Massachusetts Institute of Technology
Department of Electrical Engineering and Computer Science

6.012 Microelectronic Devices and Circuits
Spring 2007

Homework #8 – Due May 11, 2007

Problem 1:

a.) To obtain V_{OUT}=0V, we need I_{OUT}=0A, so I_C must equal I_{SUP}. Using this information, we can calculate V_{BE} as well as the voltage drop across R_S. Adding these drops to the negative voltage supply yields the necessary V_{BIAS}.

$$V_{BE} = V_{th} \ln\left(\frac{I_C}{I_S}\right) = 0.025 \bullet \ln\left(\frac{100\text{uA}}{1\text{fA}}\right) = 633\text{mV}$$
$$I_B R_S = 1\text{uA} \bullet 5\text{k}\Omega = 5\text{mV}$$
$$V_{BIAS} = -2.5 + I_B R_S + V_{BE} = -1.862V$$

b.) The input resistance, unloaded voltage gain, and output resistance are given below.

$$R_{in} = r_\pi = 25\text{k}\Omega$$
$$A_{vo} = \frac{-V_A}{V_{th}} = -4000$$
$$R_{out} = \frac{V_A}{I_C} = 1\text{M}\Omega$$
$$A_v = \left(\frac{R_{in}}{R_{in} + R_s}\right) A_{vo} \left(\frac{R_L}{R_L + R_{out}}\right) = -33$$

c.) Using the given f_T, I_{SUP} and C_μ, we can solve for C_π.

$$\omega_T = 2\pi f_T = \frac{g_m}{C_\pi + C_\mu}$$
$$6.28\text{e}9 \text{ rad/s} = \frac{0.004\text{S}}{C_\pi + 100\text{fF}}$$
$$C_\pi = 536\text{fF}$$

d.) In the Miller approximation, C_μ is Miller-multiplied, to form a large equivalent capacitance. The resulting ω_{3db} is given below.

$$\omega_{3db} = \frac{1}{R_{in}^{'}}\left[\frac{1}{C_\pi + \left(1+g_m R_{out}^{'}\right)C_\mu}\right]$$
$$R_{out}^{'} = r_o \,||\, R_L = 9.9\text{k}\Omega$$
$$R_{in}^{'} = R_s \,||\, r_\pi = 4.2\text{k}\Omega$$
$$\omega_{3db} = 5.22\text{e}7 \text{ rad/s}$$

e.) The are two open circuit time constants due to the two parasitic capacitances. The inverse of the time-constant sum yields the 3db bandwidth.

$$\omega_{3db} = \frac{1}{\tau_\pi + \pi_\mu}$$
$$\tau_\pi = R_{in}^{'} C_\pi = 2.2\text{ns}$$
$$\tau_\mu = \left(R_{out}^{'} + R_{in}^{'}\left(1+g_m R_{out}^{'}\right)\right)C_\mu = 17.9\text{ns}$$
$$\omega_{3db} = 4.96\text{e}7 \text{ rad/s}$$

Problem 2:

a.) Given a source resistance and β, we can solve for the necessary I_C such that the total output resistance is 100 ohms.

$$R_{out} = \frac{1}{g_m} + \frac{R_s}{\beta} = 100\Omega$$
$$\frac{1}{g_m} = 50\Omega$$
$$I_C = 500\text{uA}$$

b.) The necessary bias voltage equals V_{BE} plus any drop across the source resistance.

$$V_{BIAS} = I_B R_s + V_{th} \ln\left(\frac{I_C}{I_S}\right) = 25\text{mV+673mV=698mV}$$

c.) C_π and C_μ can be found from device parameters.

