18.06 Problem Set 2

due Thursday, February 19, 2015, before 4:00 pm (sharp deadline) in Room E17-131

This homework has 4 questions to hand-in. Write down all details of your solutions, NOT JUST THE ANSWERS. Show your reasoning. Please staple the pages together and **clearly write your name**, your recitation section, and the name of your recitation instructor on the first page of the problem set.

Cooperation on problems is permitted, but all solutions must be written up independently and you must list your collaborators on the problem set. You should first try to solve each problem yourself, otherwise you will not learn much from hearing the solution.

This homework also has an online part and you can choose whether to do it (self-graded) on MITx with MATLAB or (human-graded) in Julia. Follow the instructions on course website.

Problem 1. Find the $PA = LU$ factorizations (and check them) for
$A = \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 2 & 3 & 4 \end{pmatrix}$ and $A = \begin{pmatrix} 1 & 2 & 0 \\ 2 & 4 & 1 \\ 1 & 1 & 1 \end{pmatrix}$

Problem 2. Suppose that S and T are lines through $(0, 0, 0, 0)$ in $\mathbb{R}^4$ (4-dimensional space). So S contains all multiples of a nonzero vector v and T contains all multiples of a nonzero vector w.

(a) When is the union $S \cup T$ of the two lines also a subspace?

(b) If $S \cup T$ is not a subspace, describe the smallest possible subspace that contains both lines S and T.

(c) If S and T are ANY subspaces of $\mathbb{R}^4$, not necessarily lines, how would you construct the smallest subspace that contains both S and T?

Problem 3. Suppose A and B are 3 by 3 matrices.

(a) If a vector v is in the column space of AB, why is v also in the column space of A? [Remember: The column space of A consists of all vectors of the form Ax for some vector x]

(b) Give an example of A and B so that $C(AB)$ is SMALLER than $C(A)$.

(c) If a vector v is in the column space of AB, this does not necessarily mean it is in the column space of B. Give an example of A and B and a vector v that is in $C(AB)$ but not $C(B)$.

Problem 4. For n by n permutation matrices: Why are there $n!$ of them ? Comparing Px with a vector x, how many 4 by 4 permutations move:

(a) no entries of x

(b) 1 entry

(c) 2 entries

(d) 3 entries

(e) all four entries of x ?

The answers should add to $4! = 24$.