18.06 Professor Edelman Quiz 1 February 28, 2020

Your easy to read printed name is: ______________________

(We need your name on every page for gradescope.)

(Exam ends at 11:55am.)

Please circle your recitation:

(1)	T 10	36-155	Yau Wing Li
(2)	T 10	36-153	Sung Woo Jeong
(3)	T 11	36-153	Sung Woo Jeon
(4)	T 12	2-146	Yau Wing Li
(5)	T 12	2-136	James Tao
(6)	T 1	2-136	James Tao
(7)	T 1	2-142	Kai Huang
(8)	T 2	2-136	Kai Huang
(9)	T 3	2-136	Yu Pan

Important Instructions: We will be using Gradescope. Please write on one side only of a page. If you need extra pages, please write continued, and on the extra pages clearly label with problem number and letter.

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1 (20 pts.)

For each set below, decide if it is or is not a vector space. Explain briefly why or why not.

1. (a) (4 pts.) All 10×2 tall-skinny orthogonal matrices.

1. (b) (4 pts.) All polynomials in x that are 0 at $x = 1806$ and $x = 2000$.

1. (c) (4 pts.) All $(n+1) \times n$ matrices of the form $\begin{pmatrix} 0 & 0 & \cdots & 0 \\ v_1 & 0 & \cdots & 0 \\ 0 & v_2 & \ldots & 0 \\ \vdots & \vdots & \cdots & \vdots \\ 0 & 0 & \cdots & v_n \end{pmatrix}$.

1. (d) (4 pts.) All functions $f(x)$ of the form $c_1 e^x + c_2 e^{-x}$, where c_1 and c_2 are real scalars.

1. (e) (4 pts.) All 5×5 symmetric matrices A (meaning $A = A^T$).

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2 (15 pts.)

A researcher measures the temperature at n points in the plane: $(x_1, y_1), (x_2, y_2), \ldots (x_n, y_n)$. The temperatures measured are $f_1, f_2, \ldots, f_n$ respectively. This researcher wants to find a best set of coefficients a, b, c, d, e, g to fit a function of the form

$$f(x, y) = ax^2 + bxy + cy^2 + dx + ey + g$$

to the data.

2. (a) (8 pts.) Set up an equation of the form $Ax \approx b$ that represents this researcher's problem.

2. (b) (4 pts.) Suppose the matrix A in the above can be written as $A = QR$, where Q is tall-skinny orthogonl and R is invertible and upper triangular. What are the dimensions of Q? and the dimensions of R?

2. (c) (3 pts.) Write the solution to the best set of coefficients in terms of possibly Q,Q^T,R, or R^{-1} and the given temperatures.

3 (15 pts.)

How many parameters are needed? We are looking for the minimum required to specify the object. Briefly explain.

3. (a) (5 pts.) The "one cold" vector has $(n-1)$ elements 1 and the remaining one 0. How many parameters are needed to represent a "one cold" vector when n is not fixed in advance?

3. (b) (5 pts.) How many parameters are required to represent a rank-1 two by two matrix? (Possible hint: it may be easier to see the correct answer with the svd, though this problem can be done without the svd if you think carefully.)

3. (c) (5 pts.) An anti-symmetric matrix is one where $A^T = -A$. How many parameters are required to represent a 4 x 4 anti-symmetric matrix?

4 (10 pts.)

A square matrix A has first column and last column all ones. Why can't it have an inverse?

5 (20 pts.)

The rank-r SVD of

$$A = \begin{pmatrix} 1 & 3 & 1 \\ 3 & 8 & 2 \\ 5 & 12 & 2 \end{pmatrix}$$

is numerically computed with Julia to be $A = U\Sigma V^T$, where

$$U = \begin{pmatrix} -0.203600 & -0.585801 \\ -0.543021 & -0.599144 \\ -0.814662 & 0.545769 \end{pmatrix}$$

$$\Sigma = \begin{pmatrix} 6.136942826453964 & \\ & 0.7740001393771697 \end{pmatrix}$$

$$V = \begin{pmatrix} -0.365991 & 0.446524 \\ -0.912869 & -0.00172137 \\ -0.180887 & -0.89477 \end{pmatrix}$$

5. (a) (5 pts.) What is the rank of A?

5. (b) (6 pts.) A linearly transforms the unit sphere $\{x : \|x\| = 1\}$ into a filled ellipse in a plane, not an ellipsoid. What are the lengths of the semi-axes of this ellipse?

5. (c) (9 pts.) Circle the (chopped) numbers in U, Σ, V below that would figure in the best rank-1 approximation to A.

$$U = \begin{pmatrix} -0.2036 & -0.5858 \\ -0.5430 & -0.5991 \\ -0.8147 & 0.5458 \end{pmatrix} \quad \Sigma = \begin{pmatrix} 6.1369 & \\ & 0.7740 \end{pmatrix} \quad V = \begin{pmatrix} -0.3659 & 0.4465 \\ -0.9129 & -0.0017 \\ -0.1809 & -0.8948 \end{pmatrix}$$

6 (20 pts.)

The matrix $E = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 2 & 1 & 0 & 0 \\ 3 & 0 & 1 & 0 \\ 4 & 0 & 0 & 1 \end{pmatrix}$ and the matrix $F = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 5 & 1 & 0 \\ 0 & 6 & 0 & 1 \end{pmatrix}$.

Ideally without working too hard, calculate $E^{-1}F^{-1}$.

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