18.06 Problem Set 3

Due Thursday, February 26, 2015, before 4:00 pm (sharp deadline) in Room E17-131

This homework has 4 questions to hand-in. Write down all details of your solutions, NOT JUST THE ANSWERS. Show your reasoning. Please staple the pages together and **clearly write your name**, your recitation section, and the name of your recitation instructor on the first page of the problem set.

Cooperation on problems is permitted, but all solutions must be written up independently and you must list your collaborators on the problem set. You should first try to solve each problem yourself, otherwise you will not learn much from hearing the solution.

This homework also has an online part and you can choose whether to do it (self-graded) on MITx with MATLAB or (human-graded) in Julia. Follow the instructions on course website.

Problem 1. Suppose an unknown matrix has these special solutions in its nullspace:

$$\mathbf{s_1} = (1, 0, 5, 0, 1) \quad \mathbf{s_2} = (2, 0, 2, 1, 0) \quad \mathbf{s_3} = (-1, 2, 0, 0, 0).$$

(a) Find the row-reduced echelon matrix R with exactly these special solutions;
(b) Find one more matrix with exactly those special solutions.
(c) Any sequence of row operations has the effect of multiplying A by a ___ matrix. So how can you find all possible matrices with those 3 special solutions?

Problem 2 (The Counting Theorem)**.** Show that if A is a $m \times n$ matrix, we have

$$(\#\text{pivot columns of } A) + (\#\text{special solutions in the nullspace of } A) = n$$

The pivot columns are a basis for which subspace?
The special solutions are a basis for which subspace?
Translate this into the Counting Theorem:

$$\text{Dimension of } \_\_\_ + \text{Dimension of } \_\_\_ = n$$

Problem 3. A is a m by n matrix of rank r. Suppose there are right sides $\mathbf{b}$ for which $A\mathbf{x} = \mathbf{b}$ has *no solution.*

(a) What are all inequalities ($<$ or $\leq$) that must be true between m, n and r?
(b) How do you know that $A^T\mathbf{y} = \mathbf{0}$ has solutions other than $\mathbf{y} = 0$?

Problem 4. Find the complete solution (also called the *general solution*) to

$$\begin{bmatrix} 1 & 3 & 1 & 2 \\ 2 & 6 & 4 & 8 \\ 0 & 0 & 2 & 4 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ t \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \\ 1 \end{bmatrix}.$$