

# Lecture 7

## PN Junction and MOS Electrostatics(IV) Metal-Oxide-Semiconductor Structure (contd.)

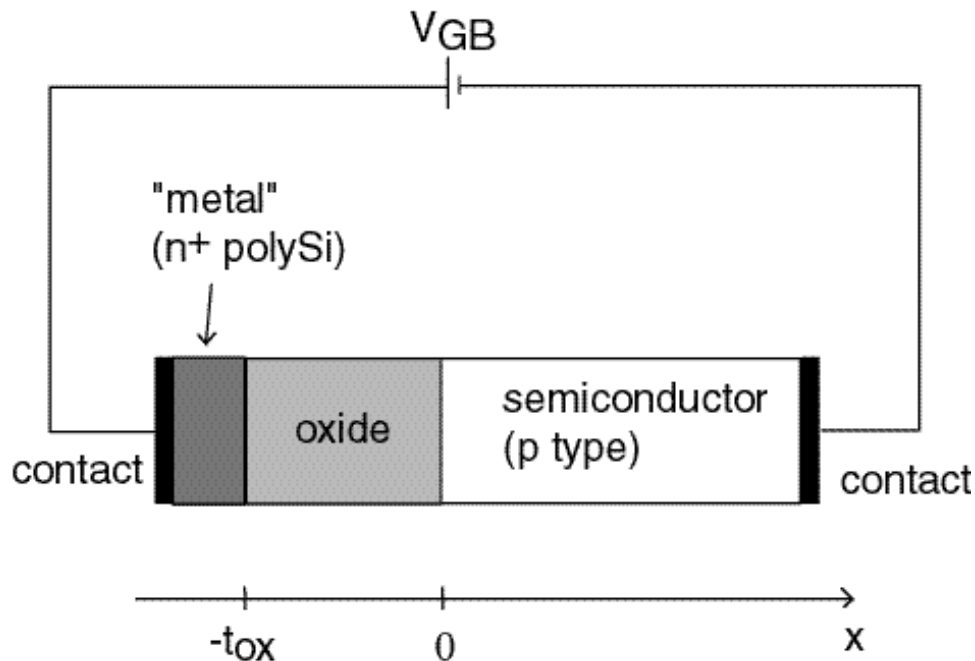
### Outline

1. Overview of MOS electrostatics under bias
2. Depletion regime
3. Flatband
4. Accumulation regime
5. Threshold
6. Inversion regime

### **Reading Assignment:**

Howe and Sodini, Chapter 3, Sections 3.8-3.9

# 1. Overview of MOS electrostatics under bias



Application of bias:

- Built-in potential across MOS structure increases from  $\phi_B$  to  $\phi_B + V_{GB}$
- Oxide forbids current flow  $\Rightarrow$ 
  - $J=0$  everywhere in semiconductor
  - Need **drift = -diffusion** in SCR
- Must maintain boundary condition at Si/SiO<sub>2</sub> interface
  - $E_{ox} / E_s \approx 3$

How can this be accommodated simultaneously?  $\Rightarrow$  **quasi-equilibrium situation** with potential build-up across MOS equal to  $\phi_B + V_{GB}$

## Important consequence of quasi-equilibrium:

⇒ Boltzmann relations apply in semiconductor

[they were derived starting from  $J_n = J_p = 0$ ]

