18.06 Problem Set 1 - SOLUTIONS

Problem 1.

(a) If j and r are different then $A(i,j)A(r,s)$ gives the zero matrix. If $j=r$ then $A(i,j)A(r,s)$ has 1 in row i and column s: It is $A(i,s)$.

One approach (many ways to get there) is that multiplying a matrix B by $A(i,j)$ puts row j of B into row i of the product. When that matrix B is $A(r,s)$, row j will be zero unless $j=r$. When $j=r$, the row has 1 in position s. Then the product puts it into row i to give $A(i,s)$.

Alternative solution: Let e_k be the vector in $\mathbb{R}^3$ with a 1 in the k^{th} position and zeros elsewhere. We write $A(i,j)$ as vector of rows: the i^{th} row is e_j and all the others are zero. We write $A(r,s)$ as a vector of columns: the s^{th} column is e_r and all the others are zero. From this description we see that $A(i,j)A(r,s)$ can only have a nonzero entry in the (i,s) position. Moreover, the entry here is $e_j \cdot e_r$.

$$A(i,j)A(r,s) = (e_j \cdot e_r)A(i,s).$$

(b) If j and r are different, part (a) says $A(i,j)A(r,s)$ is the zero matrix. So i and s must also be different to get the zero matrix in $A(r,s)A(i,j)$.

If $j=r$ then part (a) says $A(i,j)A(r,s)=A(i,s)$ is not zero. Then we need $i=s$ so the other product is not zero. And we need $j=r=i=s$ so the two products are the same.

Alternative solution: Suppose that $A(i,j)A(r,s)=A(r,s)A(i,j)$. Then $i=s$ if and only if $j=r$, since these conditions ensure that the right and left hand side are nonzero, respectively. When $i \neq s$ and $j \neq r$ we certainly have equality because both sides are zero. When $i=s$ and $j=r$ we also require that $i=r$ and $j=s$ so that $i=j=r=s$.

Problem 2.

(a) ... plane ... origin.
(b) Two planes passing through the origin intersect in either a line or a plane passing through the origin, both of which contain points other than the origin.

Problem 3.
The key point is that there are many choices for the particular solution. ANY SOLUTION CAN BE THE PARTICULAR SOLUTION.

Since the solutions to $Ax_n = 0$ form a geomtric object such as a point, line, plane or hyperplane, all passing through the origin, the set of all solutions to $Ax=b$ is a geometric object such as a point, line, plane or hyperplane, too, but provided

$b \neq 0$, it will NOT pass through the origin; instead it passes through a particular solution. It also says that any two solutions differ by a null solution.

(a) $x_p = \begin{pmatrix} 3 \\ 0 \end{pmatrix}$

(b) $x_p = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$ or $x_p = \begin{pmatrix} 0 \\ 3 \end{pmatrix}$ or anything of the form $\begin{pmatrix} c \\ 3-c \end{pmatrix}$ for $c \in \mathbb{R}$.

(c) Since the coefficient matrix is singular, MATLAB would give a warning and output NaN (not a number), whereas Julia would output an error. The warning "Matrix is singular to working precision" means that MATLAB has noticed that your matrix could either be exactly singular or almost singular, but to machine precision it is singular. Here we only asked MATLAB/Julia to solve a system that actually has infinitely many solutions and we did not specify how to choose one. For instance, later in the class we will learn how to compute the least squares solution, for which MATLAB/Julia have a more specific command.