

6.89 I: Computational Evolutionary Biology

R.C. Berwick & a cast of thousands
Today: the forces of evolution



The forces of evolution, II

The deterministic model:

$F=ma$ for gene dynamics: review

The algebra of natural selection: the lab

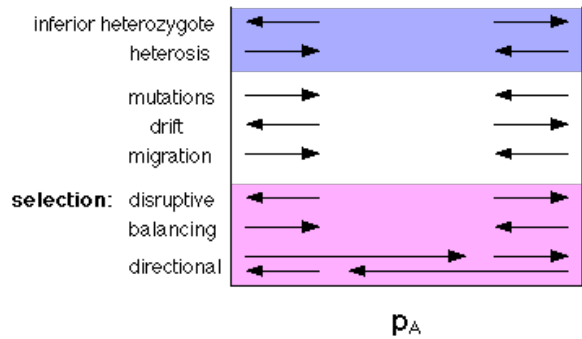
Why biology is not like physics: what goes off the rails - frequency dependent fitness

Does selection maximize fitness?

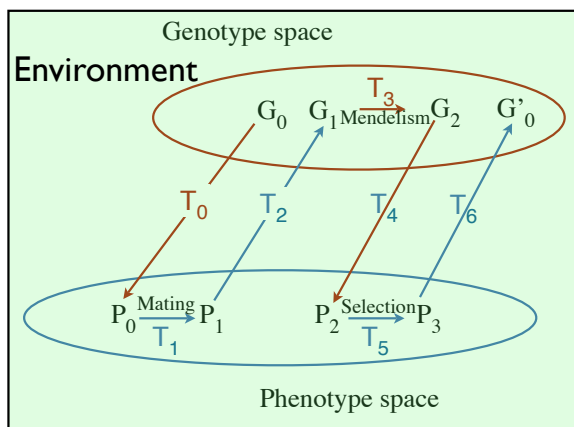
Does sex make you fitter?

The multivariate case: sickle cell anemia example

Change or die: the case for mutation



The dynamical system framework



The dynamical system framework

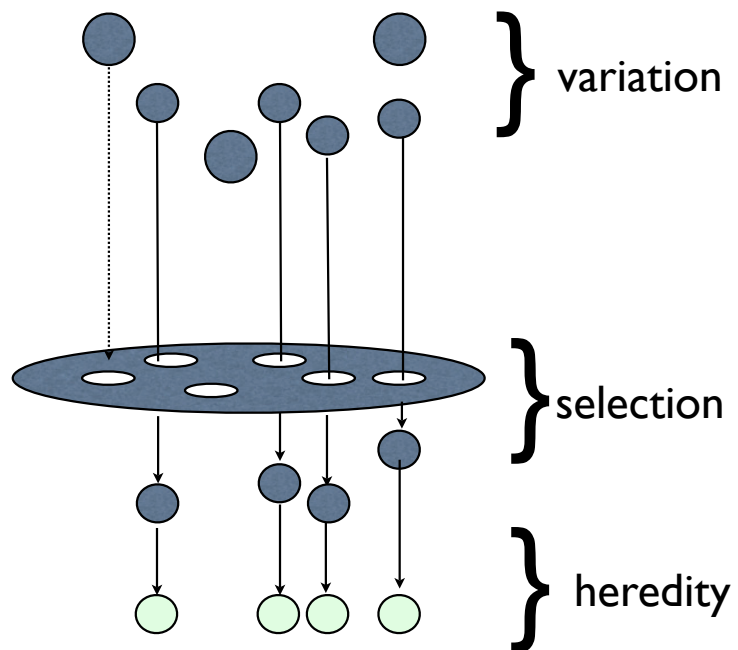
\mathbf{p} : some space

T : some mapping, $T\mathbf{p} \rightarrow \mathbf{p}$ ($\mathbf{p}' = T\mathbf{p}$)