$$n(x) = n_i e^{q\phi(x)/kT}$$

$$p(x) = n_i e^{-q\phi(x)/kT}$$

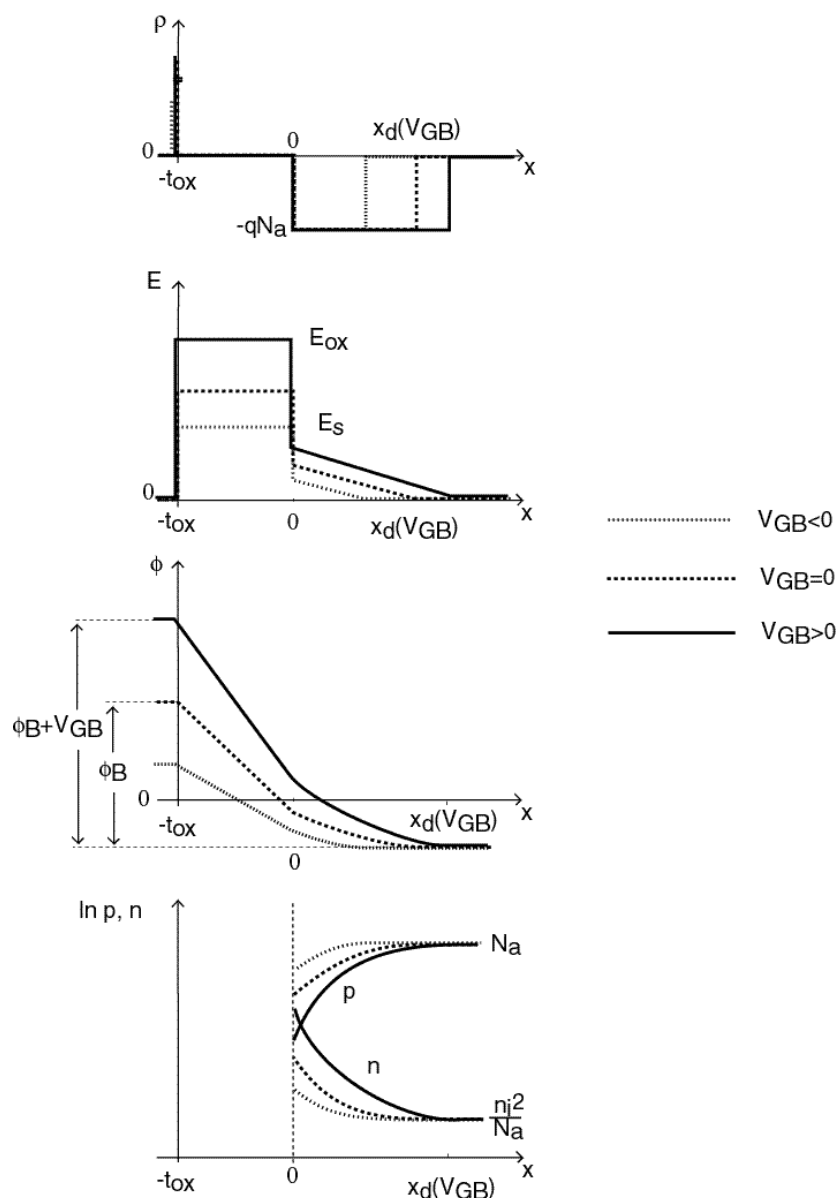
and

$$np = n_i^2 \quad \text{at every } x$$

## 2. Depletion regime

For  $V_{GB} > 0$ , metal attracts electrons and repels holes  
 $\Rightarrow$  *Depletion region widens*

For  $V_{GB} < 0$ , metal repels electrons and attracts holes  
 $\Rightarrow$  *Depletion region shrinks*



In depletion regime, all results obtained for thermal equilibrium apply if  $\phi_B \rightarrow \phi_B + V_{GB}$ .

For example:

Depletion region thickness:

$$x_d(V_{GB}) = \frac{\epsilon_s}{C_{ox}} \left[ \sqrt{1 + \frac{2C_{ox}^2 (\phi_B + V_{GB})}{\epsilon_s q N_a}} - 1 \right]$$

Potential drop across semiconductor SCR:

