18.06 Professor Edelman Quiz 3 May 3, 2019

Your name is: ______________________

(We need your name on every page for gradescope.)

(Note pencil can create some problems showing up in gradescope.)

Please circle your recitation:

(1)	T 10	24-307	S. Makarova
(2)	T 10	4-261	Z. Remscrim
(3)	T 11	24-307	S. Makarova
(4)	T 11	4-261	C. Hewett
(5)	T 12	4-261	C. Hewett
(6)	T 12	2-105	A. Ahn
(7)	T 1	4-149	A. Ahn
(8)	T 1	2-136	S. Turton
(9)	T 2	2-136	K. Choi
(10)	T 3	2-136	K. Choi

Important Instructions: We will be using Gradescope. Please write on one side only of a page. If you need extra pages, please write continued, and on the extra pages clearly label with problem number and letter. Note: we deleted the boxes seen on earlier tests.

Name: ______________________________

1 (25 pts.)

The matrix S has a full SVD computed with Julia, where

$$S = \begin{pmatrix} 4 & 3 & 1 & 3 \\ 3 & 2 & 1 & 1 \\ 1 & 1 & 0 & 2 \\ 3 & 1 & 2 & 0 \end{pmatrix}.$$

There are exactly three (positive) singular values:

$$\sigma_1 = 7.9487708193188125 \; \sigma_2 = 2.5425353720085857 \; \sigma_3 = 0.593764552689772.$$

The singular vectors are the columns of

$$U = \begin{pmatrix} -0.735907 & 0.34763 & 0.065278 & 0.57735 \\ -0.481518 & -0.179604 & 0.634468 & -0.57735 \\ -0.254389 & 0.527234 & -0.56919 & -0.57735 \\ -0.402328 & -0.754268 & -0.518856 & 0 \end{pmatrix} \text{ and}$$

$$V = \begin{pmatrix} -0.735907 & -0.34763 & 0.065278 & 0.57735 \\ -0.481518 & 0.179604 & 0.634468 & -0.57735 \\ -0.254389 & -0.527234 & -0.56919 & -0.57735 \\ -0.402328 & 0.754268 & -0.518856 & 0 \end{pmatrix}.$$

1.(a) (5 pts.) What is the sum of the eigenvalues of S?

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1. (b) (5 pts.) What is the product of the eigenvalues of S? (Hint: This is easy if you look at the problem correctly and time consuming if you do not.)

1. (c) (10 pts.) Ideally without much computation, what is the fourth column of the cofactor matrix of S?

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1. (d) (5 pts.) Circle the four eigenvalues of S and justify briefly each of your choices:

$$\sigma_1, \quad \sigma_2, \quad \sigma_3, \quad -\sigma_1, \quad -\sigma_2, \quad -\sigma_3, \quad 1, \quad -1, \quad 0, \quad \sigma_1^2, \quad \sigma_2^2, \quad \sigma_3^2.$$

(Checking Suggestion: compare with your answer in 1.(a).)

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2 (25 pts.)

2. (a) (6 pts.)

A matrix has all of its eigenvalues 1 . The set of matrices similar is

i)infinite ii)possibly finite iii)must be finite. (Pick and justify the best answer)

2. (b) (6 pts.)

A matrix has all of its eigenvalues real and has a real orthogonal eigenvector matrix. True or false, this matrix must be symmetric. (Pick and justify the best answer.)

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2. (c) (6 pts.)

The trace of the n by n reflection matrix $I - 2\frac{xx^T}{x^Tx}$ is

i) n ii) n+1 iii) n-1 iv) n-2. (Pick and justify the best answer.)

2. (d) (7 pts.)

A projection matrix is symmetric and satisfies $P^2 = P$. The possible eigenvalues of $\exp(P)$ are

i) 0 and 1.

ii) -1,0, and 1

iii) 0 and e

iv) 1 and e. (where e=exp(1)).

(Pick and justify the best answer)

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3 (10 pts.)

Suppose that a matrix A is not the zero matrix but A^4 is the zero matrix in 3(a) and 3(b).

