18.06 Final Exam

19 May 2016 at 9 AM

STATE YOUR NAME: ____

CIRCLE YOUR RECITATION:

R01	10–11	Sauer–Ayala
R02	10–11	Carpentier
R03	11–12	Sauer–Ayala
R04	11–12	Carpentier
R05	12–13	Hopkins
R06	12–13	Anno
R07	13–14	Hopkins
R08	13–14	Anno
R09	14–15	Fei
R10	14–15	Knizel
R11	15–16	Knizel

GRADING

1.	________	/20
2.	________	/20
3.	________	/20
4.	________	/20
5.	________	/20
6.	________	/20
7.	________	/20
8.	________	/20

TOTAL

/160

1. Clinton or Trump

For each of the following sentences, indicate whether they are true or false. (No need to justify your answer.)

(a) If A is an $n \times n$ matrix with characteristic polynomial $p_A(t) = t^n$, then $A = 0$.

(b) If A is a matrix, then any element of the kernel of A is perpindicular to any element of the image of $A^\intercal$.

(c) The only $m \times n$ matrix of rank 0 is 0.

(d) There is a orthogonal basis of $\mathbf{C}^3$ consisting of eigenvectors for the matrix

$$\begin{pmatrix} 17822 & -759i & -14795 + 69532i \\ 759i & 568347 & 385955 \\ -14795 - 69532i & 385955 & 10479 \end{pmatrix}.$$

(e) If

$$M = \begin{pmatrix} A & B \\ C & D \end{pmatrix}$$

is an $2n \times 2n$ matrix in which A, B, C, and D are all $n \times n$ blocks, then

$$\det M = (\det A)(\det D) - (\det B)(\det C).$$

2. Solve

Write a basis for the space of solutions to the system of linear equations

$$\begin{aligned} a + b + 2c + 4d + 7e &= 0; \\ a + 2b + 4c + 7d + 13e &= 0; \\ 2a + 4b + 7c + 13d + 24e &= 0; \\ 4a + 7b + 13c + 24d + 44e &= 0. \end{aligned}$$

3. Project

Compute the projection of the vector $\begin{pmatrix} 1 \\ 1 \\ 2 \\ 3 \end{pmatrix} \in \mathbf{R}^4$ onto the kernel of the matrix

$$\begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \\ -1 & 1 & -1 & 1 \\ -1 & -1 & -1 & -1 \end{pmatrix}.$$

4. Charlie Brown

Compute the inverse of the matrix

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 3 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ -4 & 5 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 3 & -9 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -4 & 1 \end{pmatrix}.$$

5. The Outer Limits

Everyone's favorite matrix is built like this: take a unit vector $\widehat{x} \in \mathbf{R}^n$, and set $P \coloneqq \widehat{x}\widehat{x}^\intercal$. In terms of $\widehat{x}$, describe the kernel of P.

What are the nonzero eigenvalues of P?

What are the corresponding eigenspaces?

6. Corny crony

Compute the characteristic polynomial of

$$\begin{pmatrix} 0 & 0 & 8 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 1 & 0 & -3 \\ 0 & 0 & 0 & 0 & 1 & -3 \end{pmatrix}.$$

7. Permute

Is the matrix

$$\begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix}$$

diagonalizable over **R**? over **C**?

8. You'll flip

Contemplate the following matrix

$$A = \begin{pmatrix} 5 & -1 & -1 \\ -1 & 5 & -1 \\ -1 & -1 & 5 \end{pmatrix}.$$

Before you compute anything, is this matrix diagonalizable over **R**? over **C**? How do you know?

Now compute the eigenvalues and eigenspaces of this matrix.