$$V_B(V_{GB}) = \frac{q N_a x_d^2}{2 \epsilon_s}$$

Surface potential

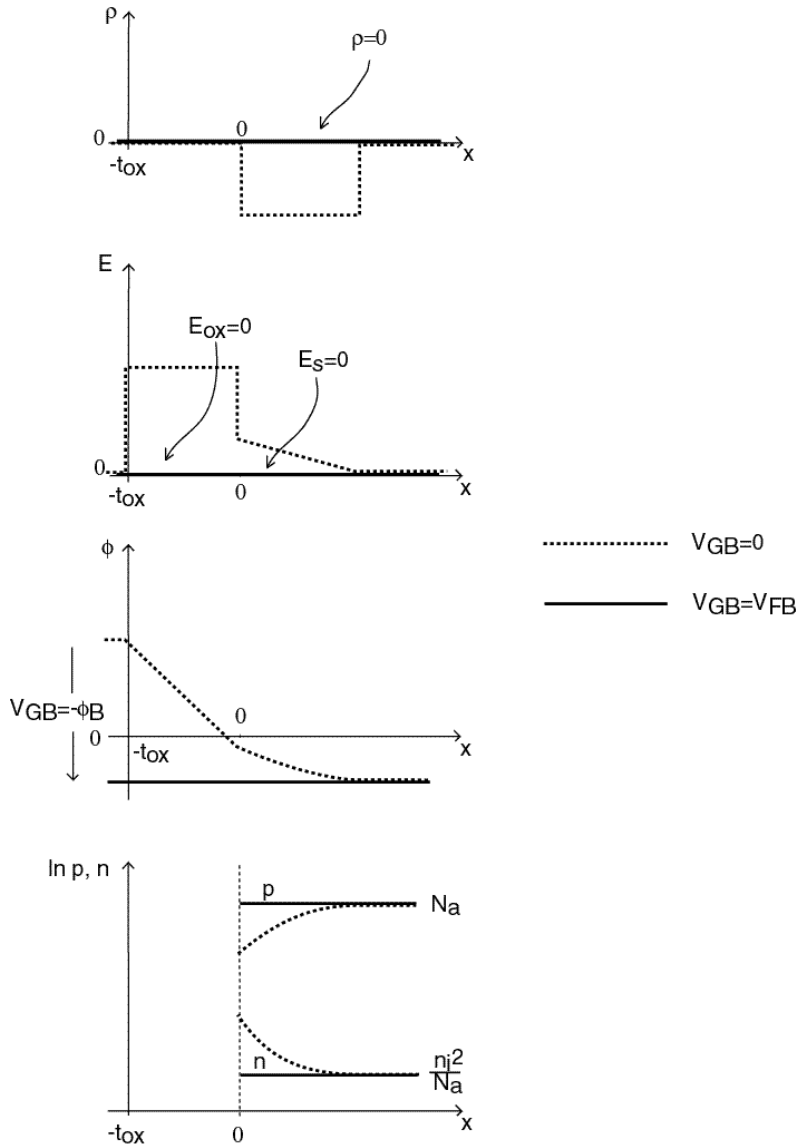
$$\phi(0) = \phi_p + V_B(V_{GB})$$

Potential drop across oxide:

$$V_{ox}(V_{GB}) = \frac{q N_a x_d t_{ox}}{\epsilon_{ox}}$$

### 3. Flatband

At a certain negative  $V_{GB}$ , depletion region is wiped out  $\Rightarrow$  *Flatband*

