

Massachusetts Institute of Technology  
Department of Electrical Engineering and Computer Science  
6.111 - Introductory Digital Systems Laboratory

**Problem Set 1 Solutions**

**Issued:** February 8, 2006

**Boolean Algebra Practice Problems** (*not graded*)

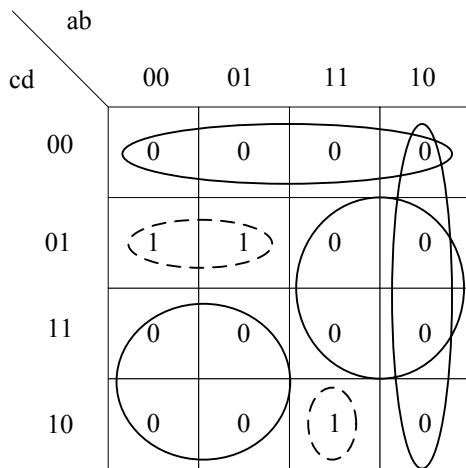
- 1)  $a + 0 = a$
- 2)  $\bar{a} \cdot 0 = 0$
- 3)  $a + \bar{a} = 1$
- 4)  $a + a = a$
- 5)  $a + ab = a(1 + b) = a$
- 6)  $a + \bar{a}b = (a + \bar{a})(a + b) = a + b$
- 7)  $a(\bar{a} + b) = a\bar{a} + ab = ab$
- 8)  $ab + \bar{a}b = b(a + \bar{a}) = b$
- 9)  $(\bar{a} + \bar{b})(\bar{a} + b) = \bar{a}\bar{a} + \bar{a}b + \bar{b}\bar{a} + \bar{b}b = \bar{a} + \bar{a}b + \bar{a}\bar{b} = \bar{a}(1 + b + \bar{b}) = \bar{a}$
- 10)  $a(a + b + c + \dots) = aa + ab + ac + \dots = a + ab + ac + \dots = a$
- 11)  $f(a, b, ab) = a + b + ab = a + b$
- 12)  $f(a, b, \bar{a} \cdot \bar{b}) = a + b + \overline{\bar{a} \cdot \bar{b}} = a + b + \bar{a} = 1$
- 13)  $f[a, b, (\overline{ab})] = a + b + \overline{ab} = a + b + \bar{a} + \bar{b} = 1$
- 14)  $y + y\bar{y} = y$
- 15)  $xy + x\bar{y} = x(y + \bar{y}) = x$
- 16)  $\bar{x} + y\bar{x} = \bar{x}(1 + y) = \bar{x}$
- 17)  $(w + \bar{x} + y + \bar{z})y = y$
- 18)  $(x + \bar{y})(x + y) = x$
- 19)  $w + [w + (wx)] = w$
- 20)  $x[x + (xy)] = x$
- 21)  $\overline{(\bar{x} + \bar{x})} = x$
- 22)  $\overline{(x + \bar{x})} = 0$
- 23)  $w + (\overline{wxyz}) = w(1 + \overline{xyz}) = w$
- 24)  $\bar{w} \cdot (\overline{wxyz}) = \bar{w}(\bar{w} + \bar{x} + \bar{y} + \bar{z}) = \bar{w}$
- 25)  $xz + \bar{x}y + zy = xz + \bar{x}y$
- 26)  $(x + z)(\bar{x} + y)(z + y) = (x + z)(\bar{x} + y)$
- 27)  $\bar{x} + \bar{y} + xy\bar{z} = \bar{x} + \bar{y} + \bar{z}$

## Problem 1: Karnaugh Maps and Minimal Expressions

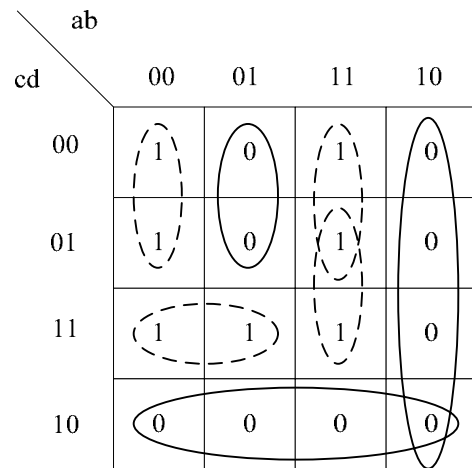
i) Truth Tables

abcd 1)	wxyz 2)
0000 0	0000 1
0001 1	0001 1
0010 0	0010 0
<u>0011 0</u>	<u>0011 1</u>
0100 0	0100 0
0101 1	0101 0
0110 0	0110 0
<u>0111 0</u>	<u>0111 1</u>
1000 0	1000 0
1001 0	1001 0
1010 0	1010 0
<u>1011 0</u>	<u>1011 0</u>
1100 0	1100 1
1101 0	1101 1
1110 1	1110 0
1111 0	1111 1

ii) Karnaugh Maps



(1)



