Using State Variable Filter to Create Lead/Lag Filters

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Introduction

The state variable filter serves as an example of a low-pass, high-pass and band-pass filter. The filter is unique because it is based on the idea that any transfer function can be created using only integrators and gain blocks as its basic components. This filter also serves as a starting point to creating other filters such as lead and lag filters used in compensators for control systems.

State-Variable Filter (SVF)

The state-variable filter uses two integrators and one gain block to create a filter that exhibits a low-pass, band-pass or high-pass characteristic based on where the output is realized. The basic state-variable filter is presented in Figure 1.

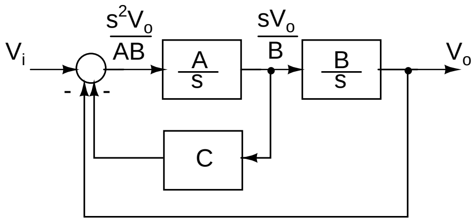

Figure 1: State-Variable Filter

The low-pass, band-pass and high-pass outputs are represented by V_o, $\frac{sV_o}{B}$ and $\frac{s^2V_o}{AB}$ respectively, which from here on in, will be referred to as V_o, V_2 and V_1. The respective transfer functions are represented by Equations 1-3.

$$\frac{V_o}{V_i} = \frac{AB}{s^2 + CAs + AB} \tag{1}$$

$$\frac{V_o}{V_2} = \frac{sA}{s^2 + CAs + AB} \tag{2}$$

$$\frac{V_o}{V_1} = \frac{s^2}{s^2 + CAs + AB} \tag{3}$$

These three equations show that it is possible to realize transfer functions with the order of the numerator less than or equal to the order of the denominator without the use of feed-forward zeros. In other words:

$$\Theta(Num) \leq \Theta(Den) \tag{4}$$

is possible using only integrators.

Modifying SVF for Lead/Lag Compensators

The lead compensator is a very useful filter in control theory. This filter is used to add phase margin to a system close to instability. The lead compensator transfer function is realized with one pole and one zero. The lead zero is at a lower frequency than that of its pole, creating a phase bump at a frequency equalling the geometric mean of the pole and zero. The lead transfer function is:

$$H = \frac{\tau_1 s + 1}{\tau_2 s + 1}, \tau_1 > \tau_2 \tag{5}$$

In order to realize the 'lead' transfer function, the low-pass transfer function from the SVF is analyzed. Some modifications are however necessary. First, the 's^2' in the denominator needs to disappear, and secondly, there needs to be a 's' as the second term in the numerator. The latter can be realized by adding a feed-forward path to the SVF as shown in Figure 2.

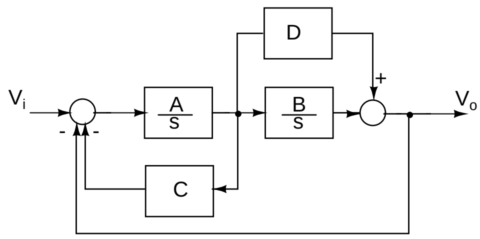

Figure 2: State-Variable Filter, Modified

The resulting transfer function of this configuration is:

$$\frac{V_o}{V_i} = \frac{A(Ds + B)}{s^2 + A(C + D)s + AB} \tag{6}$$

The next step is to now eliminate the 's^2' term. It is clear that 'A' multiplies all terms except the

's^2' term, therefore to eliminate this term, take the limit as 'A' approaches infinity, then divide by 'B' to make it the same form as Equation 5.

$$\lim_{A\to\infty} \frac{V_o}{BV_i} = \frac{\frac{D}{B}s+1}{\frac{C+D}{B}s+1} \quad (7)$$

Since the function is now in the same form as the lead function, the time constants can be determined.

$$\tau_1 = \frac{D}{B} \quad (8)$$

$$\tau_2 = \frac{C+D}{B} \quad (9)$$

There is one thing to note, however, namely $\tau_1 > \tau_2$ is required for a lead compensator. In order for this to be true, 'C' has to be negative.

$$\frac{D}{B} > \frac{C+D}{B} \quad (10)$$

$$D > C + D \quad (11)$$

$$0 > C \quad (12)$$

If 'C' is made to be positive, then a lag compensator can be realized with the same topology. The final block diagram is shown in Figure 3.