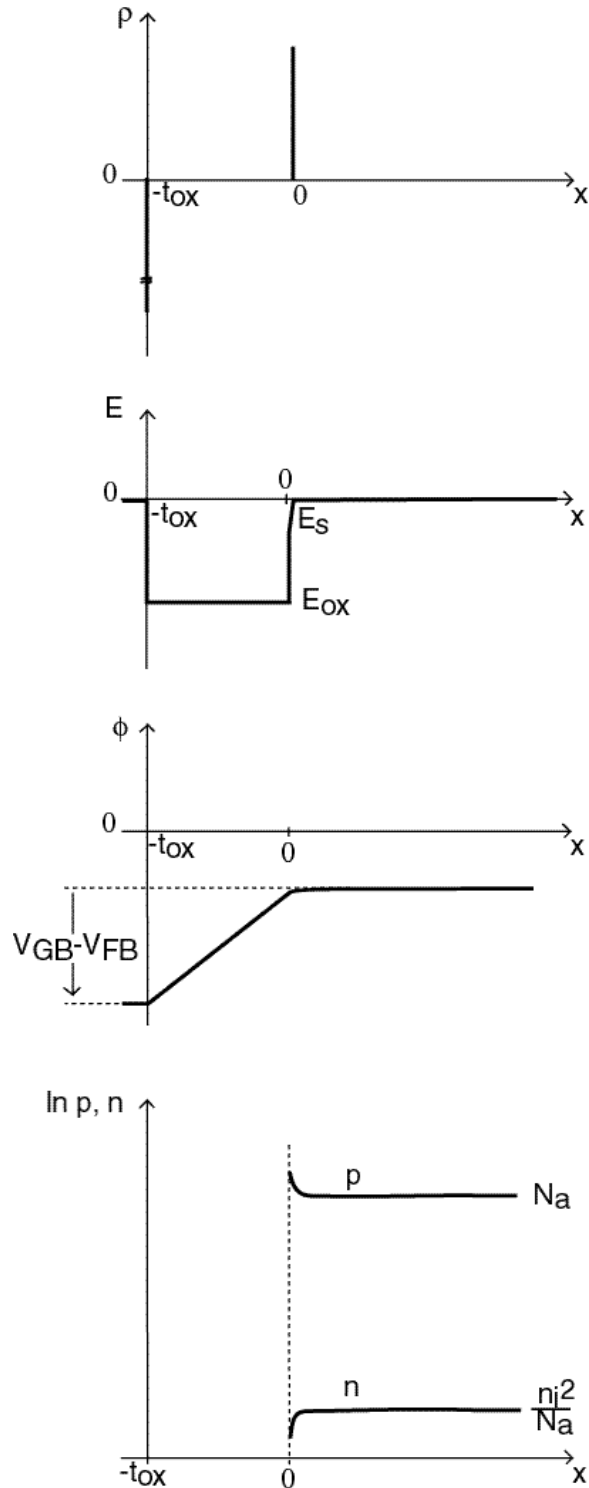


*Flatband Voltage:*

$$V_{GB} = V_{FB} = -\phi_B = -(\phi_{N^+} - \phi_p)$$

## 4. Accumulation regime

If  $V_{GB} < V_{FB}$  accumulation of holes at Si/SiO<sub>2</sub> interface

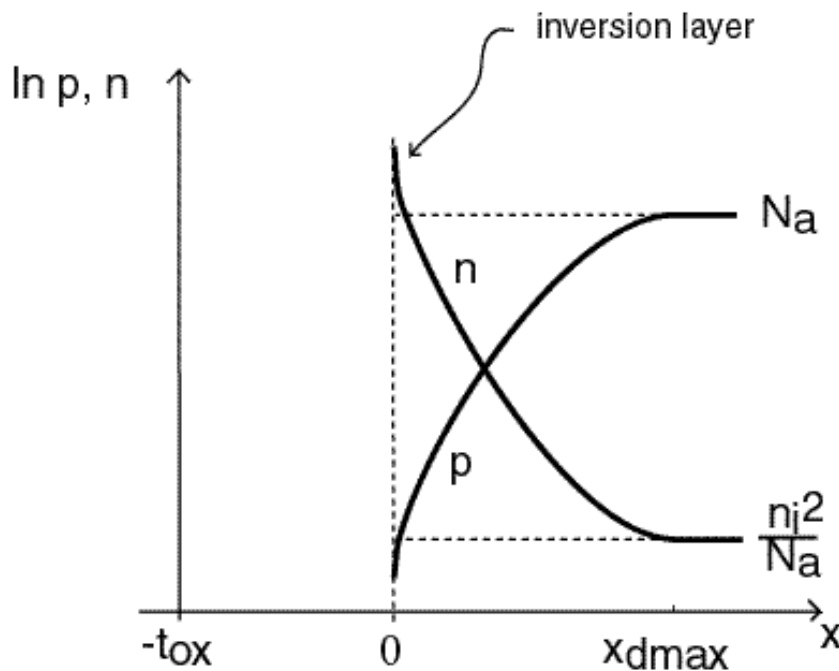


## 5. Threshold

Back to  $V_{GB} > 0$ .

For sufficiently large  $V_{GB} > 0$ , electrostatics change when  $n(0) = N_a \Rightarrow$  *threshold*.

Beyond *threshold*, we **cannot** neglect contributions of electrons towards electrostatics.



Let's compute the gate voltage (*threshold voltage*) that leads to  $n(0) = N_a$ .

**Key assumption:** use electrostatics of depletion (neglect electron concentration at threshold)



## Computation of threshold voltage.

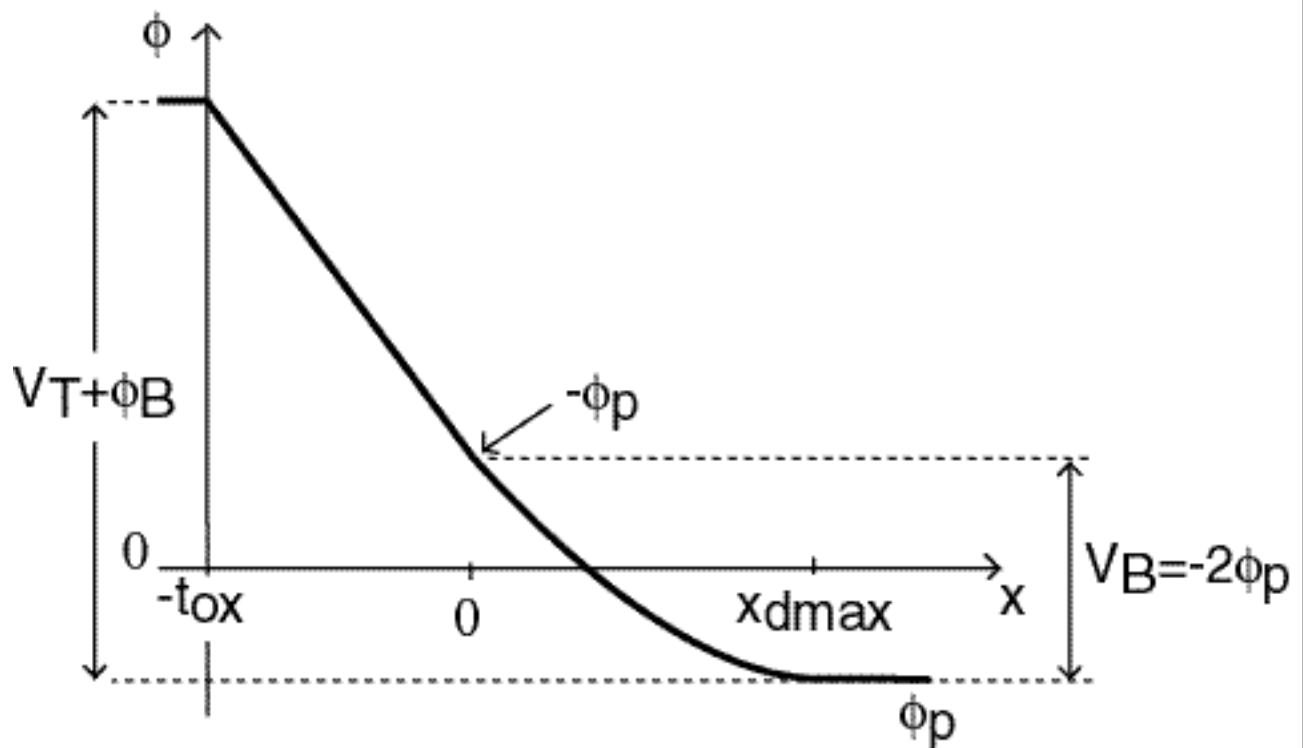
Three step process:

**First**, compute potential drop in semiconductor at threshold. Start from:

$$n(0) = n_i e^{q\phi(0)/kT}$$

Solve for  $\phi(0)$  at  $V_{GB} = V_T$ :

$$\phi(0)|_{V_{GB} = V_T} = \frac{kT}{q} \cdot \ln \frac{n(0)}{n_i} \Big|_{V_{GB} = V_T} = \frac{kT}{q} \cdot \ln \frac{N_a}{n_i} = -\phi_p$$



Hence:

$$V_B(V_T) = -2\phi_p$$

## Computation of threshold voltage (contd.)

**Second**, compute potential drop in oxide at threshold.

Obtain  $x_d(V_T)$  using relationship between  $V_B$  and  $x_d$  in depletion:

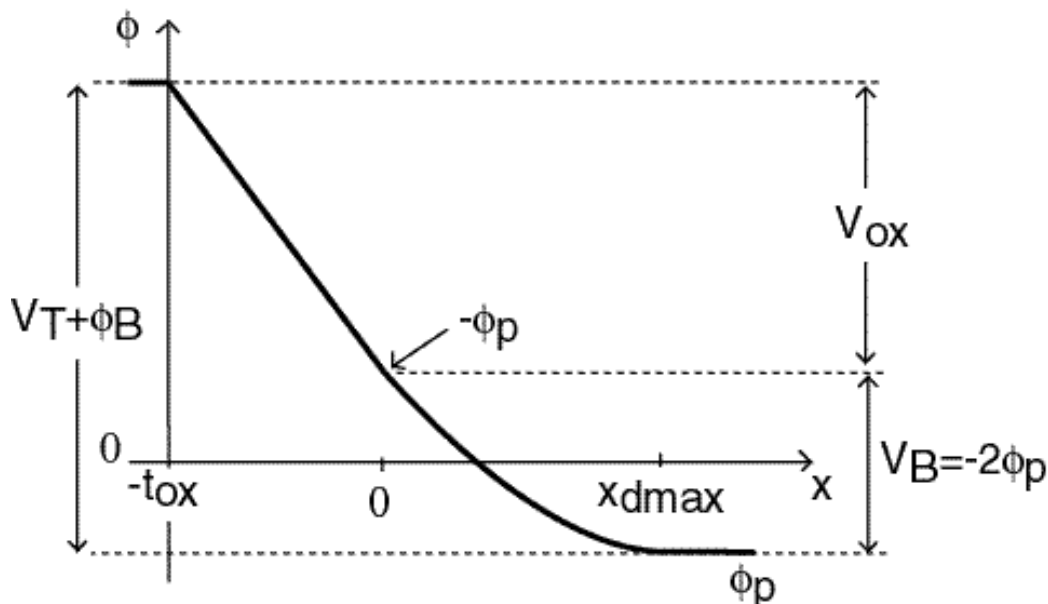
$$V_B(V_{GB} = V_T) = \frac{qN_a x_d^2(V_T)}{2\epsilon_s} = -2\phi_p$$

Solve for  $x_d$  at  $V_{GB} = V_T$ :

$$x_d(V_T) = x_{d\max} = \sqrt{\frac{2\epsilon_s(-2\phi_p)}{qN_a}}$$

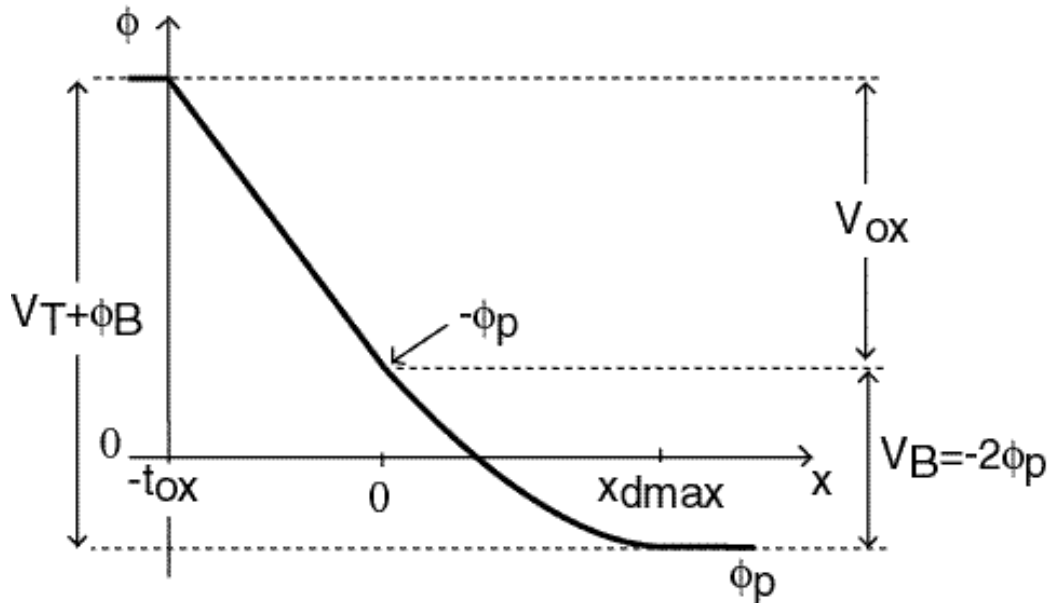
Then:

$$V_{ox}(V_T) = E_{ox}(V_T)t_{ox} = \frac{qN_a x_d(V_T)}{\epsilon_{ox}} t_{ox} = \frac{1}{C_{ox}} \sqrt{2\epsilon_s qN_a (-2\phi_p)}$$



## Computation of threshold voltage. (contd.)

**Finally**, sum potential drops across structure.



$$V_T + \phi_B = V_B(V_T) + V_{ox}(V_T) = -2\phi_p + \frac{1}{C_{ox}} \sqrt{2\epsilon_s q N_a (-2\phi_p)}$$

Solve for  $V_T$ :

$$V_{GB} = V_T = V_{FB} - 2\phi_p + \frac{1}{C_{ox}} \sqrt{2\epsilon_s q N_a (-2\phi_p)}$$

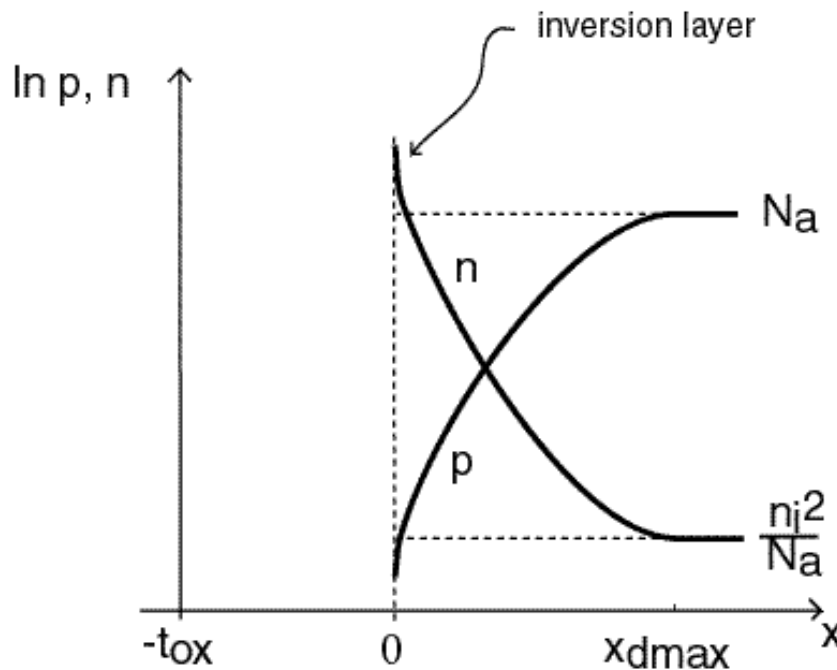
**Key dependencies:**

- If  $N_a \uparrow \Rightarrow V_T \uparrow$ . The higher the doping, the more voltage required to produce  $n(0) = N_a$
- If  $C_{ox} \uparrow$  ( $t_{ox} \downarrow$ )  $\Rightarrow V_T \downarrow$ . The thinner the oxide, the less voltage dropped across the oxide.

## 6. Inversion

What happens for  $V_{GB} > V_T$ ?

More electrons at Si/SiO<sub>2</sub> interface than acceptors  
⇒ *inversion*.



Electron concentration at Si/SiO<sub>2</sub> interface modulated by  $V_{GB} \Rightarrow V_{GB} \uparrow \rightarrow n(0) \uparrow \rightarrow |Q_n| \uparrow$ :