(2)

iii) Minimum Sum of Products

$$(1) \bar{a} \cdot \bar{c} \cdot d + a \cdot b \cdot c \cdot \bar{d}$$

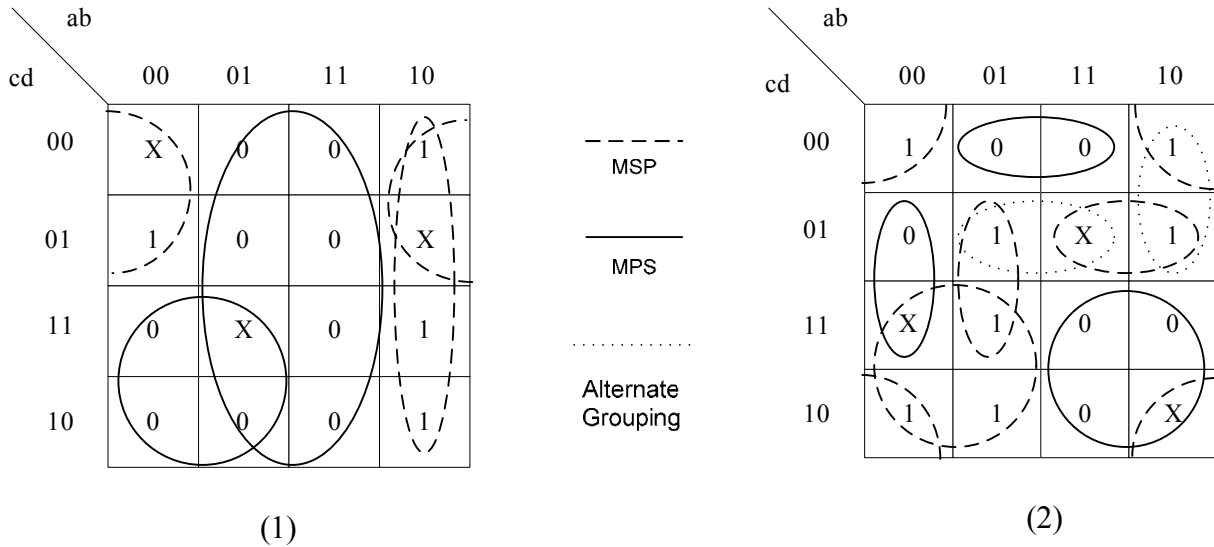
$$(2) \bar{a} \cdot \bar{b} \cdot \bar{c} + a \cdot b \cdot \bar{c} + a \cdot b \cdot d + \bar{a} \cdot c \cdot d$$

iv) Minimum Product of Sums

$$(1) (c + d)(\bar{a} + b)(\bar{a} + \bar{d})(a + \bar{c})$$

$$(2) (\bar{c} + d)(\bar{a} + b)(a + \bar{b} + c)$$

### Problem 2: Karnaugh Maps with “Don’t Cares”



(1)

- i.  $\bar{b} \cdot \bar{c} + a \cdot \bar{b}$
- ii.  $\bar{b} \cdot (\bar{c} + a)$
- iii. Both solutions are unique.
- iv. Yes, MSP = MPS

(2)

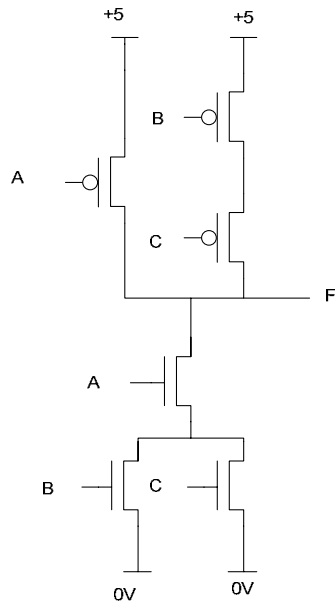
- i.  $\bar{b} \cdot \bar{d} + c \cdot \bar{a} + \bar{a} \cdot b \cdot d + a \cdot \bar{c} \cdot d$
- ii.  $(\bar{b} + c + d)(\bar{a} + \bar{c})(a + b + \bar{d})$
- iii. The MPS solution is unique, the MSP is not.  
 In the MSP:  $\bar{a} \cdot b \cdot d$  can be replaced with  $\bar{c} \cdot b \cdot d$ .  
 $a \cdot \bar{c} \cdot d$  can be replaced with  $a \cdot \bar{c} \cdot \bar{b}$ .
- iv. No, MSP  $\neq$  MPS.

### Problem 3: DeMorgan’s Theorem

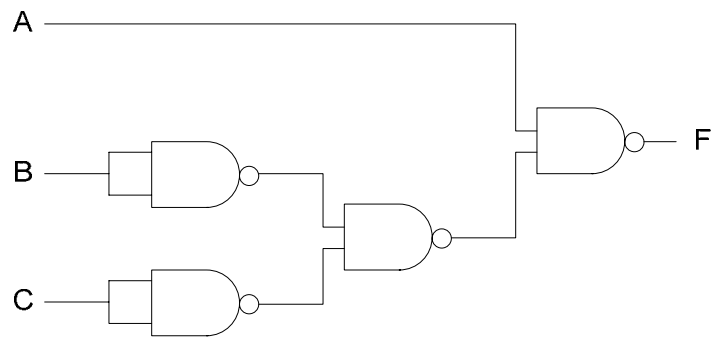
- 1)  $\overline{(a \cdot b \cdot c \cdot d)} = a + \bar{b} + c + d$
- 2)  $\overline{(a + \bar{b} + c + \bar{d})} = a \cdot b \cdot \bar{c} \cdot d$
- 3)  $(a \cdot \bar{d}) \cdot (\bar{b} \cdot c) \cdot (c \cdot \bar{d}) = a \cdot \bar{b} \cdot c \cdot \bar{d}$

### Problem 4: Transistor/Gate Level Synthesis

- 1) Transistor implementation



2) NAND gate implementation



**Problem 5: Setup and Hold times for D Flip-Flop**

- 1) The setup time is twice the delay of the inverter, the hold time is zero.
- 2) The new memory element is a negative edge triggered flip-flop.
- 3) The setup time is  $2t_{inv}$ , the hold time is zero, and the clock to Q delay is  $2t_{inv}$ .