Lecture 10

MOSFET (III)

MOSFET Equivalent Circuit Models

Outline

- Low-frequency small-signal equivalent circuit model
- High-frequency small-signal equivalent circuit model

Reading Assignment:
Howe and Sodini; Chapter 4, Sections 4.5-4.6

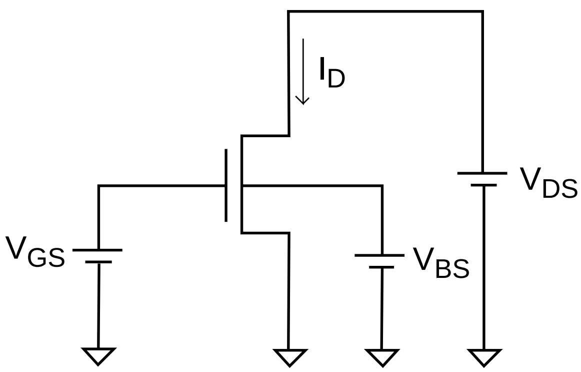
Announcements:

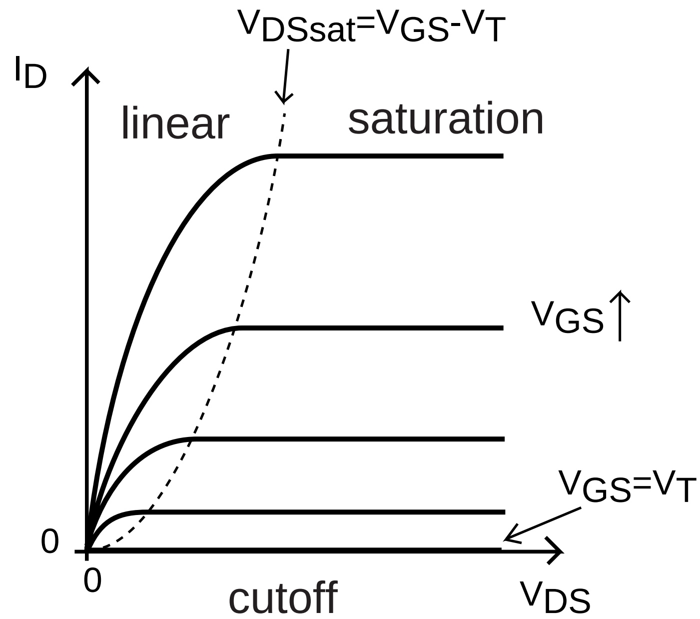
1. Quiz#1: March 14, 7:30-9:30PM, Walker Memorial; covers Lectures #1-9; open book; **<u>must have calculator</u>**

- No Recitation on Wednesday, March 14: instructors or TA's available in their offices during recitation times

Large Signal Model for NMOS Transistor

Regimes of operation:





- Cut-off

$$\mathbf{I_D = 0}$$

- Linear / Triode:

$$\mathbf{I_D} = \frac{\mathbf{W}}{\mathbf{L}} \mu_n \mathbf{C_{ox}} \left[\mathbf{V_{GS}} - \frac{\mathbf{V_{DS}}}{2} - \mathbf{V_T} \right] \bullet \mathbf{V_{DS}}$$

- Saturation

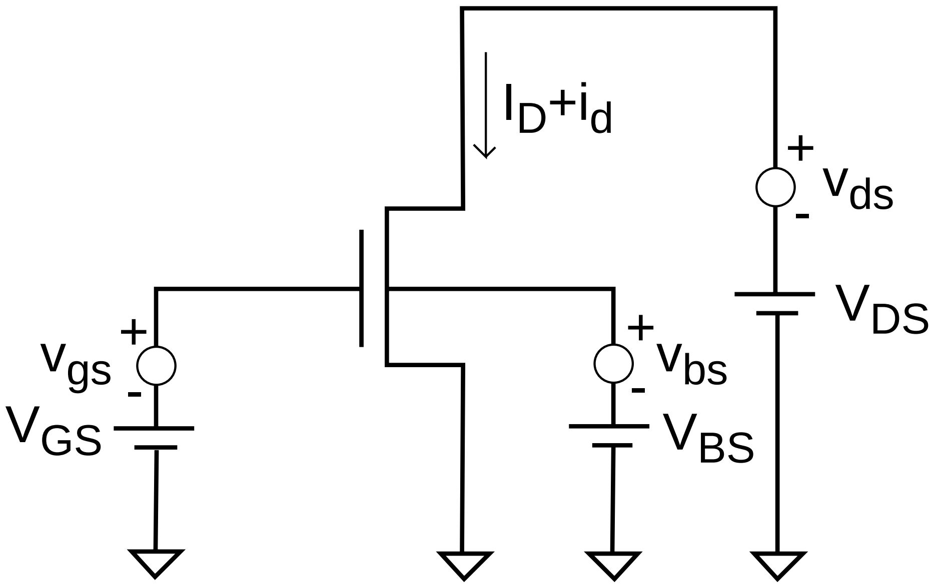
$$I_D = I_{Dsat} = \frac{W}{2L} \mu_n C_{ox} \left[V_{GS} - V_T \right]^2 \bullet \left[1 + \lambda V_{DS} \right]$$