**Field-effect control of mobile charge density!**

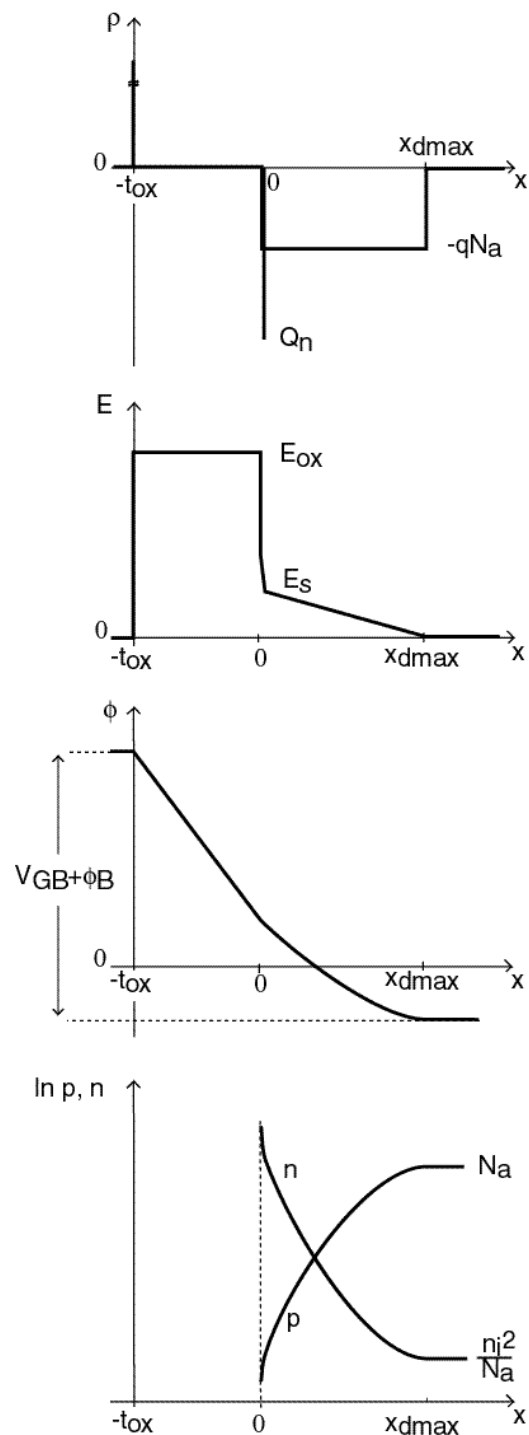
[essence of MOSFET]

Want to compute  $Q_n$  vs.  $V_{GB}$  [*charge-control relation*]

Make *sheet charge approximation*: electron layer at Si/SiO<sub>2</sub> is much thinner than any other dimension in problem ( $t_{ox}$ ,  $x_d$ ).

# Charge-Control Relation

To derive the charge-control relation, let's look at the overall electrostatics:



## Charge-Control Relation (contd.)

### Key realization:

$$n(0) \propto e^{q\phi(0)/kT}$$

$$qN_a x_d \propto \sqrt{\phi(0)}$$

Hence, as  $V_{GB} \uparrow$  and  $\phi(0) \uparrow$ ,  $n(0)$  will change a lot, but  $|Q_d|$  will change very little.

### Several consequences:

- $x_d$  does not increase much beyond threshold:

$$x_d(\text{inv.}) \approx x_d(V_T) = \sqrt{\frac{2\varepsilon_s(-2\phi_p)}{qN_a}} = x_{d,\max}$$

- $V_B$  does not increase much beyond  $V_B(V_T) = -2\phi_p$   
(*a thin sheet of electrons does not contribute much to  $V_B$ .*):

$$V_B(\text{inv.}) \approx V_B(V_T) = -2\phi_p$$

## Charge-Control Relation (contd..)

- All extra voltage beyond  $V_T$  used to increase inversion charge  $Q_n$ . Think of it as capacitor:
  - Top plate: metal gate
  - Bottom plate: inversion layer

