Lecture 22

Frequency Response of Amplifiers (II)

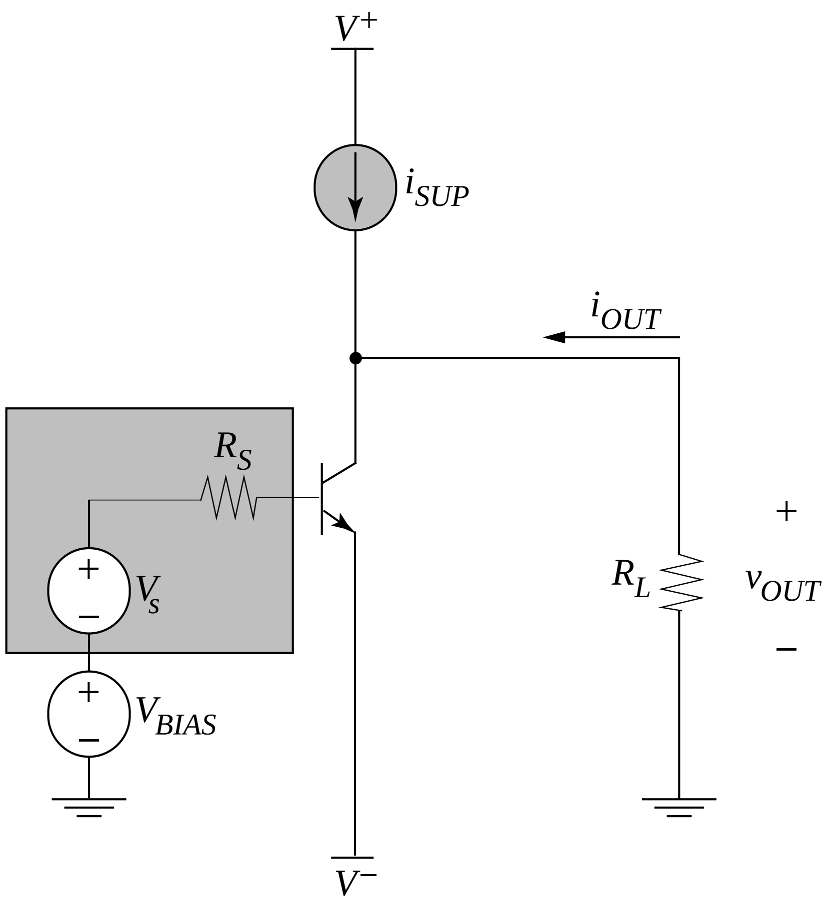
VOLTAGE AMPLIFIERS

Outline

1. Full Analysis
2. Miller Approximation
3. Open Circuit Time Constant

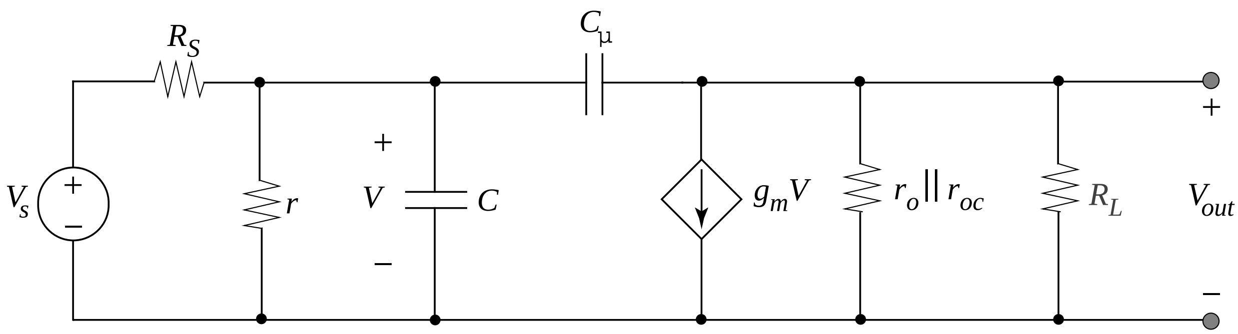
Reading Assignment:
Howe and Sodini, Chapter 10, Sections 10.1-10.4

