#### MASSACHUSETTS INSTITUTE OF TECHNOLOGY

Department of Electrical Engineering and Computer Science

# 6.302 Feedback Systems

Fall 2004 Filter Blocks WebLab Issued: September 17, 2004 Due: Friday, September 24, 2004

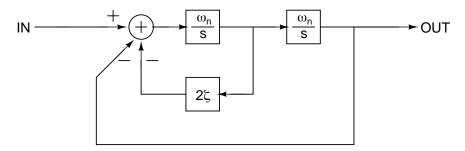
### Introduction

The purpose of this lab is to further examine and explore the response of a real canonical second-order system in terms of its system parameters  $\zeta$  and  $\omega_n$ . However, in lieu of going into lab and building many different systems, you will use a web applet to make your measurements. The web applet communicates with a server which controls a single circuit in lab. Throughout the course of this lab you will learn more about the inner-workings of this system.

Be warned: The server manages the queue of job requests, so don't wait until the "night before" to make your measurements unless you are prepared to wait your turn!

#### Pre-Lab Calculations

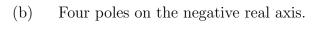
1. Consider the following system:

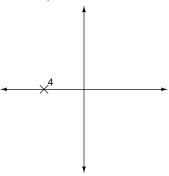


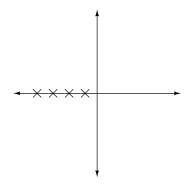
What is the closed-loop transfer function of this system? Have you seen this transfer function before? What does the closed-form of this feedback system implement?

2. Now suppose that we cascade two copies of this system and have the freedom to control the parameters  $\zeta$  and  $\omega_n$  for each system. For each of the following pole-zero plots specify the parameters  $\zeta_1$ ,  $\zeta_2$ ,  $\omega_{n1}$  and  $\omega_{n2}$  which will generate a closed-loop transfer function with the corresponding pole-zero plot. (Note: Many solutions exist. Limit  $\zeta \leq 1.9$  and  $2\pi \cdot 100$  rad/s  $\leq \omega_n \leq 2\pi \cdot 10$  krad/s.)

(a) Four concurrent poles at  $s = -2\pi \cdot 10^3 \text{ rad/s}.$ 

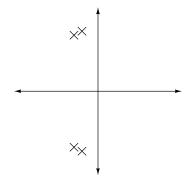


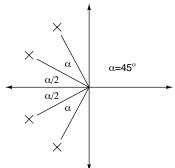




(c) Two conjugate pairs of poles with  $\zeta \leq 0.15$  for each system.

4th-Order Butterworth filter. All poles a distance  $2\pi \cdot 10^3$  rad/s from origin.





3. Use MATLAB to generate the corresponding Bode plot and step response for each system in Problem 2.

(d)

(MATLAB Hint: Use the subplot command to put multiple responses on one page.)

4. Figure 1 is a simplified block diagram of the circuit running at the server of this lab. It uses voltage inputs  $AO_0 \to AO_3$  and multiplier chips (Analog Devices' AD532J) to enable you to tune the closed-loop response to match specific values of  $\zeta$  and  $\omega_n$ .

Figure 2 is the actual circuit, shown here at half size — inputs  $AO_{0,2}$  for the first system correspond to inputs  $AO_{1,3}$  for the second system. Generate a block diagram for this circuit (in the general form of Figure 1) and determine the following relationships. (Assume the OP27 is ideal and the AD532J performs a  $Z = \frac{A \times B}{10}$  function on two input voltages.)

- (a)  $AO_0$  and  $\omega_{n1}$
- (b)  $AO_1$  and  $\omega_{n2}$
- (c)  $AO_2$  and  $\zeta_1$
- (d)  $AO_3$  and  $\zeta_2$

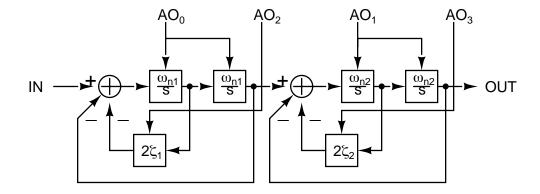


Figure 1: Simplified block diagram of weblab circuit.

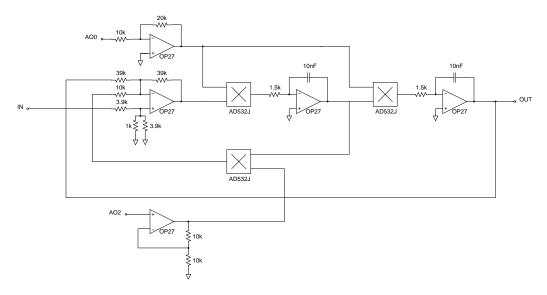


Figure 2: Actual circuit.

5. Parts (a)-(d). Calculate the input voltages that will result in a system that matches your solutions to Problem 2, parts (a)-(d). Note that the input voltages are constrained to  $0 < AO_n \le 5$  V.

### Measurements

You are now ready to go into lab. Log in at http://i-lab.mit.edu and click the "Launch Client" button to begin the lab.

1. Run a frequency sweep for each set of voltages in Problem 5 of the prelab. Sweep between 2 and 3 decades in frequency, insuring that you measure all vital parts of the response while keeping your server request manageable. After each sweep make sure you download the data.

It is possible that some valid combinations for the prelab actually saturate the circuit

- in this part. You will know if this has happened, and should reduce the magnitude peaking of your solution until it no longer saturates the server circuit.
- 2. Use MATLAB to plot and print out your measured responses. Ambitious students can include the theoretical response on the same plots.
  - (MATLAB Hint: This can be achieved by using the form of the bode command that returns magnitude, phase and frequency information. Use the db command so that the theoretical magnitude data matches the downloaded data.)
- 3. Butterworth filters are systems that exhibit no magnitude peaking and roll off with some slope depending on the order of the filter. Generate a plot in MATLAB that compares the magnitude response of systems (a) and (d). What do you notice?
- 4. **Optional**. Use the pole-zero applet at http://web.mit.edu/6.302/pz/ to find a pole-zero plot of a third-order Butterworth filter using system (d) as a guide. Do your best to simulate this system with the circuit in lab and print out a plot comparing your measurement with the theoretical response of the ideal third-order Butterworth filter. Make sure you turn in the result of your pole-zero applet efforts.

## Write Up

The write up for this lab should be short, simple and informal. Do your best to conserve paper when printing from MATLAB, but do avoid cluttering any one plot excessively.