Effect of back bias

$$\mathbf{V_T(V_{BS}) = V_{To}} + \gamma \left[\sqrt{-2\phi_p - \mathbf{V_{BS}}} - \sqrt{-2\phi_p} \right]$$

Small-signal device modeling

In many applications, we are only interested in the response of the device to a ***small-signal*** applied on top of a bias.



Key Points:

- Small-signal is ***small***
  - $\Rightarrow$ response of non-linear components becomes linear
- Since response is linear, lots of linear circuit techniques such as superposition can be used to determine the circuit response.
- Notation: $i_D = I_D + i_d$ ---Total = DC + Small Signal

Mathematically:

$$i_D(V_{GS}, V_{DS}, V_{BS}; v_{gs}, v_{ds}, v_{bs}) \approx$$

$$I_D(V_{GS}, V_{DS}, V_{BS}) + i_d(v_{gs}, v_{ds}, v_{bs})$$

With i_d linear on small-signal drives:

$$i_d = g_m v_{gs} + g_o v_{ds} + g_{mb} v_{bs}$$

Define:

$g_m \equiv$ ***transconductance*** [S]
$g_o \equiv$ ***output or drain conductance*** [S]
$g_{mb} \equiv$ ***backgate transconductance*** [S]

Approach to computing g_m, g_o, and g_{mb}.

$$\mathbf{g_m} \approx \left.\frac{\partial \mathbf{i_D}}{\partial \mathbf{v_{GS}}}\right|_{\mathbf{Q}}$$

$$\mathbf{g_o} \approx \left.\frac{\partial \mathbf{i_D}}{\partial \mathbf{v_{DS}}}\right|_{\mathbf{Q}}$$

$$\mathbf{g_{mb}} \approx \left.\frac{\partial \mathbf{i_D}}{\partial \mathbf{v_{BS}}}\right|_{\mathbf{Q}}$$

$$\mathbf{Q} \equiv [\mathbf{v_{GS} = V_{GS}, v_{DS} = V_{DS}, v_{BS} = V_{BS}}]$$

Transconductance

In saturation regime:

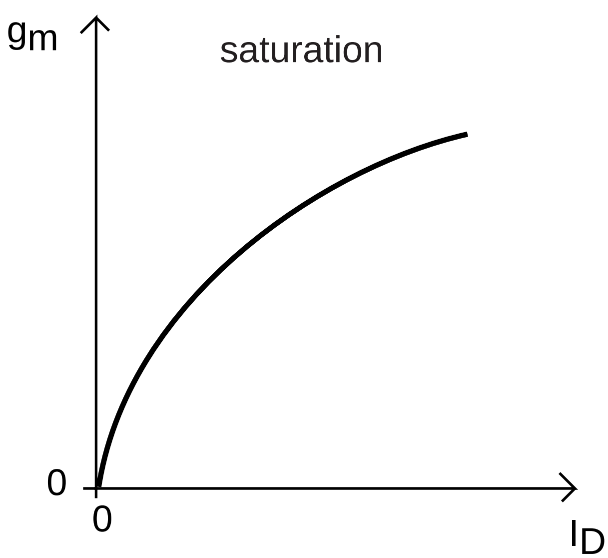
$$i_D = \frac{W}{2L}\mu_n C_{ox}\left[v_{GS} - V_T\right]^2 \bullet \left[1 + \lambda V_{DS}\right]$$

Then (neglecting channel length modulation) the transconductance is:

$$g_m = \left.\frac{\partial i_D}{\partial v_{GS}}\right|_Q \approx \frac{W}{L}\mu_n C_{ox}\left(V_{GS} - V_T\right)$$

