Practice final exam 18.06. Spring 2016

Problem 1

Let A be an $n \times n$ matrix. True or False:

- A is invertible if and only if $Ker(A) = 0$
- A is invertible if and only if the rows of A span $\mathbb{R}^n$
- If A is similar to $2A$, then $A = 0$
- Similar matrices have the same set of eigenvalues
- If all the eigenvalues of A are zero, then $A = 0$
- The rank of A is equal to the number of nonzero eigenvalues of A counted with multiplicity

Problem 2

Consider the system of equations

$$\begin{cases} x + 3y + 5z = a \\ x + 2y + 2z = b \\ x + \ \ y - \ \ z = c \end{cases}$$

- Find the general solution of the homogeneous equation.
- Let $a = 0$, $b = 0$, and $c = -2$. Find the most general solution of these inhomogeneous equations.
- Find values of a, b, and c such that these equations have no solution.

Problem 3

For what values of k is the matrix

$$A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 2 & k \\ 0 & 0 & k \end{pmatrix}$$

diagonalizable over $\mathbb{R}$, over $\mathbb{C}$?

Problem 4

Compute the determinant of the following matrix

$$A = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 2 & 3 & 4 & 5 \\ 3 & 3 & 3 & 4 & 5 \\ 4 & 4 & 4 & 4 & 5 \\ 5 & 5 & 5 & 5 & 5 \end{pmatrix}.$$

Problem 5

Let

$$A = \begin{pmatrix} 3 & 2-i & -3i & 4 \\ 2+i & 0 & 1-i & 3 \\ 3i & 1+i & 0 & 2 \\ 0 & 0 & 0 & 1 \end{pmatrix}.$$

- Find the eigenvalues and eigenspaces of A
- Find the determinant of $A^3 + 2A$

Problem 6

Find the projection of $v^T = (1, 2, 3)$ onto the coimage of

$$A = \begin{pmatrix} 1 & 1 & -2 \\ 1 & 2 & -3 \\ 0 & 1 & 1 \\ 0 & 0 & 2 \end{pmatrix}.$$