18.06 Problem Set 8

Due Thursday, April 23, 2015, before 4:00 pm (sharp deadline) in Room E17-131

This homework has several questions to hand-in. Write down all details of your solutions, NOT JUST THE ANSWERS. Show your reasoning. Please staple the pages together and **clearly write your name**, your recitation section, and the name of your recitation instructor on the first page of the problem set.

Cooperation on problems is permitted, but all solutions must be written up independently and you must list your collaborators on the problem set. You should first try to solve each problem yourself, otherwise you will not learn much from hearing the solution.

This homework also has an online part and you can choose whether to do it (self-graded) on MITx with MATLAB or (human-graded) in Julia. Follow the instructions on course website.

Problem 1. Section 6.5, Problem 2, page 350.
Which of A_1, A_2, A_3, A_4 has two positive eigenvalues? Use the test, don't compute the λ's. Find an $\boldsymbol{x}$ so that $\boldsymbol{x}^T A_1 \boldsymbol{x} < 0$, so A_1 fails the test.

$$A_1 = \begin{bmatrix} 5 & 6 \\ 6 & 7 \end{bmatrix} \quad A_2 = \begin{bmatrix} -1 & -2 \\ -2 & -5 \end{bmatrix} \quad A_3 = \begin{bmatrix} 1 & 10 \\ 10 & 100 \end{bmatrix} \quad A_4 = \begin{bmatrix} 1 & 10 \\ 10 & 101 \end{bmatrix}$$

Problem 2. Section 6.5, Problem 20, page 352.
Give a quick reason why each of these statements is true:
(a) Every positive definite matrix is invertible.
(b) The only positive definite projection matrix is $P = I$.
(c) A diagonal matrix with positive diagonal entries is positive definite.
(d) A symmetric matrix with a positive determinant might not be positive definite!

Problem 3. Section 6.5, Problem 28, page 353.
Without multiplying $A = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} 2 & 0 \\ 0 & 5 \end{bmatrix} \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$, find
(a) the determinant of A
(b) the eigenvalues of A
(c) the eigenvectors of A
(d) a reason why A is symmetric positive definite.

Problem 4. Section 6.6, Problem 20, page 362.
Why are these statements all true?
(a) If A is similar to B then A^2 is similar to B^2.
(b) A^2 and B^2 can be similar when A and B are not similar (try $\lambda = 0, 0$).

(c) $\begin{bmatrix} 3 & 0 \\ 0 & 4 \end{bmatrix}$ is similar to $\begin{bmatrix} 3 & 1 \\ 0 & 4 \end{bmatrix}$
(d) $\begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$ is not similar to $\begin{bmatrix} 3 & 1 \\ 0 & 3 \end{bmatrix}$
(e) If we exchange rows 1 and 2 of A, and then exchange columns 1 and 2, **the eigenvalues stay the same**. In this case $M =$ ____________.