MIT 18.06 Exam 1, Spring 2022
Johnson

Your name: ______________________________

Recitation: ______________________________

problem	**score**
1	/26
2	/24
3	/25
4	/25
total	/100

Problem 0 (∞ points): Honor code

Copy the following statement with your signature into your solutions:

> *I have completed this exam* ***closed-book/closed-notes*** *entirely on my* ***own.***
>
> [your signature]

Problem 1 (26 points):

Suppose

$$A = \begin{pmatrix} 1 & 2 & 1 & 2 \\ 2 & 4 & 2 & 5 \\ 1 & 2 & 1 & 1 \end{pmatrix}.$$

(a) Give a basis for $N(A)$.

(b) For what value or values (if any) of α does $Ax = \begin{pmatrix} 1 \\ 2\alpha \\ \alpha \end{pmatrix}$ have any solution x?

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Problem 2 (24 points):

Give a **basis** for the **nullspace** $N(A)$ and a basis for the **column space** $C(A)$ for each of the following matrices:

(a) The one-column matrix $A = \begin{pmatrix} 1 \\ 2 \\ 3 \\ 4 \end{pmatrix}$.

(b) The one-row matrix $A = \begin{pmatrix} 1 & 2 & 3 & 4 \end{pmatrix}$.

(c) The 100-row matrix $A = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 1 & 2 & 3 & 4 \\ \vdots & \vdots & \vdots & \vdots \\ 1 & 2 & 3 & 4 \end{pmatrix}$ in which every row is $\begin{pmatrix} 1 & 2 & 3 & 4 \end{pmatrix}$.

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Problem 3 (25 points):

Suppose that we are solving $Ax = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$. In each of the parts below, a **complete** solution x is proposed. For each possibility, say **impossible** if that could *not* be a *complete* solution to such an equation, **or** give the the **size** $m \times n$ and the **rank** of the matrix A if x is possible.

(a) $\vec{x} = \begin{pmatrix} 1 \\ 2 \\ 3 \\ 4 \end{pmatrix}$

(b) $\vec{x} = \begin{pmatrix} 1 \\ 2 \\ 3 \\ 4 \end{pmatrix} + \alpha_1 \begin{pmatrix} 1 \\ -1 \\ 5 \\ 17 \end{pmatrix} + \alpha_2 \begin{pmatrix} 1 \\ 0 \\ 0 \\ 1 \end{pmatrix}$ for all real numbers $\alpha_1, \alpha_2 \in \mathbb{R}$

(c) $\vec{x} = \begin{pmatrix} 1 \\ 2 \end{pmatrix} + \alpha \begin{pmatrix} 1 \\ 2 \end{pmatrix}$ for all real numbers $\alpha \in \mathbb{R}$

(d) $\vec{x} = \begin{pmatrix} 1 \\ 2 \end{pmatrix} + \alpha \begin{pmatrix} 1 \\ -1 \end{pmatrix}$ for all real numbers $\alpha \in \mathbb{R}$

(e) $\vec{x} = \begin{pmatrix} 1 \\ 2 \end{pmatrix} + \alpha_1 \begin{pmatrix} 1 \\ -1 \end{pmatrix} + \alpha_2 \begin{pmatrix} 1 \\ -1 \end{pmatrix}$ for all real numbers $\alpha_1, \alpha_2 \in \mathbb{R}$

Problem 4 (25 points):

Let

$$B = \begin{pmatrix} 1 & & \\ 1 & 1 & \\ 1 & 1 & 1 \end{pmatrix}, \qquad C = \begin{pmatrix} 2 & -1 & -1 \\ & 2 & -1 \\ & & 2 \end{pmatrix}, \qquad b = \begin{pmatrix} 5 \\ -8 \\ -4 \end{pmatrix}.$$

Compute:

$$(CB)^{-1}b.$$

(Hint: Remember what I said in class about inverting matrices!)

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