18.06 Exam II: The Examining

11 March 2016

NAME: _____

RECITATION: _____

GRADING	
1. _____	/20
2. _____	/20
3. _____	/20
4. _____	/20
5. _____	/20
TOTAL	/100

1. Yay or nay

For each of the following matrices, answer YES or NO: are they invertible? (You do *not* have to justify your answer.)

(a) $\begin{pmatrix} 1 & 4 \\ 2 & 5 \\ 3 & 6 \end{pmatrix}$

(b) $\begin{pmatrix} 5 & 1 \\ 25 & 5 \end{pmatrix}$

(c) $\begin{pmatrix} 1 & -1 \\ 6 & 5 \end{pmatrix}$

(d) $\begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}$

(e) $\begin{pmatrix} 1 & 0 & 6 \\ 0 & 7 & 8 \\ 0 & 0 & 3 \end{pmatrix}$

(f) $\begin{pmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \end{pmatrix}$

(g) $\begin{pmatrix} 1 & 2 & 3 & 4 \\ 0 & 5 & 6 & 7 \\ 0 & 0 & 0 & 8 \\ 0 & 0 & 0 & 9 \end{pmatrix}$

(h) $\begin{pmatrix} 1 & 3 & 5 & 7 & 9 \\ 2 & 7 & 12 & 17 & 22 \\ 0 & 0 & 3 & 0 & 0 \\ 0 & 0 & 4 & 16 & 0 \\ 0 & 0 & 5 & 25 & 45 \end{pmatrix}$

(i) $\begin{pmatrix} 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 \\ 2 & 2 & 2 & 1 & 1 & 2 \\ 2 & 2 & 2 & 1 & 2 & 3 \\ 2 & 2 & 2 & 2 & 3 & 5 \end{pmatrix}$

(j) $\begin{pmatrix} 0 & 1 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 & 1 & 0 \end{pmatrix}$

2. Solve

Find a basis for the space of solutions to the following system of linear equations in the seven variables $x_1, x_2, x_3, x_4, x_5, x_6, x_7$:

$$\begin{aligned} x_1 + x_2 + x_3 + x_4 &= 0 \\ x_2 + x_3 + x_4 + x_5 &= 0 \\ x_3 + x_4 + x_5 + x_6 &= 0 \\ x_4 + x_5 + x_6 + x_7 &= 0 \end{aligned}$$

3. Rank and file

Compute the rank of the following matrix:

$$\begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 3 & 4 & 5 \\ 3 & 4 & 5 & 6 \\ 4 & 5 & 6 & 7 \\ 5 & 6 & 7 & 8 \\ 6 & 7 & 8 & 9 \\ 7 & 8 & 9 & 10 \end{pmatrix}$$

4. Null at tea

Find a basis for the kernel of the following matrix:

$$\begin{pmatrix} 1 & 1 & 2 & 5 & 14 & 42 \\ 1 & 2 & 5 & 14 & 42 & 132 \\ 2 & 5 & 14 & 42 & 132 & 429 \end{pmatrix}$$

5. Inversion invasion

Compute the inverse of this matrix:

$$\begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 2 & 3 & 4 & 5 \\ 0 & 0 & 1 & 3 & 6 & 10 \\ 0 & 0 & 0 & 1 & 4 & 10 \\ 0 & 0 & 0 & 0 & 1 & 5 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$