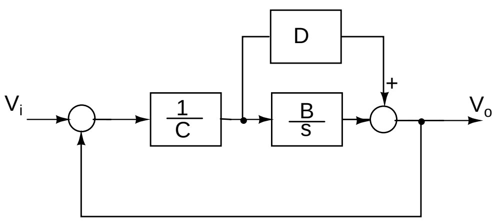

Figure 3: Lead/Lag Filter

From this block diagram, it is easy to see how a lead or lag filter can be constructed with the use of one integrator and two gain stages. Basic analog building blocks can now be used to realize this filter. A more convenient way to synthesize this filter is to just use transconductance amplifiers (OTA) and capacitors, and no resistors.

Beginning with Figure 3, the next step is to assign voltages and currents to each line in the block diagram. The most convenient assignments, if OTAs will be used in place of operational amplifiers, are shown in Figure 4.

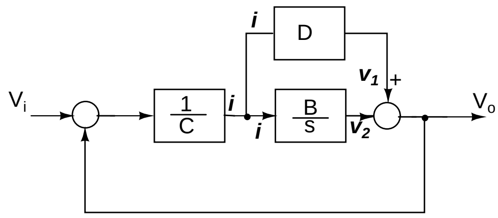

Figure 4: Lead/Lag Filter with Variables as Currents and Voltages

Circuit Implementation of Lead/Lag Filter

Since the error voltage changes to a current, it is clear that an OTA is used there. One thing to notice in the block diagram is that at the output of the '$\frac{1}{C}$' block is sampled and not split into two currents. The error voltage therefore needs to be changed to a current twice, one for the gain block and one for the integrator. The implementation of this block diagram is shown in Figure 5.

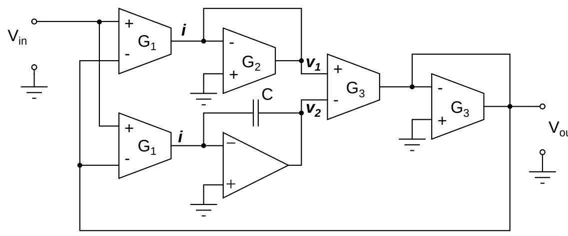

Figure 5: Lead/Lag Filter Implementation

In this configuration, any OTAs that have the same subscript have the same bias currents. A short hand-analysis of this circuit will show the transfer function for the lead/lag filter.

$$i = G_1(V_{in} - V_{out}) \quad (13)$$

$$V_1 = \frac{i}{G_2} = \frac{G_1(V_{in} - V_{out})}{G_2} \quad (14)$$

$$V_2 = -\frac{i}{Cs} = -\frac{G_1(V_{in} - V_{out})}{Cs} \tag{15}$$

$$V_{out} = \frac{G_3(V_1 - V_2)}{G_3} = V_1 - V_2 \tag{16}$$

After a little algebra, the transfer function is realized as:

$$\frac{V_{out}}{V_{in}} = \frac{\frac{Cs}{G_2} + 1}{\frac{(G_1+G_2)Cs}{G_1 G_2} + 1} \tag{17}$$

$$\tau_1 = \frac{C}{G_2} \tag{18}$$

$$\tau_2 = \frac{(G_1 + G_2)C}{G_1 G_2} \tag{19}$$

In this configuration, the circuit is a lag filter, because $\tau_2 > \tau_1$. To make this circuit into a lead filter, the inputs to the G_1 filter need to be switched, specifically, V_{in} connects to the negative terminal and V_{out} connects to the positive terminal. Two birds with one stone!