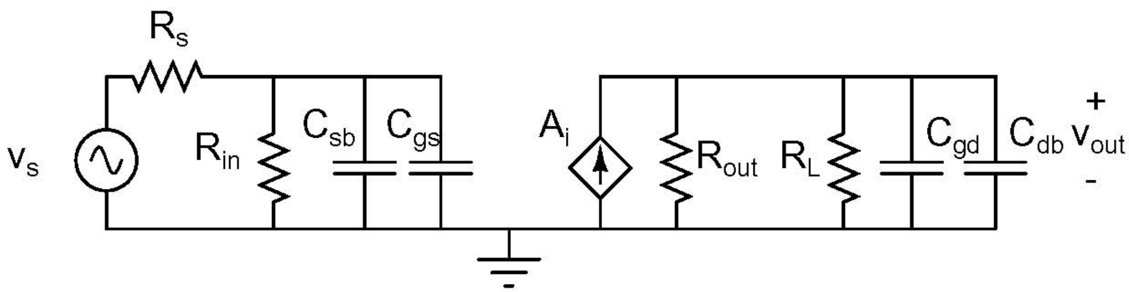
$$C_\pi = g_m \tau_f + \sqrt{2} C_{jeo} = 2.14\text{pF}$$
$$C_\mu = \frac{C_{\mu 0}}{\sqrt{1 + \frac{V_{CB}}{\Phi_{Bc}}}} = \frac{200\text{fF}}{\sqrt{1+\frac{2.5\text{-}0.698}{0.75}}} = 108\text{fF}$$

d.) We use the Miller approximation and open-circuit time constants to find the 3db bandwidth of the common-collector.

$$C_T = C_\pi\left(\frac{1/g_m}{1/g_m + R_L}\right) + C_\mu = 112\text{fF}$$
$$R_T = R_s \,||\, R_{in} = 5\text{k}\Omega \,||\, \left(r_\pi + (\beta+1)R_L\right) = 5\text{k}\Omega$$
$$\tau = R_T C_T = 0.56\text{ns}$$
$$\omega_{3\text{db}} = \frac{1}{\tau} = 1.8\text{e}9 \text{ rad/s}$$

Problem 3:

a.) Since the bulk is tied to a DC potential, C_{db} appears between the drain and AC ground. The relevant two-port model is given below. Students did not need to consider C_{sb}, although its position is shown in the two-port model. The rest of the problem ignores C_{sb}. In addition, we use the zero-bias value of C_{db} which gives a conservative estimate of the 3db bandwidth.



$$R_{in}^{'} = R_s \,||\, \frac{1}{g_m}$$

$$R_{out}^{'} = R_L \,||\, r_o\left(1 + g_m R_s\right)$$

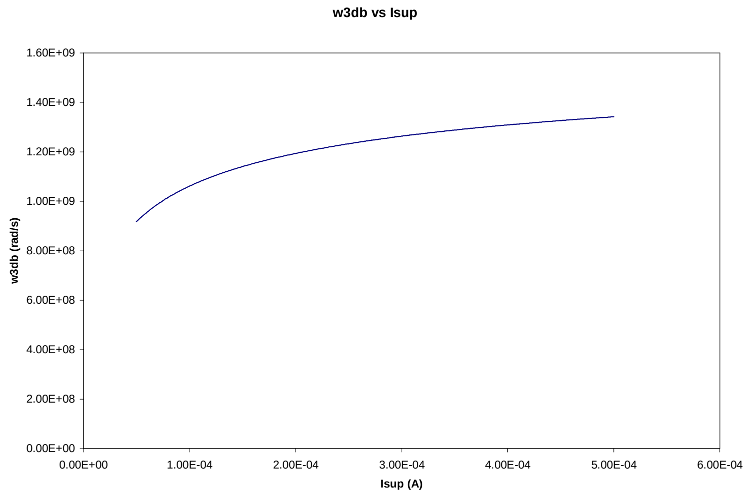
$$\omega_{3db} = \frac{1}{C_{gs} R_{in}^{'} + \left(C_{gd} + C_{db}\right) R_{out}^{'}}$$

b.) The relevant formulas for the parasitic capacitances are given below. As mentioned above, we ignored C_{sb}, and used the zero-bias value for C_{db}.

$$C_{gs} = \frac{2}{3} W L C_{ox} + W C_{ov} = 178\text{fF}$$

$$C_{gd} = W C_{ov} = 25\text{fF}$$

$$C_{db} = C_j W L_{diff} + C_{jsw}\left(W + 2L_{diff}\right) = 61\text{fF}$$



c.) Increasing I_{SUP} improves the frequency response of the amplifier by minimizing the time constant due to C_{gs}. However, as I_{SUP} increases, the dominant time constant becomes the one due to C_{gd} and C_{db}. Since the equivalent resistance associated with these capacitors is R_L due to the high output resistance of the common-gate stage, increasing I_{SUP} does not modify these time constants. Increasing I_{SUP} has the drawback of increased power dissipation with diminishing returns.