Massachusetts Institute of Technology
Department of Electrical Engineering and Computer Science

6.012 Microelectronic Devices and Circuits
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Homework #7 – Due May 4, 2007

Problem 1:

a.) To obtain V_{OUT}=0V, we need I_{OUT}=0A, so I_C must equal I_{SUP}. Using this information, we can calculate V_{BE} as well as the voltage drop across R_S. Adding these drops to the negative voltage supply yields the necessary V_{BIAS}.

$$V_{BE} = V_{th} \ln\left(\frac{I_C}{I_S}\right) = 0.025 \bullet \ln\left(\frac{100\text{uA}}{1\text{fA}}\right) = 633\text{mV}$$

$$I_B R_S = \frac{I_C}{\beta} R_S = \left(\frac{100\text{uA}}{100}\right) \bullet 1\text{k}\Omega = 1\text{mV}$$

$$V_{BIAS} = V_{BE} + I_B R_S - 2.5\text{V} = -1.866\text{V}$$

b.) The input resistance for a common-emitter equals r_π.

$$R_{in} = \frac{\beta V_{th}}{I_C} = \frac{100 \bullet 0.025\text{V}}{100\text{uA}} = 25\text{k}\Omega$$

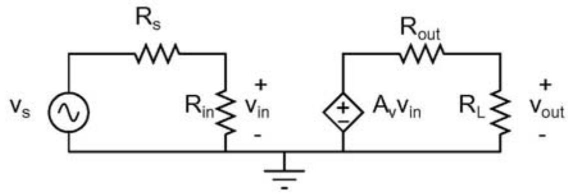
c.) The unloaded voltage gain A_{vo} for a common-emitter equals $-g_m r_o$.

$$A_{vo} = -g_m r_o = \frac{-I_C}{V_{th}} \bullet \frac{V_A}{I_C} = -\frac{25}{25\text{mV}} = -1000$$

d.) The output resistance for a common-emitter equals r_o.

$$R_{out} = r_o = \frac{V_A}{I_C} = \frac{25}{100\text{uA}} = 250\text{k}\Omega$$

e.) The two-port model is given below. From this model, we can write the overall loaded voltage gain using the voltage-divider rule.



$$\frac{v_{out}}{v_s} = \left(\frac{R_{in}}{R_{in} + R_S}\right)(A_{vo})\left(\frac{R_L}{R_L + R_{out}}\right)$$

$$\frac{v_{out}}{v_s} = \left(\frac{25\text{k}\Omega}{25\text{k}\Omega + 1\text{k}\Omega}\right)(-1000)\left(\frac{10\text{k}\Omega}{10\text{k}\Omega + 250\text{k}\Omega}\right) = -37$$

f.) We are given a value of λ at a length of 1.5um. Using a length of 1.5um, we find that the output resistance is 100kΩ. We need an output resistance of 250kΩ, or 2.5 times what is provided by a length of 1.5um. Since λ is inversely proportional to length, increasing the length by a factor of 2.5 will result in the necessary output resistance. We need L=3.75um.

$$R_{out} = r_o = \frac{1}{\lambda I_D} = 100\text{k}\Omega @ L = 1.5\text{um}$$

g.) We want to choose W such that the two amplifiers have the same loaded voltage gain. Since they have equal output resistances and load resistances, the voltage dividers created by R_L and R_{out} are equal for both amplifiers. Since the NMOS has infinite input resistance, the voltage divider consisting of R_S and R_{in} equals 1 for the NMOS, versus 25/26 for the NPN. We can solve for the necessary g_m and then the necessary W.

$$A_v = \frac{25}{26} \bullet 1000 = 961.5 = g_m R_{out} = g_m \bullet 250\text{k}\Omega$$

$$g_m = 0.003846 = \sqrt{2 \bullet \frac{W}{3.75\text{um}} \bullet \frac{50\text{uA}}{\text{V}^2} \bullet 100\text{uA}}$$

$$W = 5547\text{um}$$

h.) We want to again bias the amplifier such that all of the I_{SUP} is sunk by the NMOS. This means that $I_D = I_{SUP}$. Since there is infinite input resistance for an NMOS, there is no drop across R_S.