Common Emitter Amplifier



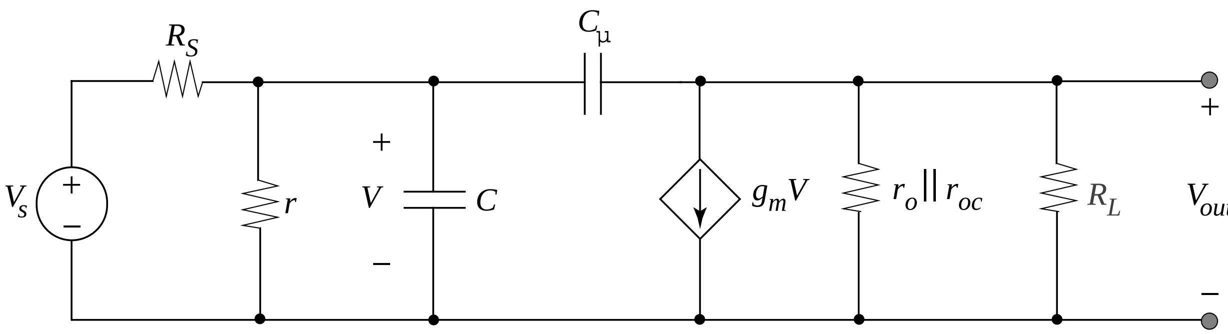
- Operating Point Analysis
  - v_s=0, R_S = 0, $r_o \to \infty$, $r_{oc} \to \infty$, $R_L \to \infty$
  - Find V_{BIAS} such that I_C=I_{SUP} with the BJT in the forward active region

Frequency Response Analysis of the Common Emitter Amplifier

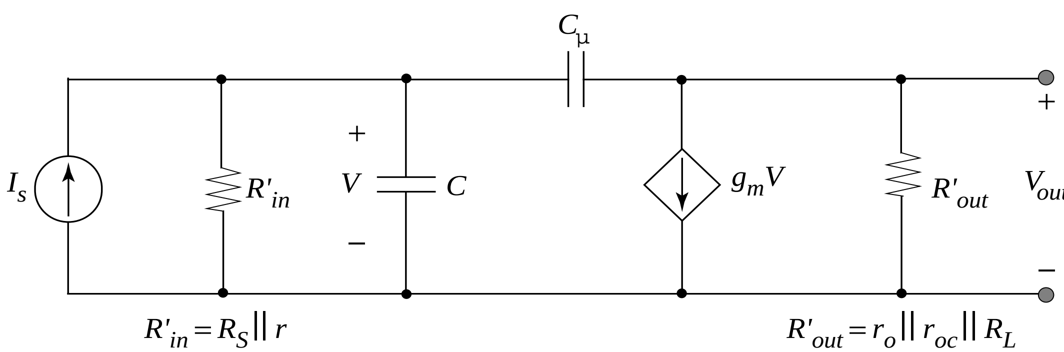


- Frequency Response
  - Set $V_{BIAS} = 0$.
  - Substitute BJT small signal model (with capacitors) including R_S, R_L, r_o, r_{oc}
  - Perform impedance analysis

1. Full Analysis of CE Voltage Amplifier



Replace voltage source and resistance with current source and resistance using Norton Equivalent



Node 1:

$$\mathbf{I_s = \frac{V_\pi}{R'_{in}} + j\omega C_\pi V_\pi + j\omega C_\mu (V_\pi - V_{out})}$$

Node 2:

$$\mathbf{g_m V_\pi + \frac{V_{out}}{R'_{out}} = j\omega C_\mu (V_\pi - V_{out})}$$

Full Frequency Response Analysis (contd.)