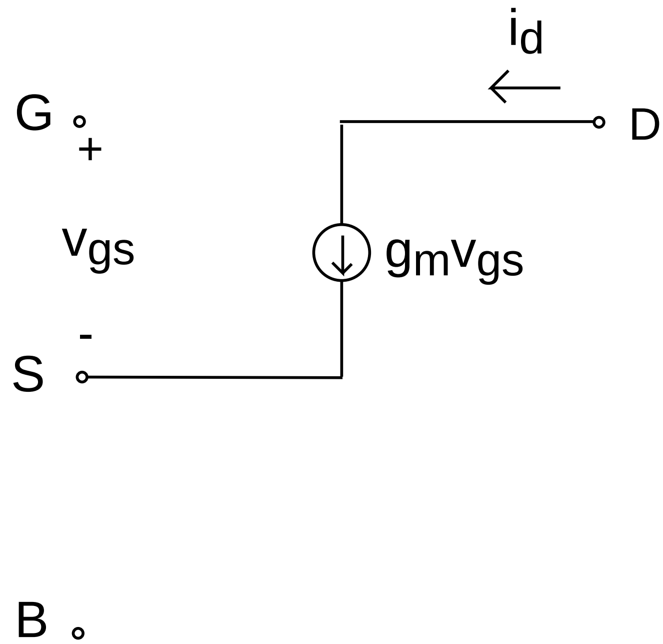
Rewrite in terms of I_D:

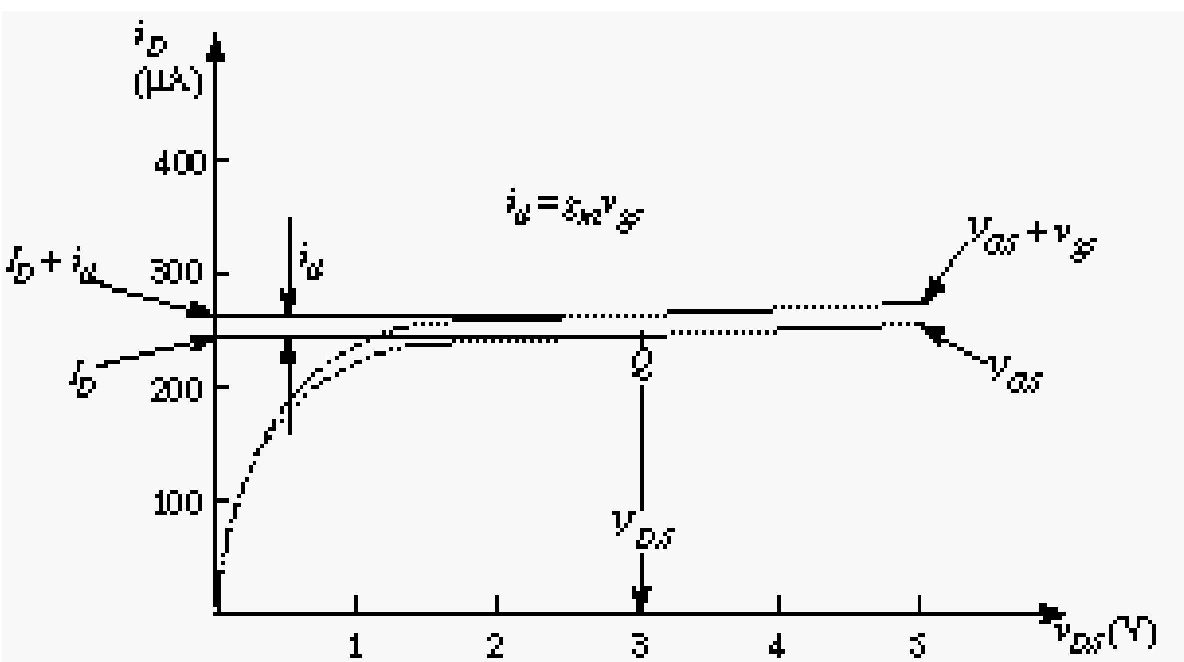
$$g_m = \sqrt{2\frac{W}{L}\mu_n C_{ox} I_D}$$



Transconductance (contd.)

