate [h <sup>-1</sup> ]
4.5	0.574	4.194	0.9825	63.1°	816	125387
3.0	0.262	2.860	0.9837	69.8°	530	655337
2.3	0.154	2.217	0.9844	73.2°	400	1556920

Table 1: Kinematics and count rates of the luminosity control measurement for three beam energies at  $\theta_e = 10^\circ$ . The assumed solid angle is 22.5 msr.

### 3.4 Count Rate Estimate

Figure 7 shows the expected number of counts in any given angle bin and for various beam

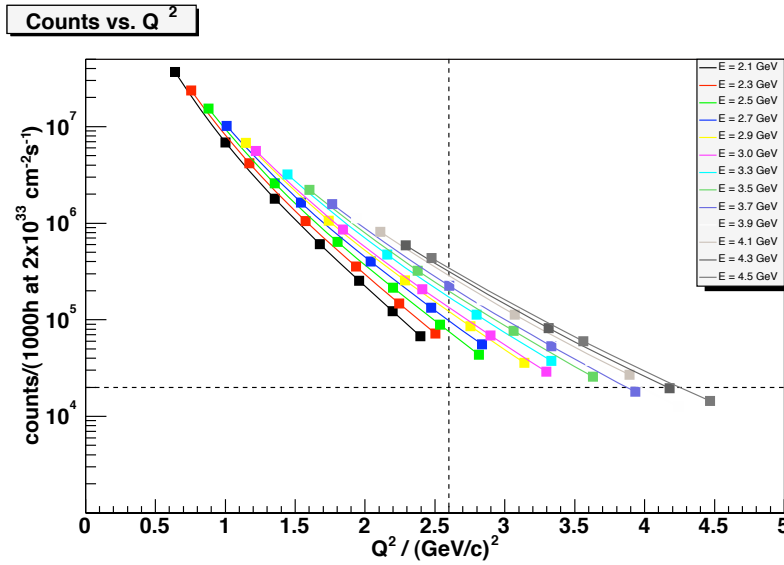


Figure 7: Expected distribution of counts per marked angle bin for the BLAST detector for various beam energies, as a function of  $Q^2$ . The assumed luminosity is  $2 \cdot 10^{33}/(\text{cm}^2\text{s}) \times 1000$  hours.

energies for a canonical run of 1000 h at a luminosity of  $2 \cdot 10^{33}/(\text{cm}^2\text{s})$  as a function of  $Q^2$ . Höhler form factor [33] based cross sections were used for this estimate, good to within 10% for both  $e^+$  and  $e^-$  up to  $Q^2 \approx 3$  (GeV/c)<sup>2</sup>. We see that for  $Q^2 \simeq 2.6$  (GeV/c)<sup>2</sup>, the number of counts per angle bin ranges between  $\approx 7 \cdot 10^4$  (at 2.3 (GeV/c)<sup>2</sup> and smallest  $\epsilon$ ) and  $\approx 4 \cdot 10^5$  (at 4.5 (GeV/c)<sup>2</sup> and highest  $\epsilon$ ).

Figure 8 shows the expected number of counts in any given angle bin and for various beam energies versus  $\epsilon$ . Generally, lowest  $\epsilon$  values at reasonable counts of  $> 10^4$  are possible down to  $\epsilon \approx 0.4$ , for which the beam energy should not exceed  $\approx 2.5$  GeV. At higher energies, the lowest value of  $\epsilon$  reachable with the rearmost scattering angle increases, while at the same time the count rate decreases.

Measurements at three beam energies, as listed in Table 2, will yield precise ratios of  $e^+p$  and  $e^-p$  cross sections at  $Q^2 = 2.6$  (GeV/c)<sup>2</sup> for a wide range of  $\epsilon$ . The counts for each  $Q^2$  point in the table are in excess of  $\approx 7 \cdot 10^4$  counts. In combination with world

