

## 18.06 PROBLEM SET 9

Due Thursday, April 30, 2015, before 4:00 pm (sharp deadline) in Room E17-131

This homework has 5 questions to hand-in. Write down all details of your solutions, NOT JUST THE ANSWERS. Show your reasoning. Please staple the pages together and **clearly write your name**, your recitation section, and the name of your recitation instructor on the first page of the problem set.

Cooperation on problems is permitted, but all solutions must be written up independently and you must list your collaborators on the problem set. You should first try to solve each problem yourself, otherwise you will not learn much from hearing the solution.

**Problem 1.** Let  $\sigma_{\max}(A)$  be the largest singular value of a matrix  $A$ . Show that  $\sigma_{\max}(A^{-1})\sigma_{\max}(A) \geq 1$  for any square invertible matrix  $A$ .

**Problem 2.** Suppose  $A$  has orthogonal columns  $w_1, w_2, \dots, w_n$  of lengths  $\sigma_1, \sigma_2, \dots, \sigma_n$ . What are  $U$ ,  $\Sigma$ , and  $V$  in the SVD?

**Problem 3.** If  $A = QR$  with an orthogonal matrix  $Q$ , the SVD of  $A$  is almost the same as the SVD of  $R$ . Which of the three matrices  $U$ ,  $\Sigma$ ,  $V$  is changed because of  $Q$ ?

**Problem 4.** Let  $n > 1$ . Show that there is no  $n$  by  $n$  matrix  $A$  such that  $AM = M^T$  for every  $n$  by  $n$  matrix  $M$ .

**Problem 5.** Let  $V$  be a vector space and  $T : V \rightarrow V$  a linear transformation. Suppose that for every linear transformation  $S : V \rightarrow V$ , we have  $S(T(v)) = T(S(v))$  for all vectors  $v \in V$ . Show that there exists a scalar  $c$  such that  $T(v) = cv$  for all vectors  $v \in V$ .