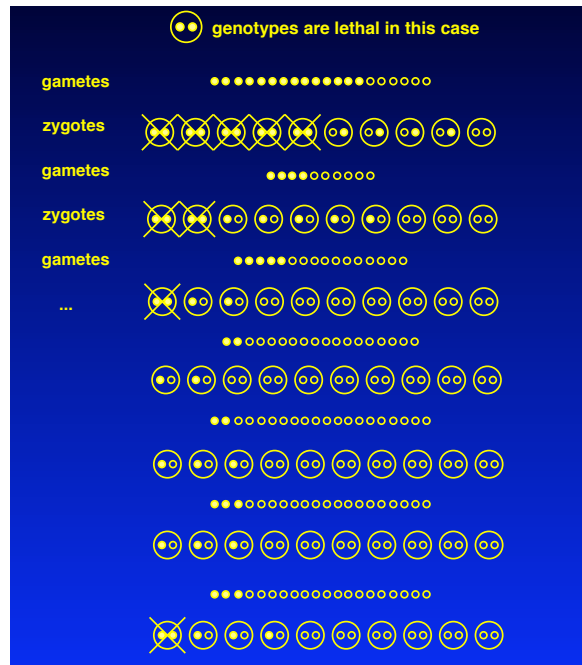
sequence $\mathbf{p}, T\mathbf{p}, T(T\mathbf{p}), \dots, T^k(\mathbf{p}) = \text{orbit of } \mathbf{p}$

$\mathbf{p}' - \mathbf{p} = T\mathbf{p} - \mathbf{p}$

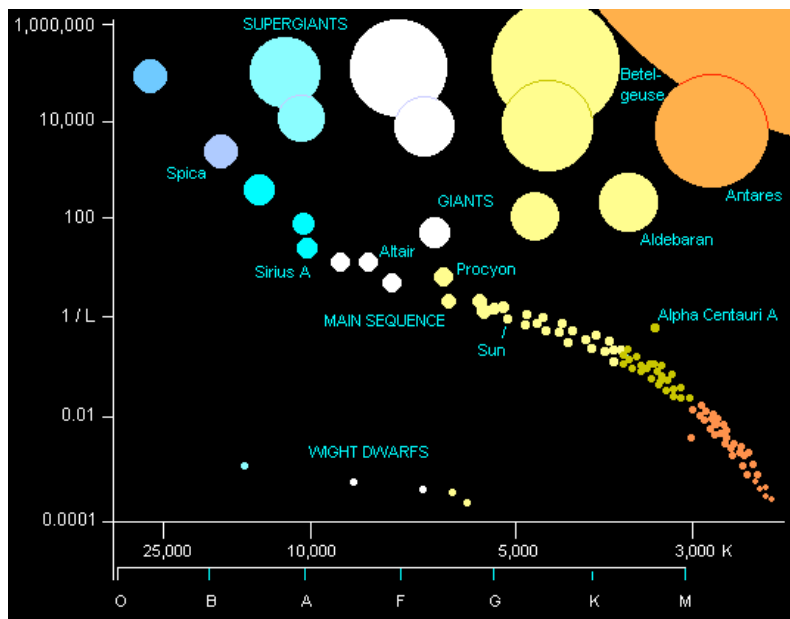
A selectional model of evolution



New reality TV show: "Survivor"



Transformational Evolution - the main sequence



	Mortal	Immortal
Variational		
Transformational		

Fisher's proof of mud slides

x = 1st parent's deviation from mean

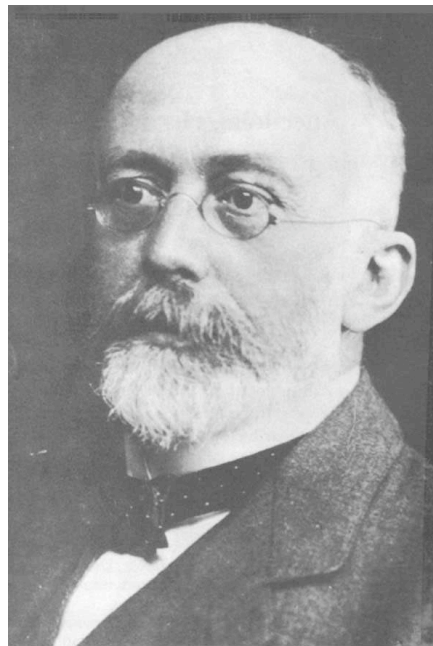
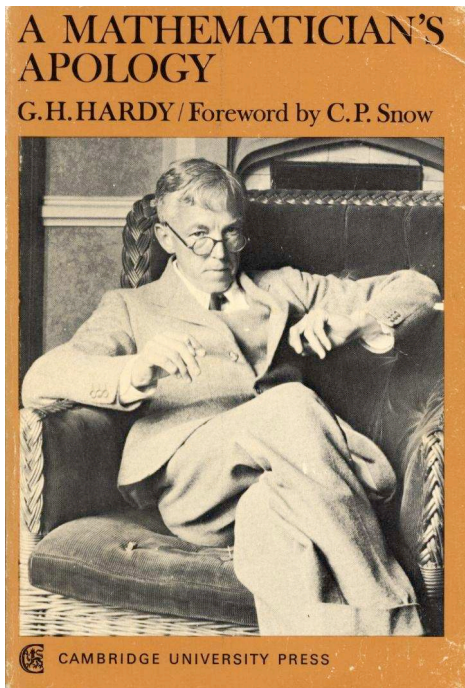
y = 2nd parent's deviation from mean

