18.06 Problem Set 7

Due Thursday, April 16, 2015, before 4:00 pm (sharp deadline) in Room E17-131

This homework has 3 questions to hand-in. Write down all details of your solutions, NOT JUST THE ANSWERS. Show your reasoning. Please staple the pages together and **clearly write your name**, your recitation section, and the name of your recitation instructor on the first page of the problem set.

Cooperation on problems is permitted, but all solutions must be written up independently and you must list your collaborators on the problem set. You should first try to solve each problem yourself, otherwise you will not learn much from hearing the solution.

This homework also has an online part and you can choose whether to do it (self-graded) on MITx with MATLAB or (human-graded) in Julia. Follow the instructions on course website.

Problem 1. Consider the matrix

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}.$$

Find the eigenvectors, the eigenvalues, and the kth power of A. As a side remark, the matrix A is the adjacency matrix of the oriented graph below, so the (i, j) entry of the matrix A^k counts the number of k-step paths from i to j.
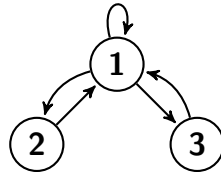
Problem 2. The matrix in this question is skew-symmetric ($A^T = -A$):

$$\frac{d\mathbf{u}}{dt} = \begin{bmatrix} 0 & c & -b \\ -c & 0 & a \\ b & -a & 0 \end{bmatrix} \mathbf{u} \quad \text{or} \quad \begin{matrix} u_1' = cu_2 - bu_3 \\ u_2' = au_3 - cu_1 \\ u_3' = bu_1 - au_2 \end{matrix}$$

- the derivative of $\|\mathbf{u}(t)\|^2 = u_1^2 + u_2^2 + u_3^2$ is $2u_1u_1' + 2u_2u_2' + 2u_3u_3'$. Substitute u_1', u_2', u_3' to obtain *zero*. This implies that $\|\mathbf{u}(t)\|^2$ stays equal to $\|\mathbf{u}(0)\|^2$.
- Show that for any skew symmetric matrix A, the exponential $Q = e^{At}$ is orthogonal. To do this, prove along the way that $Q^T = e^{-At}$.

Problem 3. Suppose B is a 2×2 with real entries such that $B^4 = I$. What are the possible pairs of eigenvalues (λ_1, λ_2) of B? Provide an example for each of them.