- Re-arrange **2** and obtain an expression for V_π
- Substituting it into **1** and with some manipulation, we can obtain an expression for V_{out} / I_s:

$$\frac{V_{out}}{I_s} = \frac{-R'_{in}R'_{out}\left(g_m - j\omega C_\mu\right)}{1 + j\omega\left(R'_{out}C_\mu + R'_{in}C_\mu + R'_{in}C_\pi + g_m R'_{out}R'_{in}C_\mu\right) - \omega^2 R'_{out}R'_{in}C_\mu C_\pi}$$

Changing input current source back to a voltage source:

$$\frac{V_{out}}{V_s} = \frac{-g_m R'_{out}\left(\dfrac{r_\pi}{R_S + r_\pi}\right)\left(1 - j\omega\dfrac{C_\mu}{g_m}\right)}{1 + j\omega\left(R'_{out}C_\mu + R'_{in}C_\mu\left(1 + g_m R'_{out}\right) + R'_{in}C_\pi\right) - \omega^2 R'_{out}R'_{in}C_\mu C_\pi}$$

where $R'_{in} = \mathbf{R_S} \,\|\, \mathbf{r_\pi}$ and $R'_{out} = \mathbf{r_o} \,\|\, \mathbf{r_{oc}} \,\|\, \mathbf{R_L}$

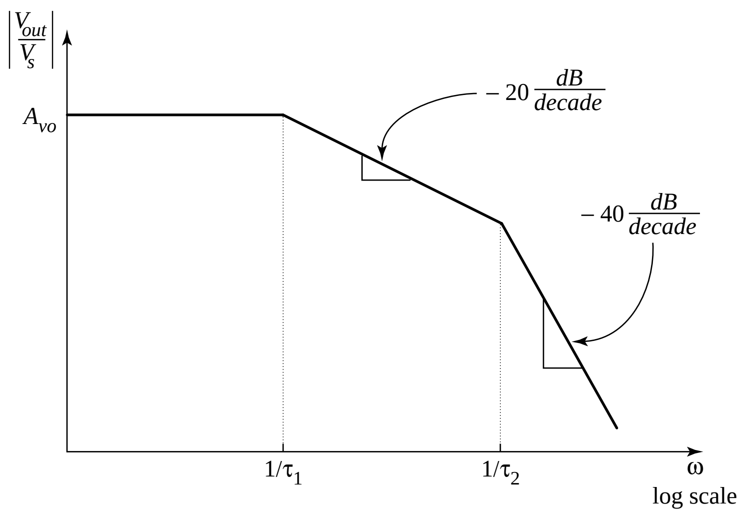
We can ignore zero at g_m/C_μ because it is higher than ω_T.

The gain can be expressed as:

$$\frac{V_{out}}{V_s} = \frac{A_{vo}}{(1 + j\omega\tau_1)(1 + j\omega\tau_2)} = \frac{A_{vo}}{1 - j\omega(\tau_1 + \tau_2) - \omega^2\tau_1\tau_2}$$

where A_{vo} is the gain at low frequency and τ_1 and τ_2 are the two time constants associated with the capacitors

Denominator of the System Transfer Function



$$\tau_1 + \tau_2 = R'_{out} C_\mu + R'_{in} C_\mu \left(1 + g_m R'_{out}\right) + R'_{in} C_\pi$$

$$\tau_1 \bullet \tau_2 = R'_{out} R'_{in} C_\mu C_\pi$$

We could solve for τ_1 and τ_2 but is algebraically complex.

- However, if we assume that $\tau_1 >> \tau_2$,$\Rightarrow$ $\tau_1 + \tau_2 \approx \tau_1$.
- This is a conservative estimate since the *true* τ_1 is actually smaller and hence the *true* bandwidth is actually larger than:

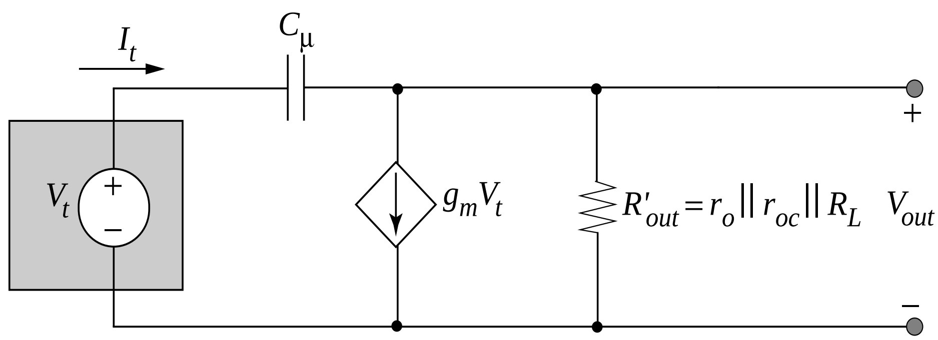
$$\tau_1 \approx R'_{in} \left[C_\pi + C_\mu \left(1 + g_m R'_{out}\right)\right] + R'_{out} C_\mu$$