$$100\text{uA} = \frac{W}{2L}\mu_n C_{ox}\left(V_{GS} - V_T\right)^2$$

$$V_{GS} = V_T + 0.052\text{V} = 1.052\text{V}$$

$$V_{BIAS} = -2.5\text{V} + V_{GS} = -1.448\text{V}$$

Problem 2:

a.) Here, we find the input resistance, current gain, and output resistance of the common-base amplifier. From the two-port model, we can write the overall current gain i_{out}/i_S.

$$R_{in} = \frac{1}{g_m} = 125\Omega$$

$$A_i = \frac{\beta}{\beta+1} = 0.99$$

$$R_{out} = r_o\left(1+g_m\left(r_\pi \| R_S\right)\right) = 125\Omega\left(1+0.008\text{S}\left(12.5\text{k}\Omega \| 5\text{k}\Omega\right)\right)$$

$$R_{out} = 3.7\text{M}\Omega$$

$$\frac{i_{out}}{i_s} = \left(\frac{R_S}{R_S+R_{in}}\right)\left(A_i\right)\left(\frac{R_{out}}{R_L+R_{out}}\right) = 0.975\bullet 0.99\bullet 0.987 = 0.953$$

b.) Here, we find the output resistance and transconductance of the common-emitter amplifier with emitter degeneration. The emitter degeneration resistor is R_S, since the other current sources are ideal. Since there is no source resistance associated with the voltage source v_b, the entire voltage v_b is dropped across R_{in}.

$$R_{out} = r_o\left(1+g_m R_S\right) = 125\text{k}\Omega\left(1+0.008\text{S}\bullet 5\text{k}\Omega\right) = 5.125\text{M}\Omega$$

$$G_M = \frac{g_m}{1+g_m R_E} = \frac{0.008\text{S}}{1+0.008\text{S}\bullet 5\text{k}\Omega} = 195\text{uS}$$

$$\frac{i_{out}}{v_b} = -\left(G_M\right)\left(\frac{R_{out}}{R_{out}+R_L}\right) = 193\text{uS}$$

c.) Here, we rewrite the answers to parts a and b, and divide to find the ratio.

$$i_{out1} = 0.953 i_s$$

$$i_{out2} = 0.000193\text{S}\bullet v_b$$

$$\frac{i_{out1}}{i_{out2}} = 4938\Omega\bullet\frac{i_s}{v_b}$$

d.) Increasing R_S will increase the amount of emitter degeneration. This will decrease the transconductance of the common-emitter amplifier with emitter degeneration, decreasing i_{out2} and increasing the ratio of part c.

Changing R_L will have a negligible impact on the ratio in part c, since it only appears in the current-dividers. Since the output resistance of the common base and common emitter amplifiers is very large, the impact of R_L is insignificant on this ratio.

Finally, increasing I_{SUP} will increase ratio in part c. Increasing I_{SUP} has a negligible impact on G_M of the common-emitter, due to the emitter degeneration. Increasing I_{SUP} decreases R_{in} of the common-base, improving the input current-divider, and increasing the ratio in part c.