variance = $E(x^2)$

$$\text{var}\left(\frac{1}{2}(x+y)\right) = E\left(\left[\frac{1}{2}(x^2 + y^2)\right]^2\right) = E\left[\frac{1}{4}(x^2 + 2xy + y^2)\right] =$$

$$E\left[\frac{1}{4}(2x^2)\right] = \frac{1}{2}E(x^2)$$

Gregor Mendel saves Darwin?



Terminology

A, a = different forms of the same gene
AKA “alleles”

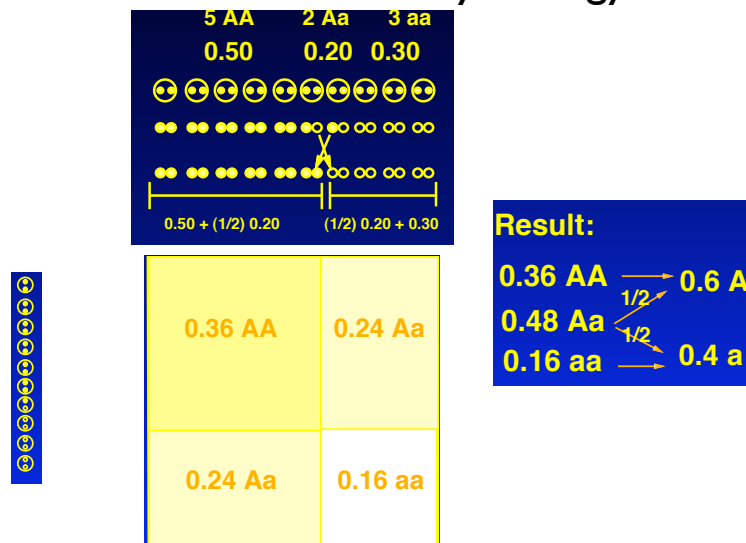
p, q = frequencies of alleles
two chromosomes in each eukaryotic
cell - diploid - so possible genotypes
are:

AA = homozygote A

Aa = heterozygote A, a

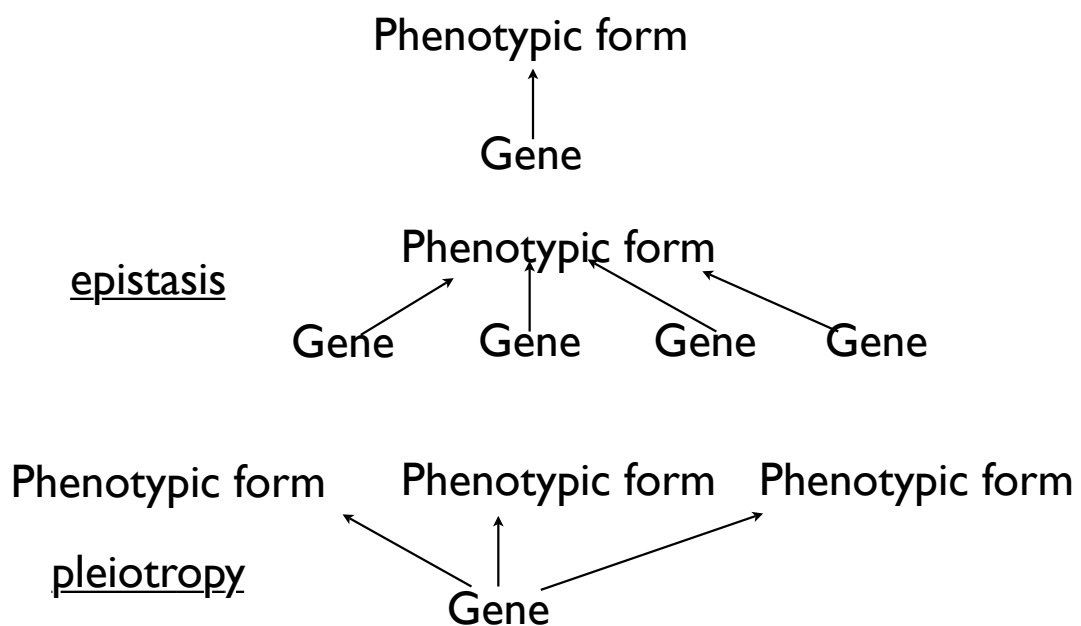
aa = homozygote a

Hardy-Weinberg as the simplest
evolutionary dynamical system: “Newton’s First
Law” of evolutionary biology