Then:

$$\boxed{\omega_{3dB} = \frac{1}{\tau_1} = \frac{1}{R'_{in} \left[C_\pi + C_\mu \left(1 + g_m R'_{out}\right)\right] + R'_{out} C_\mu}}$$

2. The Miller Approximation

Effect of C_μ on the Input Impedance:



The input impedance Z_i is determined by applying a test voltage V_t to the input and measuring I_t:

$$V_{out} = -g_m V_t R'_{out} + I_t R'_{out}$$

The Miller Approximation assumes that current through C_μ is small compared to the transconductance generator

$$I_t << |g_m V_t|$$

$$V_{out} \approx -g_m V_t R'_{out}$$

We can relate V_t and V_{out} by

$$V_t - V_{out} = \frac{I_t}{j\omega C_\mu}$$

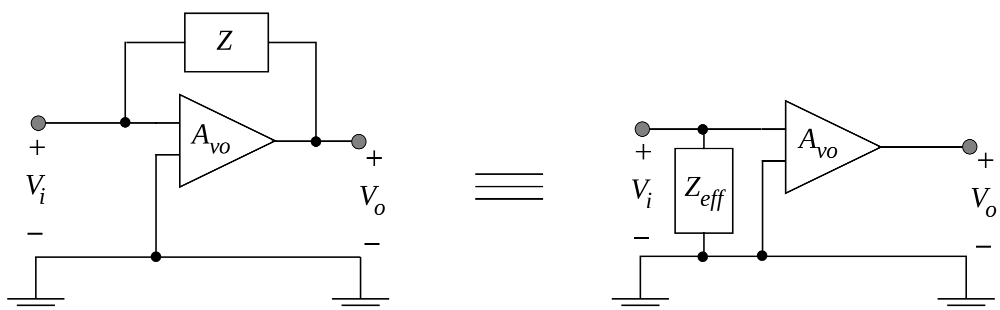
The Miller Approximation (contd.)

After some Algebra:

$$\frac{V_t}{I_t} = Z_{eff} = \frac{1}{j\omega C_\mu \left(1 + g_m R'_{out}\right)} = \frac{1}{j\omega C_\mu \left(1 - A_{vC_\mu}\right)}$$

The effect of C_μ at input is that C_μ is "Miller multiplied" by $(1-A_{vC\mu})$

Generalized "Miller Effect"

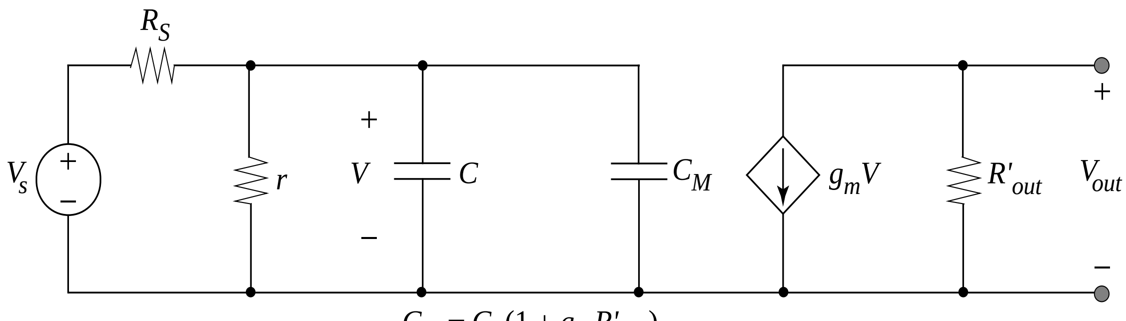


$Z_{eff} = Z/(1 - A_{vo})$

- An impedance connected across an amplifier with voltage gain A_{vo} can be replaced by an an impedance to ground … divided by $(1-A_{vo})$
- A_{vo} is large and negative for common-emitter and common-source amplifiers
- Capacitance at input is magnified.