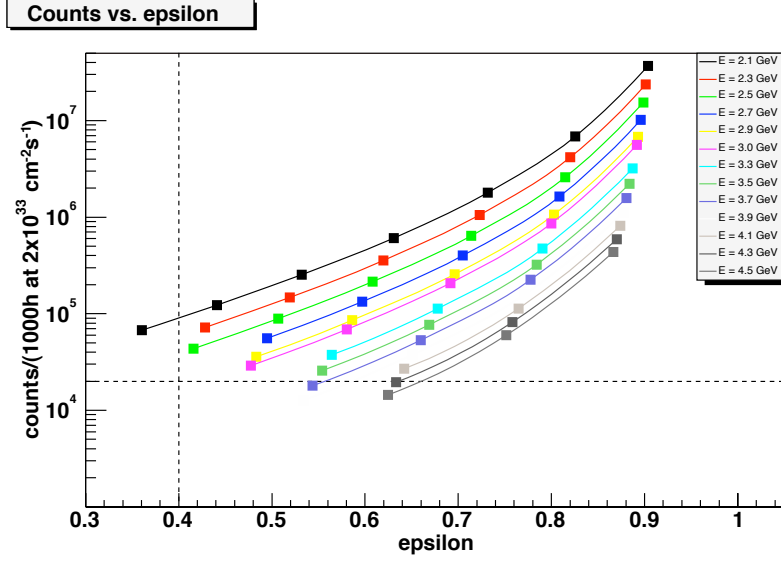


Figure 8: Expected distribution of counts per marked angle bin for the BLAST detector for various beam energies, as a function of  $\epsilon$ . The assumed luminosity is  $2 \cdot 10^{33}/(\text{cm}^2\text{s}) \times 1000$  hours.

$E_0$ [GeV]	$Q^2$ [(GeV/c) <sup>2</sup> ]	$\theta_e$	$p_{e'}$ [GeV/c]	$\epsilon$	$\theta_p$	$p_p$ [GeV/c]
4.5	2.6	24.9°	3.114	0.86	38.0°	2.125
3.0	2.6	43.0°	1.614	0.65	31.2°	2.125
2.3	2.6	67.6°	0.914	0.39	23.4°	2.125

Table 2: Kinematics for three beam energies and  $Q^2 = 2.6$  (GeV/c)<sup>2</sup>.

electron-proton cross section data this allows for a precise Rosenbluth separation of the elastic positron-proton cross section.

At  $\epsilon = 0.4$  and  $Q^2 = 2.6$  (GeV/c)<sup>2</sup>, the effect on  $\sigma(e^+/e^-)$  is expected to be of the order 4%. For a 1% statistical error of the cross section ratio, about  $2 \cdot 10^4$  counts are required for both electron and positron measurements. For a 1000h measurement each with electron and positron beams, the cross section ratio will be determined statistically to better than 1% throughout the BLAST acceptance, except for the largest angle at the highest energy. In particular, the precision in the low- $\epsilon$  region for  $Q^2$  between 2 and 4 (GeV/c)<sup>2</sup> will not be limited by statistics.

It should be emphasized that the large angular acceptance of BLAST includes a wide distribution of  $\epsilon$  values in a single measurement. Table 3 summarizes kinematics and expected count rate per angle bin for the three beam energies.

Figure 9 shows the projected uncertainties, assuming a luminosity of  $2 \cdot 10^{33}/(\text{cm}^2\text{s})$  and a running time of 1000 h for both  $e^+$  and  $e^-$ . The theory curves are evaluated for constant beam energy as a function of  $\epsilon$ . The colors of the curves correspond to the colors of the projected data.