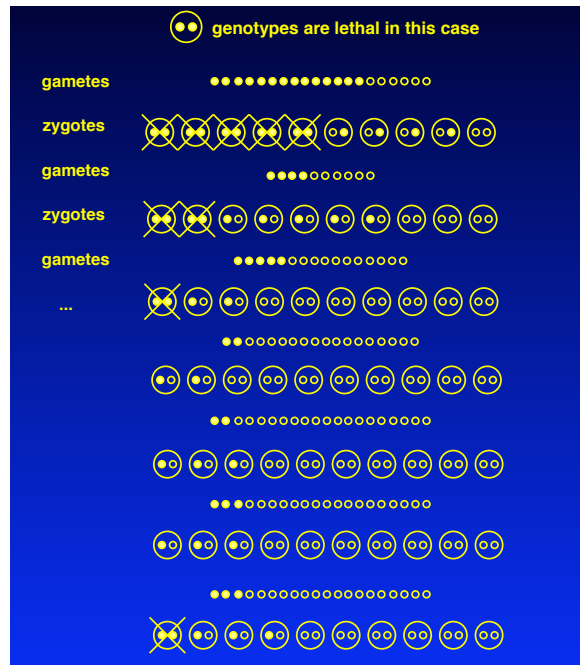


How realistic are the H-W assumptions?
What biological model are we proposing?

Model myth vs. reality



New reality TV show: "Survivor"



Survivor by the ratings numbers

Genotypes:	AA	Aa	aa
relative fitnesses:	1	1	0.7 (assume these are viabilities)
Initial gene frequency of A = 0.2			
Initial genotype frequencies (from Hardy-Weinberg)			
(newborns)	0.04	0.32	0.64
	x 1	x 1	x 0.7
Survivors (these are relative viabilities)			
	0.04	+ 0.32	+ 0.448 = Total: 0.808
genotype frequencies among the survivors: (divide by the total)			
	0.0495	0.396	0.554
gene frequency			
	A: 0.0495 + 0.5 x 0.396 = 0.2475		
	a: 0.554 + 0.5 x 0.396 = 0.7525		
genotype frequencies: (among newborns)			
	0.0613	0.3725	0.5663

The algebra of selection - J.B.S. Haldane, 1924
 1 gene in 2 different forms (alleles)

genotype	AA	Aa	aa
frequency	p^2	$2pq$	q^2
relative fitness	w_{11}	w_{12}	w_{22}
after selection			← survivors

$$\bar{w} \equiv \text{mean fitness} \equiv p^2 w_{11} + 2p(1-p)w_{12} + (1-p)^2 w_{22}$$

$$\bar{w}_1 \equiv \text{mean fitness of A} \equiv p^2 w_{11} + p(1-p)w_{12}$$

Algebra II

$$\bar{w} \equiv \text{mean fitness} \equiv p^2 w_{11} + 2p(1-p)w_{12} + (1-p)^2 w_{22}$$

$$\bar{w}_1 \equiv \text{mean fitness of A} \equiv p^2 w_{11} + p(1-p)w_{12}$$

fitness ratios (scaled):

$$\frac{p^2 w_{11}}{\bar{w}} : \frac{2pq w_{12}}{\bar{w}} : \frac{q^2 w_{22}}{\bar{w}}$$

Newton's $F=ma$ for evolutionary systems
 Basic dynamical system map: compute p' from p

$$p' = \frac{\text{A survivors}}{\text{all survivors}} = \frac{p^2 w_{11} + \frac{1}{2} \times 2p(1-p)w_{12}}{p^2 w_{11} + 2p(1-p)w_{12} + (1-p)^2 w_{11}}$$

$$p' = \frac{p(pw_{11} + p(1-p)w_{12})}{p^2 w_{11} + 2p(1-p)w_{12} + (1-p)^2 w_{11}} = p \frac{\bar{w}_1}{\bar{w}}$$

$$p' - p = \frac{p(1-p)\{w_{11}p + w_{12}(1-2p) - w_{22}(1-p)\}}{p^2 w_{11} + 2p(1-p)w_{12} + (1-p)^2 w_{11}}$$

