18.06 Professor Edelman Quiz 2 April 5, 2019

Your name is: ______________________________

(We need your name on every page for gradescope.)

(Note pencil creates some problems showing up in gradescope.)

Please circle your recitation:

(1)	T 10	24-307	S. Makarova
(2)	T 10	4-261	Z. Remscrim
(3)	T 11	24-307	S. Makarova
(4)	T 11	4-261	C. Hewett
(5)	T 12	4-261	C. Hewett
(6)	T 12	2-105	A. Ahn
(7)	T 1	4-149	A. Ahn
(8)	T 1	2-136	S. Turton
(9)	T 2	2-136	K. Choi
(10)	T 3	2-136	K. Choi

Important Instructions: We will be using Gradescope. Please write on one side only of a page. If you need extra pages, please write continued in the box, and on the extra pages clearly label with problem number and letter.

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1 (25 pts.)

A random 4x3 matrix A has a full SVD computed with Julia. The singular values are

$$2.067989079857846,\ 0.9964025831888096,\ 0.4854455453874191.$$

The singular vectors are the columns of

$$U = \begin{pmatrix} -0.534606 & 0.697017 & 0.396747 & 0.266373 \\ -0.324715 & -0.691464 & 0.539027 & 0.354805 \\ -0.650464 & -0.156256 & -0.10825 & -0.735365 \\ -0.430874 & -0.107832 & -0.735066 & 0.512247 \end{pmatrix} \text{ and}$$

$$V = \begin{pmatrix} -0.391685 & 0.466488 & 0.793077 \\ -0.729853 & 0.367332 & -0.576525 \\ -0.560265 & -0.804647 & 0.196589 \end{pmatrix}.$$

1.(a) (5 pts.) Is A^TA invertible? Why or why not?

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1. (b) (5 pts.) Find and circle a vector perpendicular to every column of A.

$$U = \begin{pmatrix} -0.534606 & 0.697017 & 0.396747 & 0.266373 \\ -0.324715 & -0.691464 & 0.539027 & 0.354805 \\ -0.650464 & -0.156256 & -0.10825 & -0.735365 \\ -0.430874 & -0.107832 & -0.735066 & 0.512247 \end{pmatrix}$$

$$V = \begin{pmatrix} -0.391685 & 0.466488 & 0.793077 \\ -0.729853 & 0.367332 & -0.576525 \\ -0.560265 & -0.804647 & 0.196589 \end{pmatrix}$$

1. (c) (5 pts.) Is AA^T invertible? Why or why not?

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1. (d) (5 pts.) How many solutions to $Ax = b$ are there for a randomly generated b such as

$$b = \begin{pmatrix} 1.2616743877482997 \\ -0.6492300349356348 \\ -1.8681658666758472 \\ -1.6717363181989333 \end{pmatrix}.$$ Explain briefly.

1. (e) (5 pts.) What is the dimension of the orthogonal complement of the row space of A? Explain briefly.

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2 (20 pts.)

2. (a) (10 pts.) Suppose $A = QR$ is the QR factorization of A into orthogonal times upper triangular. It is possible to write $\pm \det(A)$ as an expression in terms of the entries of R. What is this expression?

2. (b) (10 pts.) A fact that you can assume is that a reflector has determinant -1. Suppose you are told that Q can be written as the product of exactly 17 different reflectors, what can we say exactly about $\det(Q)$?

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3 (25 pts.) Am I a linear transformation? Briefly explain why or why not.

a. $x \to x^T A x$ where $x \in R^n$ and A is a fixed $n \times n$ matrix.

b. $A \to x^T A x$ where A is an $n \times n$ matrix and $x \in R^n$ is a fixed vector.

c. $P(x) \to P'(x)$ where $P(x)$ is a polynomial.

d. $A \to \det(A)$ where A is an $n \times n$ matrix.

e. $x \to \text{prod}(x)$ where $x \in R^n$ (and prod means compute the product of the entries in x.)

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4 (15 pts.) The $3x3$ upper triangular matrices U form a six dimensional vector space. So do the symmetric $3x3$ matrices S.

4. (a) (5 pts.) How many parameters are needed to specify a linear transformation from the $3x3$ upper triangular matrices to the symmetric $3x3$ matrices?

4. (b) (10 pts.) Give two different examples of linear transformations (other than the zero transformation) from the 3×3 upper triangular matrices to the symmetric $3x3$ matrices.

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5 (15pts.)

Consider the nonlinear matrix function $f(A) = A^T A$. It is possible to write df as a linear transformation of dA. What is that linear transformation?

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6. (Extra Credit 5 pts.) This problem is only worth five points. Some of you may see the answer right away, but others may not see it at all. We do not recommend looking at this problem unless you have extra time, as the five points may not be worth the time lost.

We have two matrices A and B:

$$A = \begin{pmatrix} 1 & 0 & 0 \\ a & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \text{ and } B = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & b & 1 \end{pmatrix}.$$

Write the products AB and BA without mechanical operations. For the five points, explain briefly your answer with words not mechanics. Graders will give points subjectively for really good expositions only.

Extra Page. Please write problem number and letter if needed.

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