$E_0$ [GeV]	$\theta_e$	$p_{e'}$ [GeV/c]	$\theta_p$	$p_p$ [GeV/c]	$Q^2$ [(GeV/c) <sup>2</sup> ]	$\epsilon$	Counts
<b>4.5</b>	<b>24</b>	<b>3.18</b>	<b>39.1</b>	<b>4.01</b>	<b>2.5</b>	<b>0.867</b>	<b>437082</b>
	<b>32</b>	<b>2.60</b>	<b>31.0</b>	<b>3.41</b>	<b>3.6</b>	<b>0.751</b>	<b>60093</b>
	<b>40</b>	<b>2.12</b>	<b>25.4</b>	<b>2.91</b>	<b>4.5</b>	<b>0.625</b>	<b>14427</b>
	48	1.74	21.2	2.51	5.2	0.505	4986
	56	1.44	18.0	2.19	5.7	0.402	2195
	64	1.22	15.5	1.94	6.2	0.318	1138
	72	1.04	13.5	1.74	6.5	0.250	662
<b>3.0</b>	<b>24</b>	<b>2.35</b>	<b>48.3</b>	<b>3.15</b>	<b>1.2</b>	<b>0.892</b>	<b>5594080</b>
	<b>32</b>	<b>2.02</b>	<b>39.7</b>	<b>2.80</b>	<b>1.8</b>	<b>0.800</b>	<b>860732</b>
	<b>40</b>	<b>1.72</b>	<b>33.2</b>	<b>2.48</b>	<b>2.4</b>	<b>0.691</b>	<b>207325</b>
	<b>48</b>	<b>1.46</b>	<b>28.2</b>	<b>2.20</b>	<b>2.9</b>	<b>0.581</b>	<b>69017</b>
	<b>56</b>	<b>1.24</b>	<b>24.2</b>	<b>1.97</b>	<b>3.3</b>	<b>0.477</b>	<b>28964</b>
	64	1.07	20.9	1.78	3.6	0.387	14356
	72	0.93	18.2	1.62	3.9	0.311	8029
<b>2.3</b>	<b>24</b>	<b>1.90</b>	<b>53.7</b>	<b>2.68</b>	<b>0.8</b>	<b>0.901</b>	<b>23563000</b>
	<b>32</b>	<b>1.68</b>	<b>45.3</b>	<b>2.44</b>	<b>1.2</b>	<b>0.820</b>	<b>4158760</b>
	<b>40</b>	<b>1.46</b>	<b>38.5</b>	<b>2.21</b>	<b>1.6</b>	<b>0.723</b>	<b>1056590</b>
	<b>48</b>	<b>1.27</b>	<b>33.1</b>	<b>2.00</b>	<b>1.9</b>	<b>0.620</b>	<b>355293</b>
	<b>56</b>	<b>1.10</b>	<b>28.6</b>	<b>1.81</b>	<b>2.2</b>	<b>0.519</b>	<b>147671</b>
	<b>64</b>	<b>0.97</b>	<b>24.9</b>	<b>1.66</b>	<b>2.5</b>	<b>0.428</b>	<b>71950</b>
	72	0.85	21.8	1.53	2.7	0.348	39498

Table 3: Kinematics for three beam energies and count estimate per 8° bin for 1000 h at  $2 \cdot 10^{33} / (\text{cm}^2\text{s})$ . For the higher beam energy the backward lepton angle acceptance is limited by the forward proton angle  $> 23^\circ$ . Bold face corresponds to kinematics within the acceptance of the current BLAST detector configuration.

### 3.5 Control of Systematics

The primary observable of this experiment is the ratio of the electron-proton and positron-proton elastic cross sections. The redundant control measurements of the luminosity will allow the  $e^+p/e^-p$  cross section ratio to be determined with high precision. As shown below, the systematic errors for individual proton and lepton acceptance and efficiency will cancel to first order.

In order to reduce the systematic errors of the cross section ratio due to uncertainties in relative luminosity, acceptance and efficiency with individual electron and positron beams, we require that the beam in DORIS be alternated between electrons and positrons, and that the BLAST magnet polarity be reversed with the same frequency.

For a given angular bin, the number of events is given by

$$N_{ij} = L_{ij} \sigma_i \kappa_{ij}^p \kappa_{ij}^l, \quad (2)$$

where  $i = e^+(e^-)$  for positrons (electrons) and  $j = +(-)$  for positive (negative) BLAST magnetic field polarity,  $L$  is the luminosity,  $\sigma$  the bin-averaged lepton-nucleon cross section,

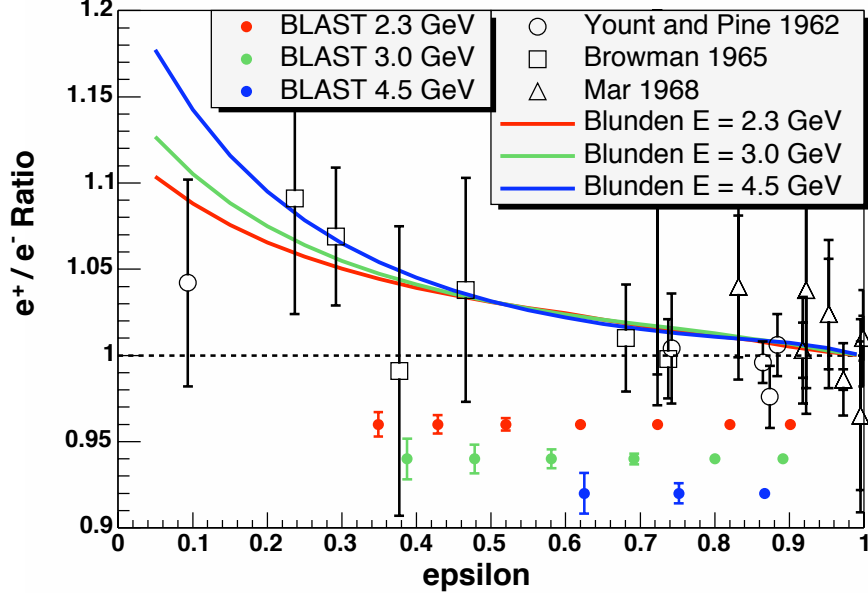


Figure 9: Projected uncertainties in the determination of the cross section ratio  $e^+p/e^-p$  for the BLAST detector for various beam energies, as a function of  $\epsilon$ . The assumed luminosity is  $2 \cdot 10^{33}/(\text{cm}^2\text{s}) \times 1000$  hours each for running with electrons and positrons, respectively.