Dynamics: Compute Δp and also w vs. p
 gives the 'jet fuel' formula for gene frequency change under selection

$$p' - p = \frac{p\bar{w}_1}{\bar{w}} - \frac{p\bar{w}}{\bar{w}} \Rightarrow \Delta p = \frac{p(\bar{w}_1 - \bar{w})}{\bar{w}} \text{ with 2 alleles,}$$

$$\bar{w} = p\bar{w}_1 + (1-p)\bar{w}_2, \text{ so substituting:}$$

$$\Delta p = \frac{p(1-p)(\bar{w}_1 - \bar{w}_2)}{\bar{w}} \text{ and now substitute for } (\bar{w}_1 - \bar{w}_2) = \frac{d\bar{w}}{dp} =$$

$$\Delta p = \frac{p(1-p)}{\bar{w}} \frac{d\bar{w}}{dp} = \frac{p(1-p)}{\bar{w}} \frac{d \ln(\bar{w})}{dp}$$

$$F=ma$$

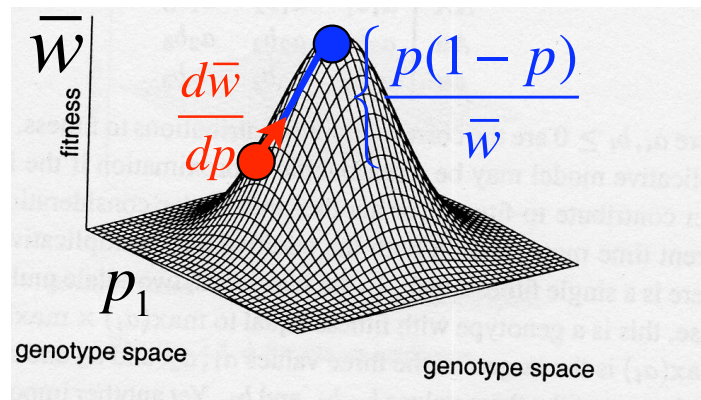
The jet fuel formula for 'evolutionary change'

$$\Delta p = \frac{p(1-p)}{\bar{w}} \frac{d\bar{w}}{dp}$$

Amount of change
in p (variance component)

Direction of change
(slope of \bar{w} wrt p , + or -)

The shape of things now



Mean fitness always increases...

$$\Delta \bar{w} = 2p(1-p)\{w_{11}p + w_{12}(1-2p) - w_{22}(1-p)\}^2 \\ \times \{w_{11}p^2 + (w_{12} + \frac{1}{2}w_{11} + \frac{1}{2}w_{22})p(1-p) + w_{22}(1-p)^2\}\bar{w}^{-2}$$

But...this is not the same things as maximizing fitness...