Equivalent circuit model representation of g_m:





Output conductance

In saturation regime:

$$i_D = \frac{W}{2L}\mu_n C_{ox}\left[v_{GS} - V_T\right]^2 \bullet \left[1 + \lambda V_{DS}\right]$$

Then:

$$g_o = \left.\frac{\partial i_D}{\partial v_{DS}}\right|_Q = \frac{W}{2L}\mu_n C_{ox}\left(V_{GS} - V_T\right)^2 \bullet \lambda \approx \lambda I_D$$

Output resistance is the inverse of output conductance:

$$r_o = \frac{1}{g_o} = \frac{1}{\lambda I_D}$$

Remember also:

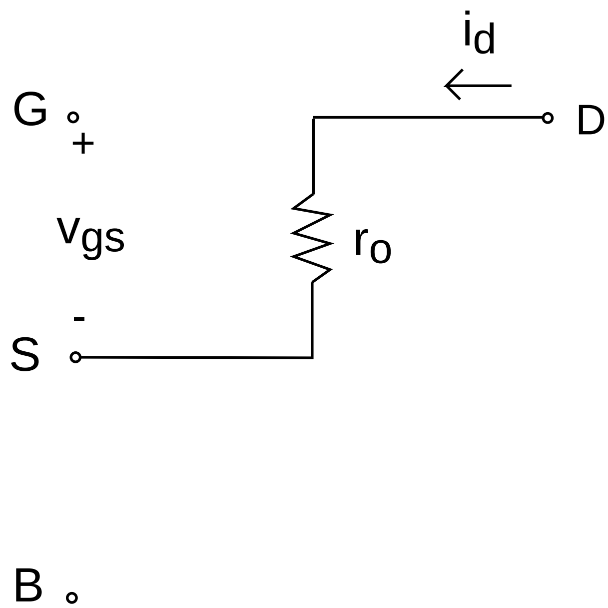
$$\lambda \propto \frac{1}{L}$$

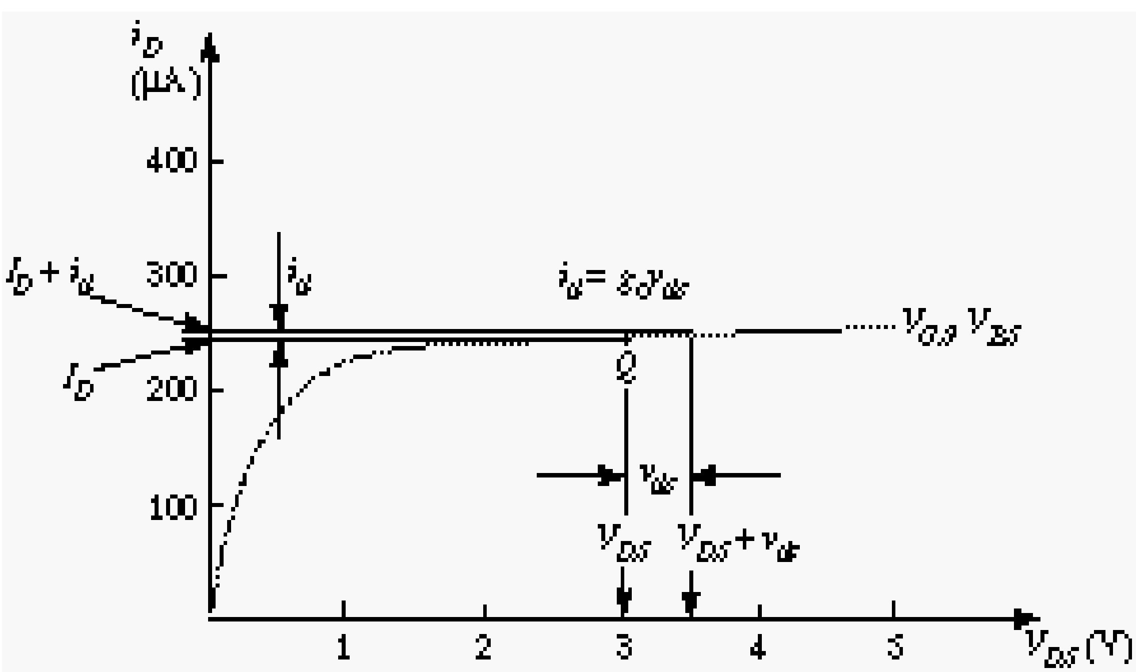
Hence:

$$r_o \propto L$$

Output conductance (contd.)