$$Z_{eff} = \frac{Z}{\left(1 - A_{vo}\right)}$$

Frequency Response of the CE Voltage Amplifier Using Miller Approximation



- The Miller capacitance is lumped together with C_π, which results in a single pole low pass filter at the input

$$\frac{V_{out}}{V_s} = -g_m\left(\frac{r_\pi}{r_\pi + R_S}\right)R'_{out}\left[\frac{1}{1 + j\omega(C_\pi + C_M)(R_S \parallel r_\pi)}\right]$$

- At low frequency (DC) the small signal voltage gain is

$$\frac{V_{out}}{V_s} = -g_m\left(\frac{r_\pi}{r_\pi + R_S}\right)R'_{out}$$

- The frequency at which the magnitude of the voltage gain is reduced by $1/\sqrt{2}$ is

$$\omega_{3dB} = \frac{1}{(R_s \parallel r_\pi)(C_\pi + C_M)} = \left[\frac{1}{(R_s \parallel r_\pi)}\right]\left[\frac{1}{C_\pi + (1 + g_m R'_{out})C_\mu}\right]$$

3. Open Circuit Time Constant Analysis

Assumptions:

- No zeros
- One "dominant" pole ($1/\tau_1 << 1/\tau_2$, $1/\tau_3$ … $1/\tau_n$)
- N capacitors

$$\frac{V_{out}}{V_s} = \frac{A_{vo}}{(1 + j\omega\tau_1)(1 + j\omega\tau_2)(1 + j\omega\tau_n)}$$

The example shows a voltage gain; however, it could be I_{out}/V_s or V_{out}/I_s.

Multiplying out the denominator:

$$\frac{V_{out}}{V_s} = \frac{A_{vo}}{1 + b_1(j\omega) + b_2(j\omega)^2 + \ldots + b_n(j\omega)^n}$$

where $b_1 = \tau_1 + \tau_2 + \tau_3 + \ldots + \tau_n$

It can be shown that the coefficient b_1 can be found exactly [see Gray & Meyer, 3rd Edition, pp. 502-506]

$$b_1 = \left(\sum_{i=1}^{N} R_{Ti} C_i \right) = \left(\sum_{i}^{N} \tau_{C_{io}} \right)$$

- τ_{Cio} is the open-circuit time constant for capacitor C_i
- C_i is the i^{th} capacitor and R_{Ti} is the Thevenin resistance across the i^{th} capacitor terminals (with all capacitors open-circuited)

Open Circuit Time Constant Analysis

Estimating the Dominant Pole

The dominant pole of the system can be estimated by:

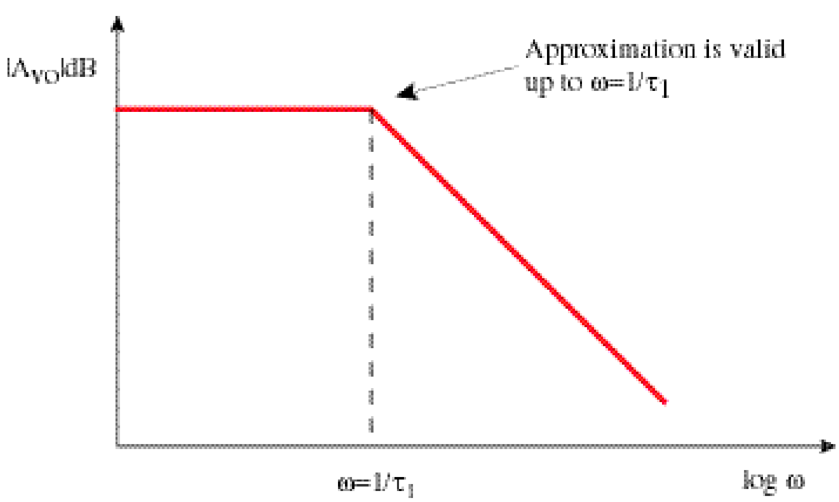
$b_1 = \tau_1 + \tau_2 + \tau_3 + \ldots + \tau_n$