Problem 3:

a.) Due to the backgate effect, we need to find the new V_{Tn}. After doing this, we can solve for the necessary V_{GS} such that $I_D = I_{SUP}$. Since the source will be at 0V, V_{BIAS} will equal V_{GS}.

$$V_{Tn} = V_{Ton} + \gamma_n\left(\sqrt{-V_{BS} - 2\Phi_p} - \sqrt{-2\Phi_p}\right)$$

$$V_{Tn} = 0.7\text{V} + 0.5\left(\sqrt{2.5 + 0.8} - \sqrt{0.8}\right) = 1.16\text{V}$$

$$200\text{uA} = \frac{W}{2L}\mu_n C_{ox}\left(V_{GS} - V_T\right)^2$$

$$V_{BIAS} = V_{GS} = V_T + 0.4\text{V} = 1.56\text{V}$$

b.) The minimum V_{OUT} is determined by the voltage requirement across the current source. Since the current source needs 0.5V across it, $V_{OUT,min}$=-2.0V. The maximum V_{OUT} is one $V_{DS,sat}$ from the positive rail. However, the value of $V_{DS,sat}$ depends on the maximum V_{OUT}, since at this point, the drain current is not necessarily equal to I_{SUP}.

$$I_D = I_{SUP} + I_{OUT} = I_{SUP} + \frac{V_{OUT}}{R_L}$$

$$V_{DS,sat} = V_{GS} - V_{Tn} = \sqrt{\frac{I_D}{\frac{W}{2L}\mu_n C_{ox}}} = \sqrt{\frac{I_{SUP} + \frac{V_{OUT}}{R_L}}{\frac{W}{2L}\mu_n C_{ox}}}$$

$$V_{OUT} = V_{DD} - V_{DS,sat} = V_{DD} - \sqrt{\frac{I_{SUP} + \frac{V_{OUT}}{R_L}}{\frac{W}{2L}\mu_n C_{ox}}}$$

$$V_{OUT} = 2.08V$$

c.) The overall voltage gain can be found by inspecting R_{in}, A_{vo}, and R_{out}. Due to the backgate effect, the backgate transconductance g_{mb} must also be calculated.

$$R_{in} = \infty$$

$$A_{vo} = \frac{g_m}{g_m + g_{mb}}$$

$$g_{mb} = g_m \left(\frac{\gamma_n}{2\sqrt{-2\Phi_p - V_{BS}}} \right) = g_m \left(\frac{0.5}{2\sqrt{2.5+0.8}} \right) = g_m (0.138)$$

$$A_{vo} = \frac{1}{1+0.138} = 0.879$$

$$R_{out} = \frac{1}{g_m + g_{mb}} = \frac{1}{g_m (1.138)}$$

$$g_m = \sqrt{2\frac{W}{L}\mu_n C_{ox} I_D} = 0.001$$

$$R_{out} = 879\Omega$$

$$\frac{v_{out}}{v_s} = (A_{vo}) \left(\frac{R_L}{R_L + R_{out}} \right) = 0.879 \bullet \left(\frac{100\text{k}\Omega}{100\text{k}\Omega + 879\Omega} \right) = 0.871$$

d.) Repeating parts a-c, we first solve for the necessary bias voltage. With no backgate effect, the threshold voltage is lower.

$$V_{Tn} = V_{Ton} = 0.7\text{V}$$

$$200\text{uA} = \frac{W}{2L}\mu_n C_{ox} (V_{GS} - V_T)^2$$

$$V_{BIAS} = V_{GS} = V_T + 0.4\text{V} = 1.1\text{V}$$

Next we check the minimum and maximum V_{OUT}. The minimum V_{OUT} remains unchanged, and is still determined by the voltage requirement of the current source. The maximum V_{OUT} also remains unchanged, since the equation that needed to be solved for V_{OUT} is independent of V_{Tn}. Finally, we solve for the the gain, with the backgate transconductance generator turned off. Note that the gain is much higher when there is no backgate effect.

$$R_{in} = \infty$$

$$A_{vo} = 1$$

$$R_{out} = \frac{1}{g_m} = 1\text{k}\Omega$$

$$\frac{v_{out}}{v_s} = (A_{vo}) \left(\frac{R_L}{R_L + R_{out}} \right) = 1 \bullet \left(\frac{100\text{k}\Omega}{100\text{k}\Omega + 1\text{k}\Omega} \right) = 0.99$$