and  $\kappa^p$  and  $\kappa^l$  are the geometric efficiencies for detecting the recoil protons and scattered leptons, accounting for the BLAST acceptance and detection efficiency.

With a given polarity of the BLAST magnetic field, the acceptance and efficiency for detecting the recoil protons will be identical for both electron and positron scattering, namely:  $\kappa_{e^{++}}^p = \kappa_{e^{-+}}^p$  and  $\kappa_{e^{+-}}^p = \kappa_{e^{--}}^p$ . Hence, for a given field polarity,  $j$ , the proton efficiencies  $\kappa_{ij}^p$  cancel in the ratio

$$\frac{N_{e^{+j}}/L_{e^{+j}}}{N_{e^{-j}}/L_{e^{-j}}} = \frac{\sigma_{e^+}}{\sigma_{e^-}} \cdot \frac{\kappa_{e^{+j}}^l}{\kappa_{e^{-j}}^l}. \quad (3)$$

However, the acceptance and efficiencies for detecting the scattered electron or positron may differ for a given BLAST magnet polarity but will be the same for opposite polarities, namely:  $\kappa_{e^{++}}^l = \kappa_{e^{--}}^l$  and  $\kappa_{e^{+-}}^l = \kappa_{e^{-+}}^l$ . By taking the product of the above ratio for opposite magnetic field polarities yields

$$\left[ \frac{N_{e^{++}}/L_{e^{++}}}{N_{e^{-+}}/L_{e^{-+}}} \cdot \frac{N_{e^{+-}}/L_{e^{+-}}}{N_{e^{--}}/L_{e^{--}}} \right]^{\frac{1}{2}} = \frac{\sigma_{e^+}}{\sigma_{e^-}}, \quad (4)$$

which measures the cross section ratio directly where all lepton and proton acceptances and efficiencies cancel to first order.

The same consideration holds for the measurement and combination of the four luminosities,  $L_{ij}$ , where the respective geometric efficiencies cancel.

Thus, frequent and random filling with both  $e^+$  and  $e^-$  beams and reversal of the BLAST field direction will minimize systematic uncertainties in the ratio from acceptance and efficiency differences as statistics are accumulated. The period for alternating beams and

magnet polarities has to be short compared to the time over which effects due to detector performance are likely to change. Some period on the order of one day would likely be sufficient.

Note the above derivation applies to the lepton detected in one sector of the BLAST detector and the proton detected in the opposite sector. During running data will be collected simultaneously for leptons and protons detected in both sectors yielding another level of redundancy and cancellation of systematic effects.

The above scheme makes use of measurements of the proton and lepton tracks in coincidence. Further information and additional checks of systematics will be obtained from proton or lepton single-arm events for which the high and low  $\epsilon$  limits of the BLAST acceptance are extended. Provided that backgrounds in single-arm elastic events can be kept at a minimum, proton single-arm ratios for electron and positron beams with the same polarity of BLAST, as well as lepton single-arm ratios with reversed field polarity also probe the  $e^+/e^-$  cross section ratio independently.



## 4 Conclusion

The current, dramatic discrepancy between recoil polarization and Rosenbluth measurements of the proton elastic form factor ratio constitutes a serious challenge to our understanding of the structure of the proton. The widely accepted explanation in terms of multiple photon exchange demands a definitive confirmation. A precision measurement of the  $e^+p/e^-p$  cross section ratio will directly test the contribution of multiple photon effects. As the prediction of the magnitude of multiple photon effect is model-dependent, the experiment described here will provide a strong constraint to theoretical calculations.

The proposed experiment described here takes advantage of unique features of the BLAST detector combined with an internal hydrogen gas target and the DORIS storage ring operated with both electrons and positrons. The systematic uncertainties are controllable at the percent level, and with the superior luminosity that can be provided at DORIS, this experiment will not be limited in statistical precision.

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