6.251/15.081J Recitation 10

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1 Examples

Example 1.1. [1], exercise 7.31.

2 Review Problems

Example 2.1. *Projections of Polyhedral Cones.*

Given a polyhedral cone $C \subseteq \mathbb{R}^{n+p}$, prove that the projection of C along its first n components, $\Pi_n(C)$, is also a polyhedral cone. Does C being pointed imply $\Pi_n(C)$ is also pointed?

Example 2.2. [1], exercise 5.8.

Example 2.3. *Elimination.*

(Adapted from [2], ch. 3.) Consider a set of teams playing matches against each other, and assume each team either wins or loses a match in which it competes. We say a team is *eliminated* if, no matter what the results of the remaining matches are, they cannot finish with the most wins. In this problem we wish to prove an equivalent condition for a team to be eliminated.

We introduce the following notation. Let T be the set of teams other than the one whose elimination status is in question. Let $w(A)$ be the total number of current wins for a subset of teams $A \subseteq T$. Finally, let r_{ij} denote the number of games remaining between any two teams $\{i,j\} \subseteq T$, and let M denote the maximum number of possible wins for the team in question.

(a) Prove that if there exists a subset of teams $A \subseteq T$ satisfying

$$w(A) + \sum_{\{i,j\}\subseteq A} r_{ij} > M|A|, \tag{1}$$

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then the team in question is eliminated. (*Hint*: You need no concepts from the class to argue this.)

(b) Prove the converse of **(a)**. In particular, this shows that 1 is an equivalent condition to the team being eliminated. (*Hint*: Formulate an appropriate maximum flow problem, then invoke the max-flow min-cut theorem.)

(c) For the following team data, construct the network flow problem from **(b)** to determine whether the Algorithms are eliminated or not.

Team	Wins	To Play	D.	A.	V.	E.	B.
Degenerates	33	8	-	1	6	1	0
Algorithms	29	4	1	-	0	3	0
Vertices	28	7	6	0	-	1	0
Ellipsoids	27	7	1	3	1	-	2
Big-O's	23	2	0	0	0	2	-

References

[1] Bertsimas, D.; Tsitsiklis, J.N. *Introduction to Linear Optimization*. Athena Scientific, 1997.

[2] Cook, W.J.; Cunningham, W.H.; Pulleyblank, W.R.; Schrijver, A. *Combinatorial Optimization*. John Wiley & Sons, Inc., 1998.