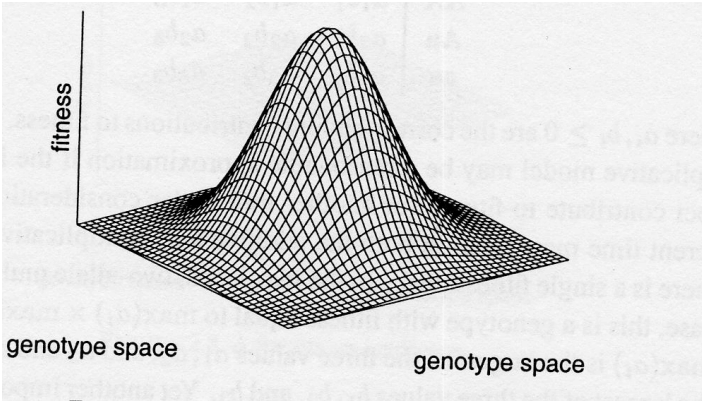
$$dp = kp(1-p)dt \text{ or}$$

$$\int \frac{1}{p(1-p)} dp = dt \text{ so}$$

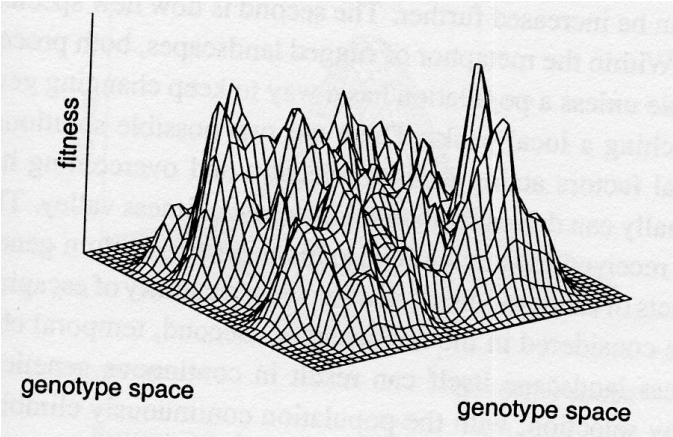
$$t(p_1, p_2) = \int_{p_1}^{p_2} \frac{1}{p(1-p)} dp$$

p	0.001-0.01	0.01-0.1	0.1-0.5	0.5-0.9	0.9-0.99	0.99-0.999
w11>w12> w22	462	480	439	439	480	462
dominance, w11=w12> w22	232	250	309	1.020	9.240	90,231

The shape of things now



The shape of things now



Solving the fundamental recurrence equation

$$p' - p = \frac{p(1-p)\{w_{11}p + w_{12}(1-2p) - w_{22}(1-p)\}}{p^2w_{11} + 2p(1-p)w_{12} + (1-p)^2w_{22}}$$

w_{11}	w_{12}	w_{22}	
$1+s$	$1+sh$	1	e.g.,
1	1	1	
$(1+s)$	$(1+s)$	1	"dominance"
1	$(1+s)$	$(1+s)$	"recessive"
$1+s$	$1+sh$	1	"heterozygote over/under dominant"

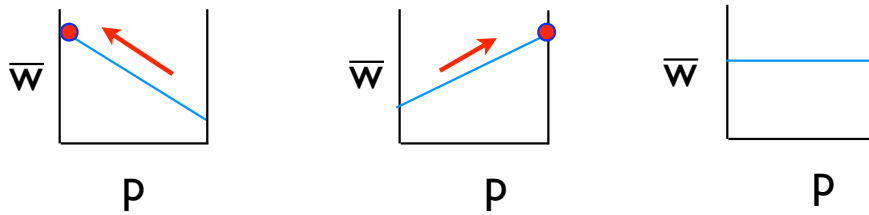
**NB: only a few
special cases have
explicit solutions!**

Dynamical system analysis of 'adaptive topography' or mean fitness vs. p

$$\bar{w} = p^2[(w_{11} - w_{12}) + (w_{22} - w_{12})] - 2p(w_{11} - w_{12}) + w_{22}$$

$$w_{12} = \frac{w_{11} + w_{22}}{2}$$

One locus, 2 allele case: 7 graphs, p vs. \bar{w}



‘Degenerate’ case: quadratic mean fitness, with
 $w_{12} = (w_{11} + w_{22})/2$

Dynamical system analysis of ‘adaptive topography’
 or mean fitness vs. p - nondegenerate case

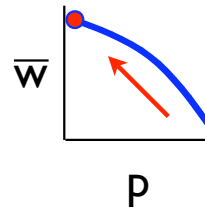
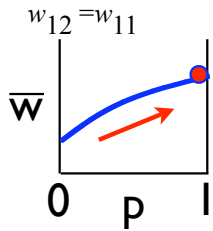
$$\bar{w} = p^2[(w_{11} - w_{12}) + (w_{22} - w_{12})] - 2p(w_{11} - w_{12}) + w_{22}$$

$w_{12} \neq \frac{w_{11} + w_{22}}{2}$ so in this case, formula for \bar{w} is a parabola.

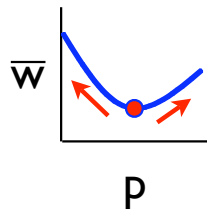
There are 4 further subcases, depending on the ordering of
 the w_{ij}

$$\hat{p} = \frac{w_{22} - w_{12}}{(w_{11} - w_{12}) + (w_{22} - w_{12})}$$

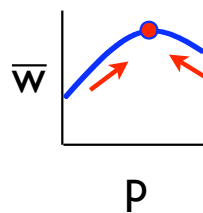
The four nonlinear cases - selection at one locus, 2 alleles - adaptive topography



$$\therefore \hat{p} = \frac{w_{22} - w_{12}}{(w_{11} - w_{12}) + (w_{22} - w_{12})} = \frac{w_{22} - w_{12}}{w_{22} - w_{12}} = 1$$



underdominance



overdominance