Equivalent circuit model representation of g_o:





Backgate transconductance

In saturation regime (neglect channel length modulation):

$$\mathbf{i_D} \approx \frac{\mathbf{W}}{2\mathbf{L}} \mu_\mathbf{n} \mathbf{C_{ox}} \left[\mathbf{v_{GS}} - \mathbf{V_T}\right]^2$$

Then:

$$\mathbf{g_{mb}} = \left.\frac{\partial \mathbf{i_D}}{\partial \mathbf{v_{BS}}}\right|_\mathbf{Q} = -\frac{\mathbf{W}}{\mathbf{L}} \mu_\mathbf{n} \mathbf{C_{ox}} (\mathbf{V_{GS}} - \mathbf{V_T}) \bullet \left(\left.\frac{\partial \mathbf{V_T}}{\partial \mathbf{v_{BS}}}\right|_\mathbf{Q} \right)$$

Since:

$$\mathbf{V_T}(\mathbf{v_{BS}}) = \mathbf{V_{To}} + \gamma \left[\sqrt{-2\phi_\mathbf{p} - \mathbf{v_{BS}}} - \sqrt{-2\phi_\mathbf{p}}\right]$$

Then :

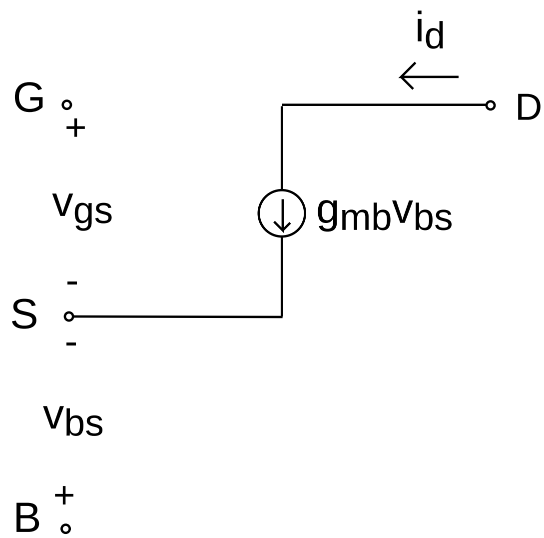
$$\left.\frac{\partial \mathbf{V_T}}{\partial \mathbf{v_{BS}}}\right|_\mathbf{Q} = \frac{-\gamma}{2\sqrt{-2\phi_\mathbf{p} - \mathbf{V_{BS}}}}$$

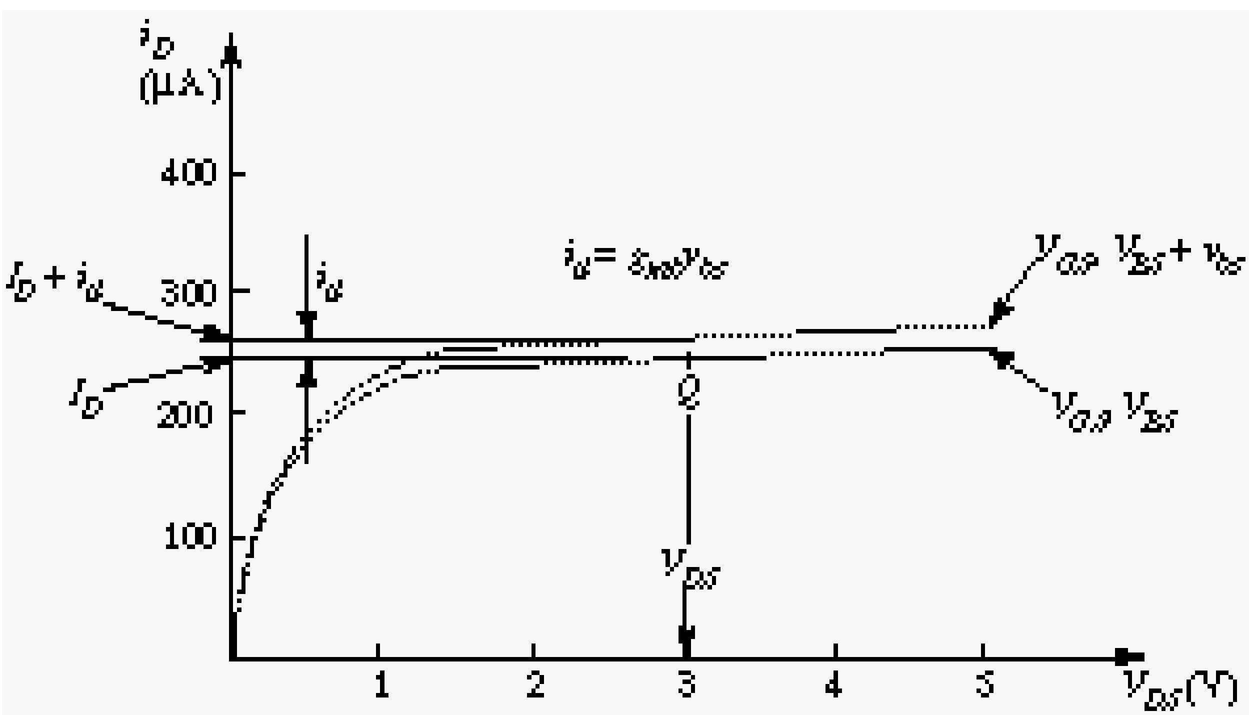
Hence:

$$\mathbf{g_{mb}} = \frac{\gamma \mathbf{g_m}}{2\sqrt{-2\phi_\mathbf{p} - \mathbf{V_{BS}}}}$$