3. (a) (5 pts.) The possible eigenvalues of A are

i) 0 and 1

ii) all 0

iii) 1,-1,$i, -i$

iv) 0,1,-1

(Pick and justify the best answer.)

3. (b) (5 pts.) The matrix A is diagonalizable. (Pick the best answer, no justification being asked for. Just right or wrong.)

i. True because the zero matrix is diagonalizable.

ii. True because A has distinct eigenvalues.

iii. False because A does not have distinct eigenvalues.

iv. False because if A were diagonalizable, and $A^4 = 0$ then A would have to be the zero matrix.

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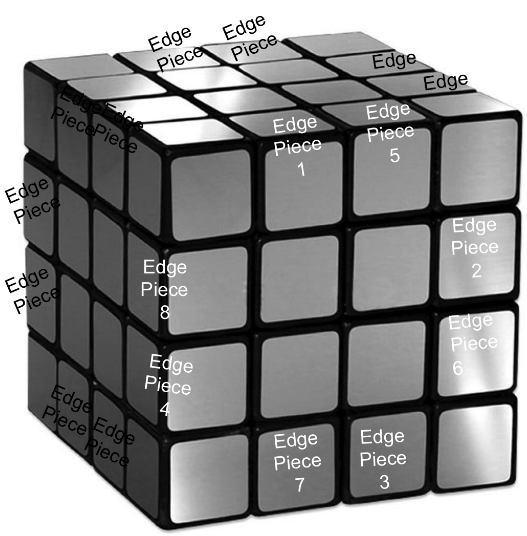
4 (20 pts.)

It is possible to buy a 4x4x4 Rubik's cube. This puzzle has 24 solid cubes called an edge piece. An edge piece has two colored stickers exposed. In the figure edge piece 1 and also edge piece 5 has a sticker on the top face and another sticker on the front face. The edge pieces on the front face are labeled 1 through 8 and are in white font.

4. (a) (10 pts) Write an 8x8 permutation matrix that describes the permutation of the 8 front face edge pieces when the front face is rotated clockwise 90 degrees. Label the exact meaning of P_{ij}.



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4. (b) (10 pts) Consider the 24x24 permutation matrix that describes the permutation of all 24 edge pieces on the puzzle. Suppose, we make 2019 random quarter turns of faces around the puzzle. What is the determinant of this matrix? Explain your reasoning. (Hint: be sure to account for the eight pieces that move with each quarter turn of a face.)

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5 (20 pts.)

The unit simplex in n dimensions is the following set: $\{x \in R^n$ with $0 \leq x_i \leq 1$ and $x_1 + x_2 + \ldots + x_n \leq 1\}$ This set has volume (area, etc.) $1/n!$. Suppose an $n \times n$ matrix A has determinant $d > 0$.

5. (a) (10 pts.). Consider the image of the simplex when we multiply by A. This is the set $\{Ax \in R^n$with $0 \leq x_i \leq 1$ and $x_1 + x_2 + \ldots + x_n \leq 1\}$. What is the volume (area, etc.) of this set?

5. (b) (10 pts.). Consider the image of the simplex when we multiply by A^{-1}. This is the set $\{A^{-1}x \in R^n$ with $0 \leq x_i \leq 1$ and $x_1 + x_2 + \ldots + x_n \leq 1\}$. What is the volume (area, etc.) of this set?

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5 (c). (Extra Credit 5 pts.) This problem is only worth five points. Some of you may see the answer right away, but others may not see it at all. We do not recommend looking at this problem unless you have extra time, as the five points may not be worth the time lost.

Suppose the eigenvalues of A in problem 5 are $\lambda_1, \ldots, \lambda_n$. What is the volume (area, etc.) of the image of the simplex under the map $A^2 + I$, that is the set $\{(A^2 + I)x$ for $x \in R^n$ with $0 \leq x_i \leq 1$ and $x_1 + x_2 + \ldots + x_n \leq 1\}$. Write your answer as an absolute value of an expression in terms of the eigenvalues. Explain briefly your answer.

Extra Page. Please write problem number and letter if needed.

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