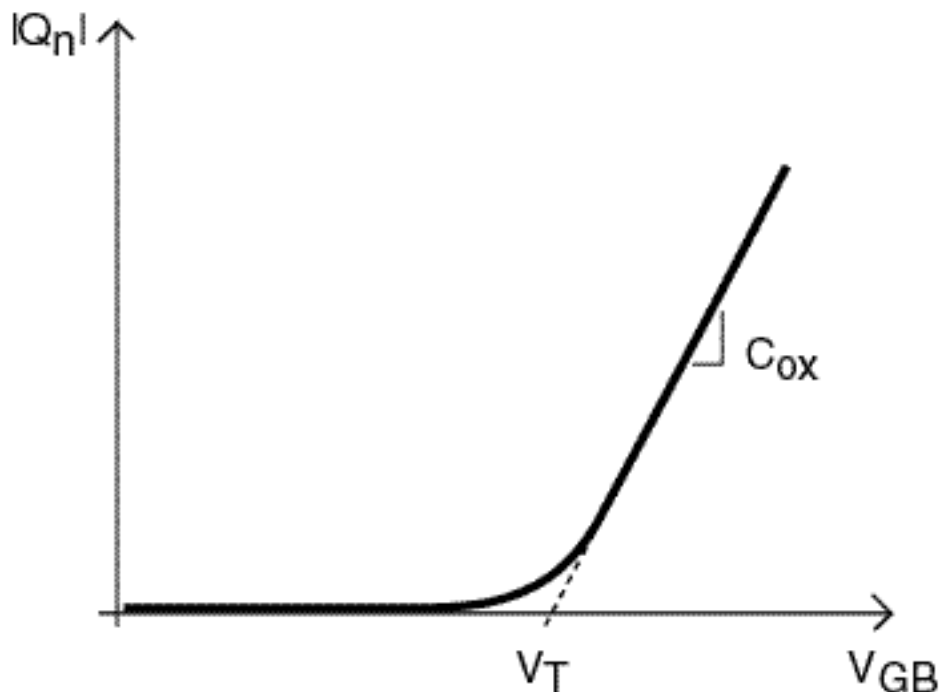
$$Q = CV$$

⇒

$$Q_n = -C_{ox} (V_{GB} - V_T) \quad \text{for } V_{GB} > V_T$$

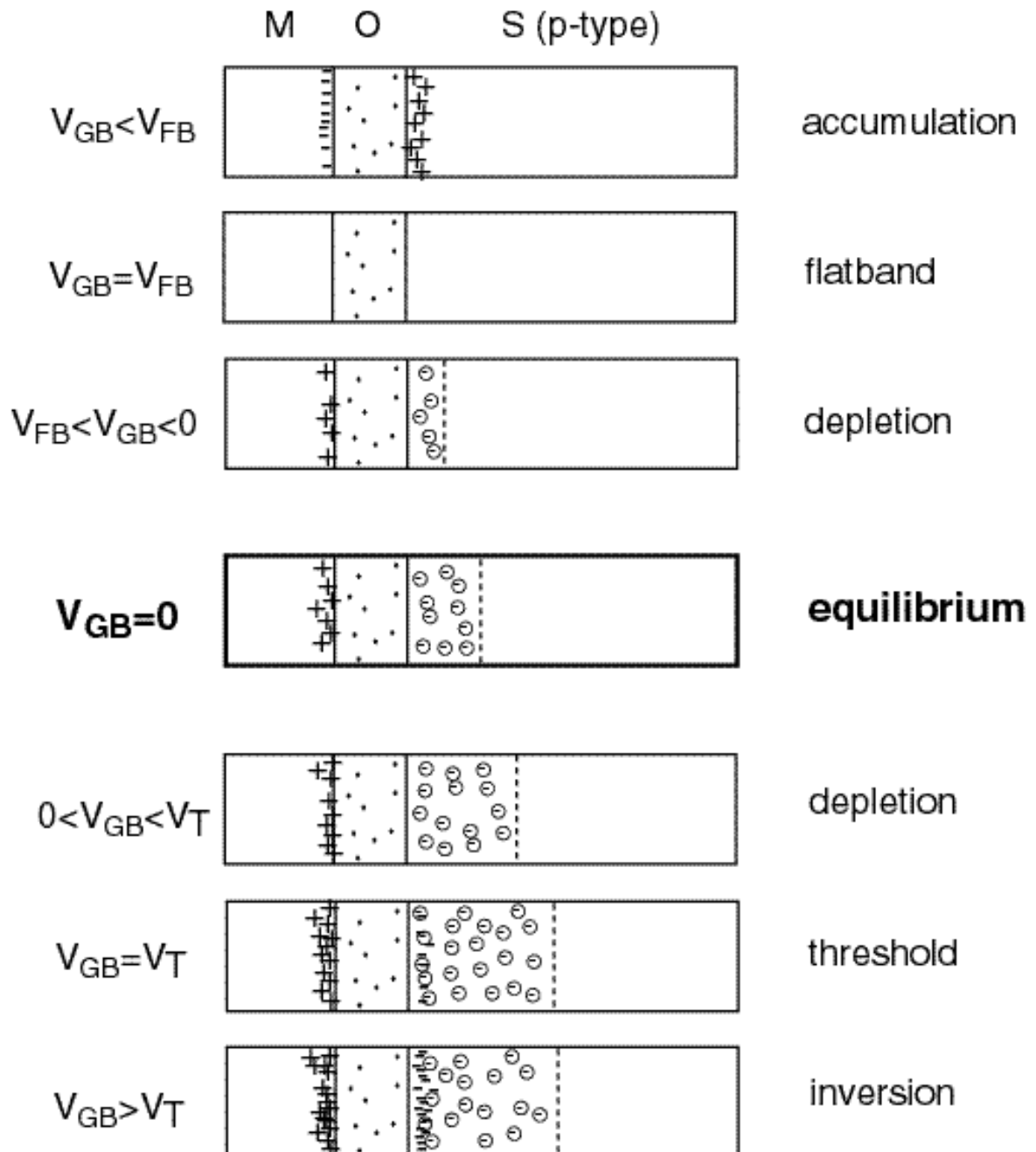
Coul/cm<sup>2</sup>

Existence of  $Q_n$  and control over  $Q_n$  by  $V_{GB}$   
⇒ key to MOS electronics



# What did we learn today?

## Summary of Key Concepts



**In inversion:**

$$|Q_n| = C_{ox} (V_{GB} - V_T) \quad \text{for } V_{GB} > V_T$$