$$\boldsymbol{b}_1 = \left(\sum_{\boldsymbol{i}=1}^{\boldsymbol{N}} \boldsymbol{R}_{\boldsymbol{Ti}} \boldsymbol{C}_{\boldsymbol{i}} \right) \approx \tau_1 = \frac{1}{\omega_1}$$

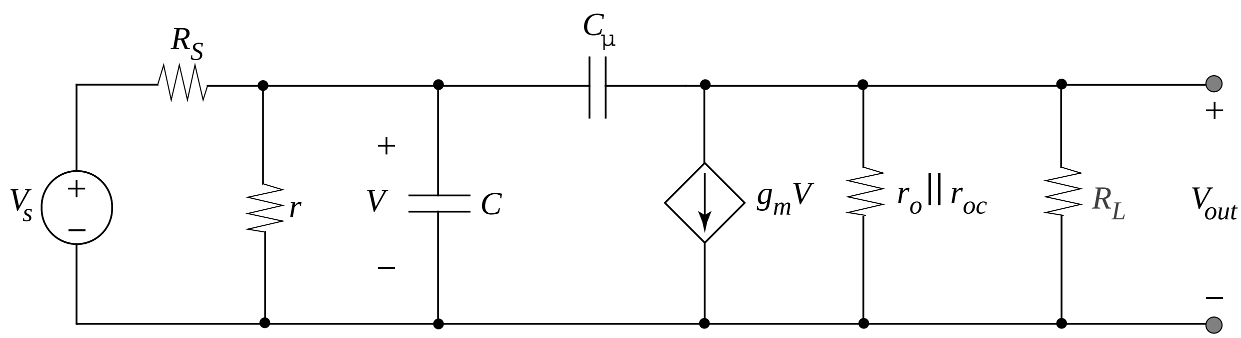
$R_{Ti}C_i$ is the **open-circuit time constant** for capacitor C_i

Power of the Technique:

- Estimates the contribution of each capacitor to the dominant pole frequency separately
- Enables the designer to understand what part of a complicated circuit is responsible for limiting the bandwidth of amplifier
- The approximate magnitude of the Bode Plot is



Common Emitter Amplifier Analysis Using OCT



From the Full Analysis

$$\frac{V_{out}}{V_s} = \frac{-g_m R'_{out}\left(\frac{r_\pi}{R_S + r_\pi}\right)\left(1 - j\omega\frac{C_\mu}{g_m}\right)}{1 + j\omega\left(R'_{out}C_\mu + R'_{in}C_\mu(1 + g_m R'_{out}) + R'_{in}C_\pi\right) - \omega^2 R'_{out}R'_{in}C_\mu C_\pi}$$

where $R'_{in} = R_S \,||\, r_\pi$ and $R'_{out} = r_o \,||\, r_{oc} \,||\, R_L$

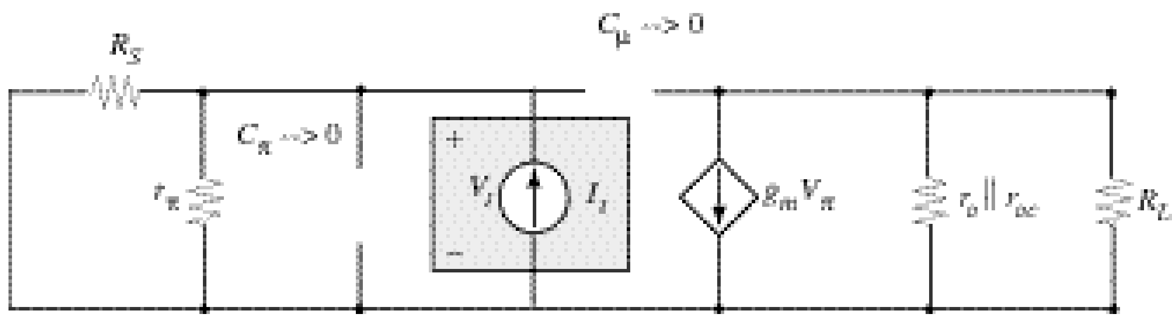
$$b_1 = R'_{out}C_\mu + R'_{in}C_\mu(1 + g_m R'_{out}) + R'_{in}C_\pi$$

