18.06 Problem Set 6

Due Thursday, April 2, 2015, before 4:00 pm (sharp deadline) in Room E17-131

This homework has 5 questions to hand-in. Write down all details of your solutions, NOT JUST THE ANSWERS. Show your reasoning. Please staple the pages together and **clearly write your name**, your recitation section, and the name of your recitation instructor on the first page of the problem set.

Cooperation on problems is permitted, but all solutions must be written up independently and you must list your collaborators on the problem set. You should first try to solve each problem yourself, otherwise you will not learn much from hearing the solution.

Problem 1. If the entries in every row of A add to zero, solve $Ax = 0$ to prove $\det A = 0$. If those entries add to one, show that $\det(A - I) = 0$. Does this mean $\det A = 1$?

Problem 2. The n by n determinant C_n has 1's above and below the main diagonal:

$$C_1 = |0|\,,\ C_2 = \begin{vmatrix} 0 & 1 \\ 1 & 0 \end{vmatrix},\ C_3 = \begin{vmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{vmatrix},\ C_4 = \begin{vmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{vmatrix}$$

(a) What are these determinants C_1, C_2, C_3, C_4?
(b) By cofactors find the relation between C_n and C_{n-1} and C_{n-2}. Find C_{10}.

Problem 3. Compute the determinants of

$$A = \begin{bmatrix} 1 & a & 0 & 0 \\ 0 & b & 0 & 0 \\ 0 & c & 1 & 0 \\ 0 & d & 0 & 1 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & a & a^2 & a^3 \\ a & 1 & a & a^2 \\ a^2 & a & 1 & a \\ a^3 & a^2 & a & 1 \end{bmatrix}$$

Problem 4. This problem shows in two ways that $\det A = 0$:

$$A = \begin{bmatrix} x_{11} & x_{12} & x_{13} & x_{14} & x_{15} \\ x_{21} & x_{22} & x_{23} & x_{24} & x_{25} \\ 0 & 0 & 0 & x_{34} & x_{35} \\ 0 & 0 & 0 & x_{44} & x_{45} \\ 0 & 0 & 0 & x_{54} & x_{55} \end{bmatrix}$$

(a) How do you know that the rows are linearly dependent?
(b) Explain why all 120 terms are zero in the big formula for $\det A$.

Problem 5. If $|\det(A)| > 1$, prove that the powers A^n cannot stay bounded. But if $|\det(A)| \leq 1$, show that some entries of A^n might still grow large. Eigenvalues will give the right test for stability, determinants tell us only one number.