6.251/15.081J Recitation 1

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1 Review

1.1 Basic Linear Algebra

1.1.1 Key Terms and Ideas

Know the formal definitions and have intuition for the following:

1. Linear independence
2. Subspace
3. Span
4. Basis
5. Dimension
6. Matrix range
7. Matrix nullspace
8. Matrix rank
9. Affine subspace

1.1.2 Commonly-Used Inversion and Transposition Properties

2 Examples

Example 2.1. (Taken from [1], chapter 3). Give an explicit solution of each of the following LPs, describing the conditions under which the problem is

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unbounded below, infeasible, etc.
(a) *Minimizing a linear function over an affine set.*

$$\begin{array}{ll} \text{minimize} & \boldsymbol{c}^T\boldsymbol{x} \\ \text{subject to} & A\boldsymbol{x} = \boldsymbol{b}. \end{array}$$

(b) *Minimizing a linear function over a halfspace.*

$$\begin{array}{ll} \text{minimize} & \boldsymbol{c}^T\boldsymbol{x} \\ \text{subject to} & \boldsymbol{a}^T\boldsymbol{x} \leq b, \end{array}$$

where $\boldsymbol{a} \neq 0$.
(c) *Minimizing a linear function over a rectangle.*

$$\begin{array}{ll} \text{minimize} & \boldsymbol{c}^T\boldsymbol{x} \\ \text{subject to} & \boldsymbol{l} \leq \boldsymbol{x} \leq \boldsymbol{u}, \end{array}$$

where $\boldsymbol{l} \leq \boldsymbol{u}$.
(d) *Minimizing a linear function over the standard simplex.*

$$\begin{array}{ll} \text{minimize} & \boldsymbol{c}^T\boldsymbol{x} \\ \text{subject to} & \boldsymbol{1}^T\boldsymbol{x} = 1 \\ & \boldsymbol{x} \geq 0. \end{array}$$

Example 2.2. (Taken from [1], chapter 3). The infinity (induced) norm of a matrix $A \in \mathbb{R}^{m\times n}$, denoted $\| A \|_{\infty,i}$, is defined as

$$\| A \|_{\infty,i} = \max_{i=1,\ldots,m} \sum_{j=1}^{n} | a_{ij} | .$$

This norm is sometimes called the max-row-sum norm.

Consider the problem of approximating a matrix, in the max-row-sum norm, by a linear combination of other matrices. That is, we are given $k+1$ matrices $A_0, \ldots, A_k \in \mathbb{R}^{m\times n}$, and need to find $\boldsymbol{x} \in \mathbb{R}^k$ that minimizes

$$\| A_0 + x_1 A_1 + \cdots + x_k A_k \|_{\infty,i}$$

Express this problem as an LP.

References

[1] Boyd, S., Vandenberghe, L. *Convex Optimization*. Course reader. Stanford University, 2001.