MASSACHUSETTS INSTITUTE OF TECHNOLOGY

Fall 2002 | 6.251/15.081

Quiz 2 | 11/18/02 (2:35-3:55 p.m., in class)

Problem 1. (30 points)

(a) Describe an algorithm for deciding whether the optimum in a max-flow problem is equal to infinity.
Note: A correct algorithm will suffice; you do **not** have to prove correctness. The algorithm should make use of the special structure of the problem; do not just say "solve a Phase I LP."

(b) Consider an uncapacitated network flow problem and let $\mathbf{B}$ be some basis. Explain why $\mathbf{B}^{-1}$ has only integer entries.
Note: Do not just say "by Theorem x.y in the book." However, a rough explanation, not a complete proof, will suffice.

(c) What is the graphical interpretation of an extreme ray in an uncapacitated network flow problem?
Note: You do not have to prove that your answer is correct.

Problem 2. (30 points)

There are n contractors and n projects. Each contractor i has a set $A(i)$ of projects that he is willing to carry out. We are looking for a matching which assigns exactly one project to each contractor.

We say that a set $S \subset \{1, \ldots, n\}$ of projects is undersubscribed if it has more elements (projects) than there are contractors who are willing to carry out a project in S.

[Mathematically, let V be the set of all i for which $A(i) \cap S$ is nonempty. The set S is undersubscribed if the cardinality of V is less than that of S.]

Show that there exists a matching in which all projects get assigned if and only if no set S is undersubscribed.

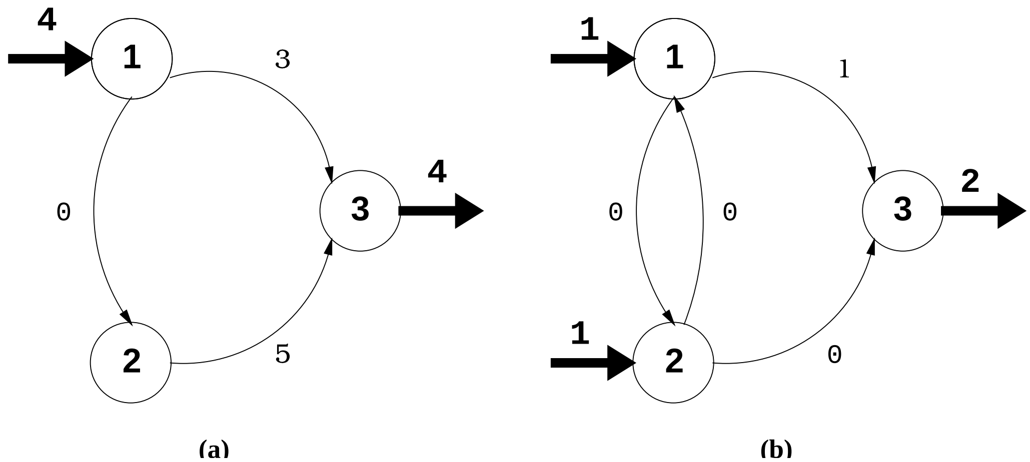

Figure 1: **(a)** Network 1. **(b)** Network 2.

Problem 3. (30 points)

We are given two uncapacitated network flow problems, illustrated in figure 1. The numbers next to each arc are the cost coefficients.

We are interested in minimizing the total cost in the two networks, in the presence of the additional constraint that the flow on arc (2,3) must be the same in both networks. We solve this problem using the Dantzig-Wolfe decomposition method.

We start the algorithm, by considering the basic feasible solution of the master problem that makes use of the extreme points

$$\mathbf{f}_1^1 = (f_{13}, f_{12}, f_{23}) = (0, 4, 4), \qquad \mathbf{f}_1^2 = (f_{13}, f_{12}, f_{23}) = (4, 0, 0)$$

for the first network and the extreme point

$$\mathbf{f}_2^1 = (f_{13}, f_{12}, f_{21}, f_{23}) = (0, 1, 0, 2)$$

for the second network.

(a) What are the numerical values of the basic variables in the master problem?

(b) Write down the basis matrix for the master problem associated with the current basic feasible solution of the master problem.

(c) Calculate the vector $[\mathbf{q}\ r_1\ r_2]$ of simplex multipliers.

(d) Solve a subproblem associated with the second network, and use its solution to carry out a simplex iteration in the master problem. Specify the values of the new basic variables in the master problem, as well as the corresponding values of the arc flows in the two networks.