$$\omega_{3dB} \approx \frac{1}{b_1} = \frac{1}{R'_{out}C_\mu + R'_{in}C_\mu(1 + +g_m R'_{out}) + R'_{in}C_\pi}$$

Common Emitter Amplifier Analysis Using OCT—Procedure

1. Eliminate all independent sources [e.g. $V_s \rightarrow 0$]
2. Open-circuit all capacitors
3. Find the Thevenin resistance by applying i_t and measuring v_t.

Time Constant for C_π



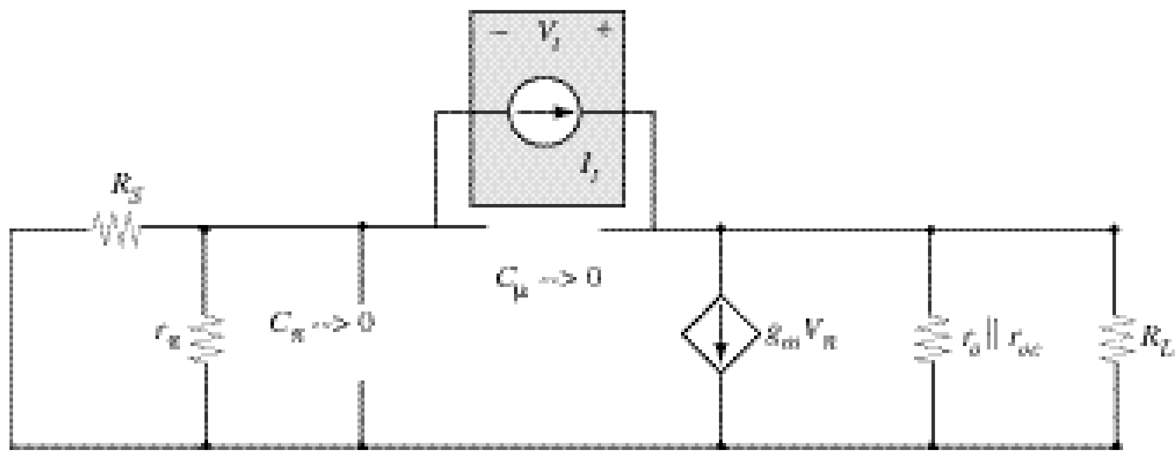
Result obtained by inspection

$$R_{T\pi} = R_S \| r_\pi$$

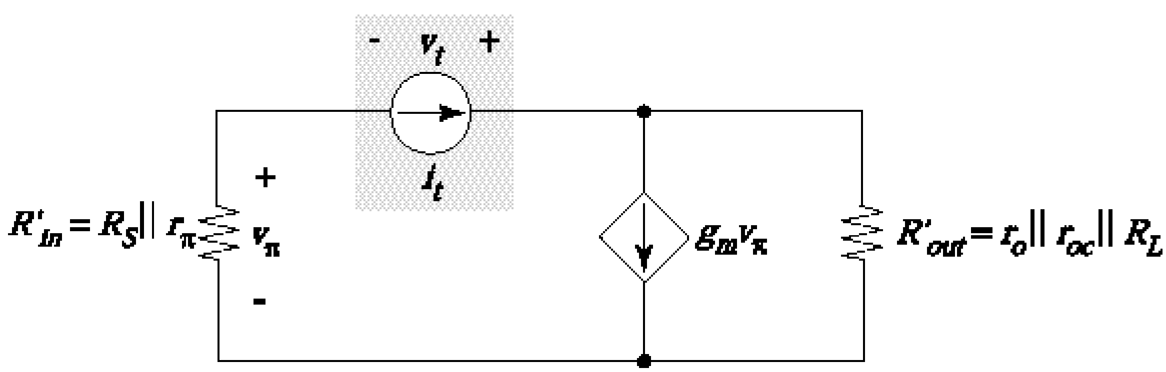
$$\tau_{C_{\pi o}} = R_{T\pi} C_\pi$$

Common Emitter Amplifier Analysis Using OCT—Time Constant for C_μ

Using the same procedure



Let $R'_{in} = \boldsymbol{R_S} \| \boldsymbol{r_\pi}$ and $R'_{out} = \boldsymbol{r_o} \| \boldsymbol{r_{oc}} \| \boldsymbol{R_L}$



$$-i_t = \frac{v_\pi}{R'_{in}} \qquad i_t = \frac{v_t + v_\pi}{R'_{out}} + g_m v_\pi$$

Eliminate v_π:

$$\frac{v_t}{i_t} = R_{T\mu} = R'_{out} + R'_{in}\left(1 + g_m R'_{out}\right)$$

$$\tau_{C_{\mu o}} = R_{T\mu} C_\mu = \left[R'_{out} + R'_{in}\left(1 + g_m R'_{out}\right)\right] C_\mu$$

Common Emitter Amplifier Analysis Using OCT—Dominant Pole

Summing individual time constants

$$b_1 = R_{T\pi}C_\pi + R_{T\mu}C_\mu$$

$$b_1 = R'_{out}C_\mu + R'_{in}C_\mu\left(1 + g_m R'_{out}\right) + R'_{in}C_\pi$$

Assume $\tau_1 >> \tau_2$

$$b_1 = \tau_1 + \tau_2 \approx \tau_1$$

$$b_1 = R'_{out}C_\mu + R'_{in}C_\mu\left(1 + g_m R'_{out}\right) + R'_{in}C_\pi$$