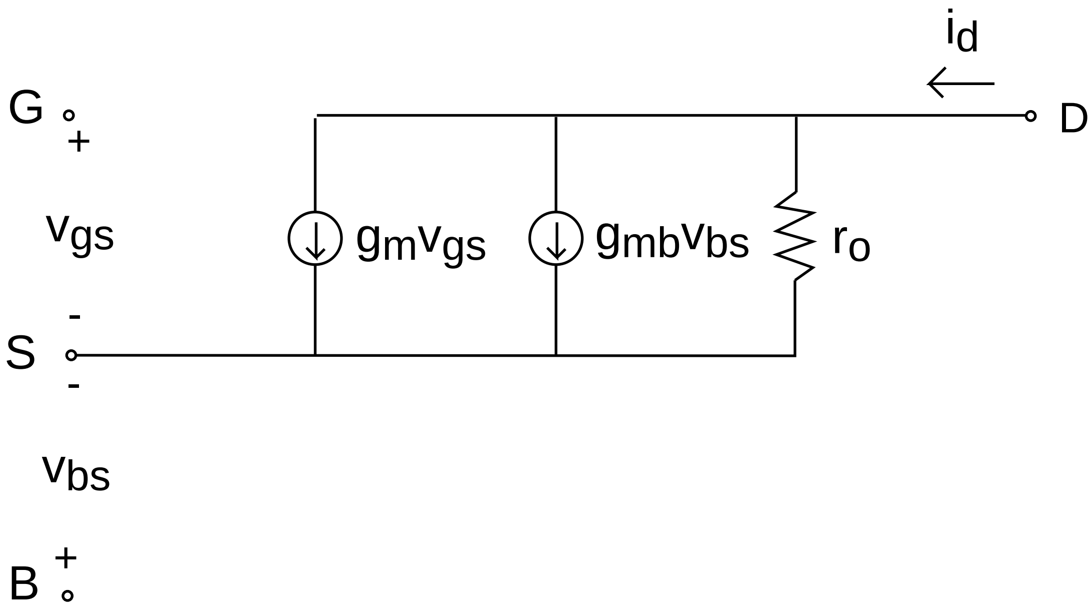
Backgate transconductance (contd.)

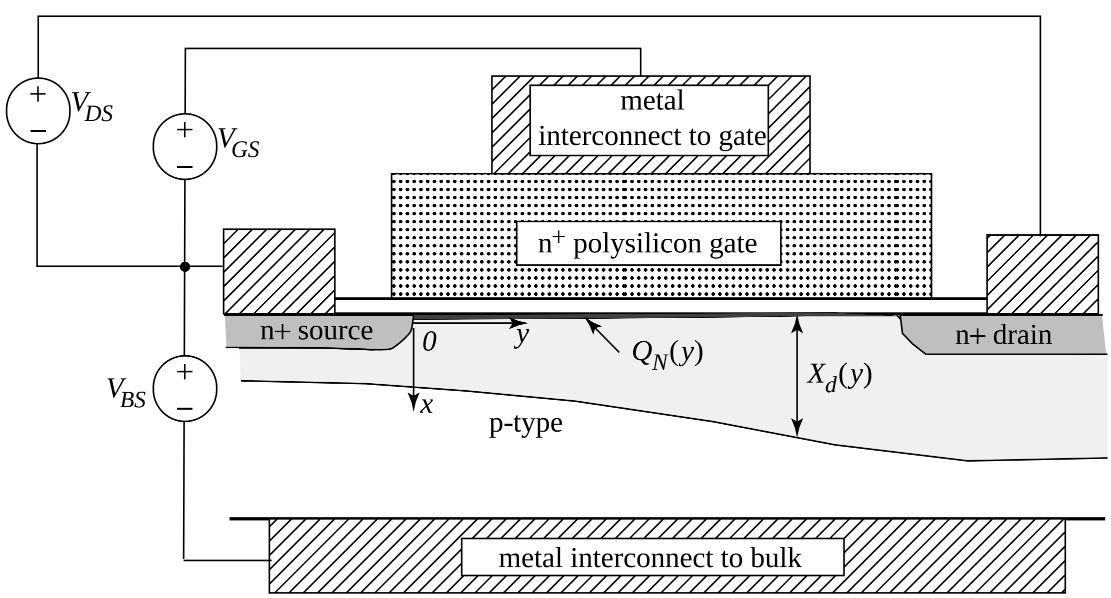
Equivalent circuit representation of g_{mb}:





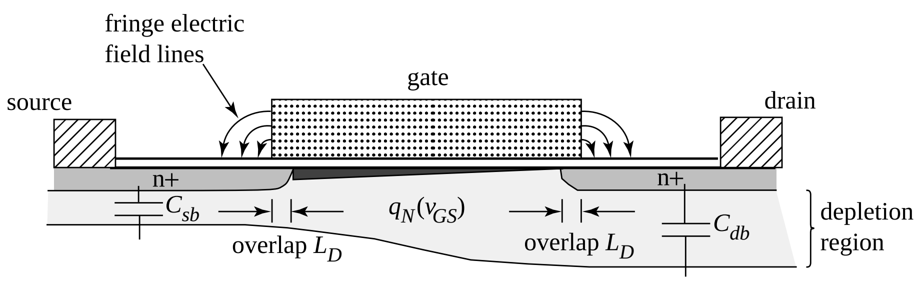
Complete MOSFET small-signal equivalent circuit model for low frequency:





2. High-frequency small-signal equivalent circuit model

Need to add capacitances. In saturation:



C_{gs} ≡ channel charge + overlap capacitance, C_{ov}
C_{gd} ≡ overlap capacitance, C_{ov}
C_{sb} ≡ source junction depletion capacitance (+sidewall)
C_{db} ≡ drain junction depletion capacitance (+sidewall)

ONLY Channel Charge Capacitance is intrinsic to device operation. All others are parasitic.

Inversion layer charge in saturation

$$q_N(v_{GS}) = W\int_0^L Q_N(y)\,dy = W\int_0^{v_{GS}-V_T} Q_N(v_C)\bullet\frac{dy}{dv_C}\bullet dv_C$$

Note that q_N is total inversion charge in the channel & $v_C(y)$ is the channel voltage. But:

$$\frac{dv_C}{dy} = -\frac{i_D}{W\mu_n Q_N(v_C)}$$

Then:

$$q_N(v_{GS}) = -\frac{W^2\mu_n}{i_D}\bullet\int_0^{V_{GS}-V_T}\left[Q_N(v_C)\right]^2\bullet dv_C$$

Remember:

$$Q_N(v_C) = -C_{ox}\left[v_{GS} - v_C(y) - V_T\right]$$

Then:

$$\boldsymbol{q_N(v_{GS}) = -\frac{W^2\mu_n}{i_D}\bullet\int_0^{v_{GS}-V_T}\left[v_{GS} - v_C(y) - V_T\right]^2\bullet dv_C}$$

Inversion layer charge in saturation (contd.)

Do integral, substitute i_D in saturation and get:

$$q_N(v_{GS}) = -\frac{2}{3} WLC_{ox}(v_{GS} - V_T)$$

Gate charge:

$$q_G(v_{GS}) = -q_N(v_{GS}) - Q_{B,\max}$$

Intrinsic gate-to-source capacitance:

$$C_{gs,i} = \frac{dq_G}{dv_{GS}} = \frac{2}{3} WLC_{ox}$$

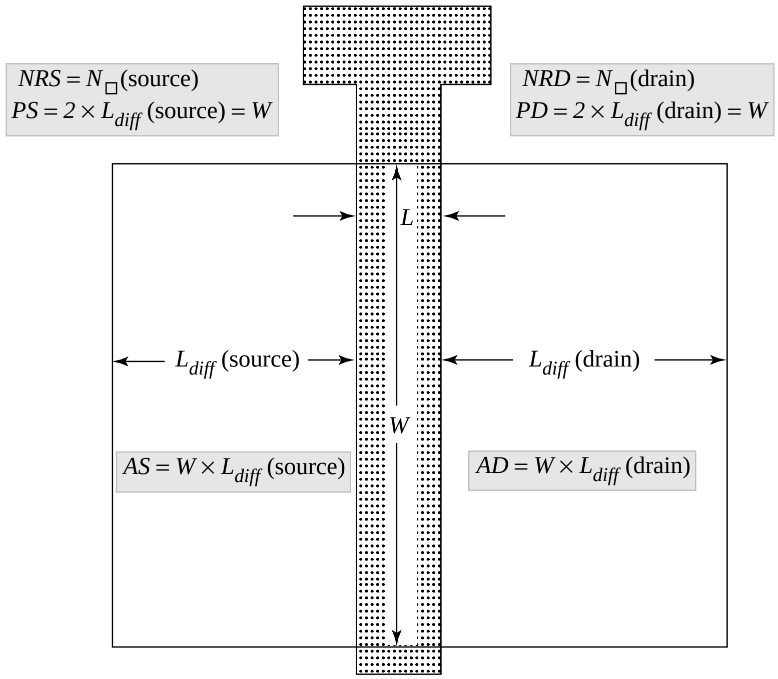
Must add overlap capacitance:

$$C_{gs} = \frac{2}{3} WLC_{ox} + WC_{ov}$$

Gate-to-drain capacitance — only overlap capacitance:

$$C_{gd} = WC_{ov}$$

Other capacitances



Source-to-Bulk capacitance:

$$C_{sb} = WL_{diff}C_j + (2L_{diff} + W)C_{jsw}$$

$$\text{where } C_j : \text{Bottom Wall at } V_{SB} (F/cm^2)$$

$$C_{jsw} : \text{Side Wall at } V_{SB} (F/cm)$$

Drain-to-Bulk capacitance:

$$C_{db} = WL_{diff}C_j + (2L_{diff} + W)C_{jsw}$$

$$\text{where } C_j : \text{Bottom Wall at } V_{DB} (F/cm^2)$$

$$C_{jsw} : \text{Side Wall at } V_{DB} (F/cm)$$

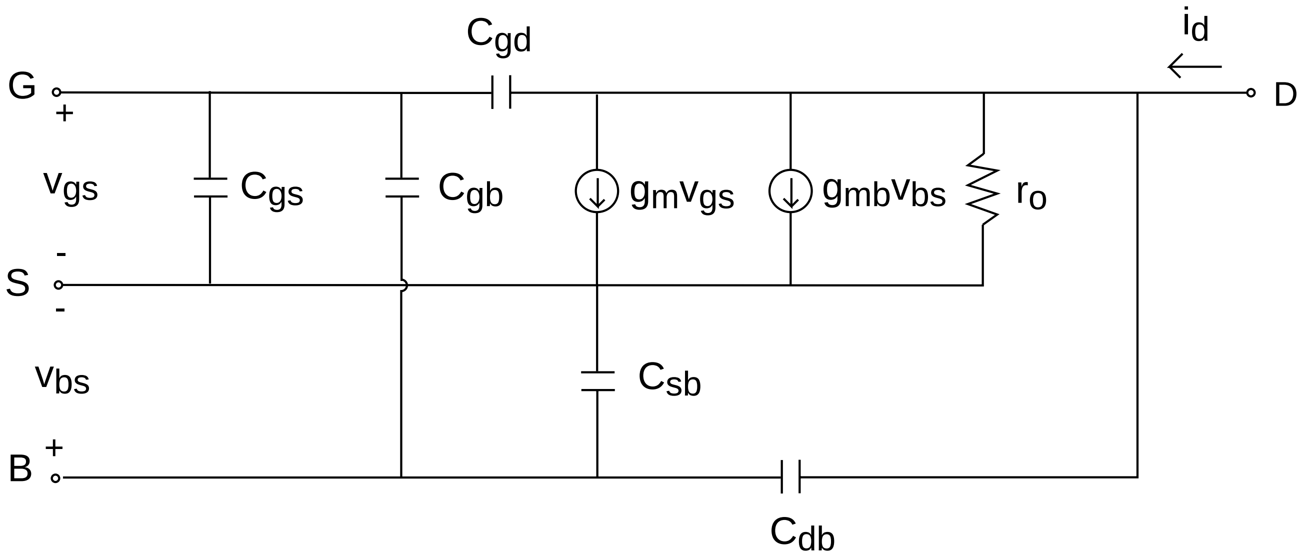
Gate-to-Bulk capacitance:

$C_{gb} \equiv$ small parasitic capacitance in most cases (ignore)

What did we learn today?

Summary of Key Concepts

High-frequency small-signal equivalent circuit model of MOSFET



In saturation:

$$g_m \propto \sqrt{\frac{W}{L} I_D}$$

$$r_o \propto \frac{L}{I_D}$$

$$C_{gs} \propto WLC_{ox}$$