$$\omega_{3dB} \approx \frac{1}{b_1} = \frac{1}{R'_{out}C_\mu + R'_{in}C_\mu\left(1 + g_m R'_{out}\right) + R'_{in}C_\pi}$$

This result is very similar to the Miller Effect calculation
Additional term $R'_{out}C_\mu$ taken into account

Compare the Three Methods of Analyzing the Frequency Response of CE Amplifier

Full Analysis—

$$\omega_{3dB} \approx \frac{1}{\tau_1} = \frac{1}{R'_{out}C_\mu + R'_{in}C_\mu\left(1 + g_m R'_{out}\right) + R'_{in}C_\pi}$$

Miller Approximation—

$$\omega_{3dB} = \left[\frac{1}{R'_{in}}\right]\left[\frac{1}{C_\pi + \left(1 + g_m R'_{out}\right)C_\mu}\right]$$

Open Circuit Time Constant—

$$\omega_{3dB} \approx \frac{1}{b_1} = \frac{1}{R'_{out}C_\mu + R'_{in}C_\mu\left(1 + g_m R'_{out}\right) + R'_{in}C_\pi}$$

What did we learn today?

Summary of Key Concepts

- Full Analysis
  - Assumes that $\tau_1 + \tau_2 \approx \tau_1$

$$\omega_{3dB} \approx \frac{1}{\tau_1} = \frac{1}{R'_{out}C_\mu + R'_{in}C_\mu\left(1 + g_m R'_{out}\right) + R'_{in}C_\pi}$$

- Miller Approximation
  - Does not take into account R'_{out}

$$\omega_{3dB} = \left[\frac{1}{R'_{in}}\right]\left[\frac{1}{C_\pi + \left(1 + g_m R'_{out}\right)C_\mu}\right]$$

- Open Circuit Time Constant (OCT)
  - Assumes a dominant pole as full analysis

$$\omega_{3dB} \approx \frac{1}{b_1} = \frac{1}{R'_{out}C_\mu + R'_{in}C_\mu\left(1 + g_m R'_{out}\right) + R'_{in}C_\pi}$$