

## 18.06 PROBLEM SET 1

due Thursday, February 12, 2015, before 4:00 pm (sharp deadline) in Room E17-131

This homework has 3 questions to hand-in. Write down all details of your solutions, not just the answers. Show your reasoning. Please staple the pages together and **clearly write your name**, your recitation section, and the name of your recitation instructor on the first page of the problem set.

Cooperation on problems is permitted, but all solutions must be written up independently and you must list your collaborators on the problem set. You should first try to solve each problem yourself, otherwise you will not learn much from hearing the solution.

This homework also has an online self-graded part in MITx, including MATLAB questions. Go to [lms.mitx.mit.edu](http://lms.mitx.mit.edu), and follow the instructions on course website.

**Problem 1.** Suppose  $A(i, j)$  is the 3 by 3 matrix with 1 in row  $i$ , column  $j$  and otherwise ALL ZERO ENTRIES.

- (a) What is the product  $A(i, j)A(r, s)$ ? The answer depends on  $i, j, r, s$ .
- (b) For which numbers  $i, j, r, s$  between 1 and 3 is it true that

$$A(i, j)A(r, s) = A(r, s)A(i, j)?$$

**Problem 2.** Suppose  $A$  is the 2 by 3 matrix  $A = \begin{pmatrix} a & b & c \\ d & e & f \end{pmatrix}$  with no zero entries.

- (a) Describe the solutions to the first equation  $ax_1 + bx_2 + cx_3 = 0$ . This is the equation of a \_\_\_\_\_ through the \_\_\_\_\_.

- (b) Say why the 2 linear equations  $Ax = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$  have a solution  $x = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$  other

than  $x = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$ .

**Problem 3.** “The solutions to  $Ax = b$  always have the form  $x = x_p + \text{any } x_n$ ” where the particular solution has  $Ax_p = b$  and the null solutions have  $Ax_n = 0$ .

This is correct but what does it say?

Let  $A$  be the matrix  $A = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$  (all ones) and  $b = \begin{pmatrix} 3 \\ 3 \end{pmatrix}$ . The null solutions  $x_n$

are all vectors of the form  $x_n = \begin{pmatrix} c \\ -c \end{pmatrix}$ .

- (a) Choose an  $x_p$  to make the statement true.
- (b) Choose a different  $x_p$  to make the statement true.
- (c) Out of curiosity what happens when you try to compute  $x = A \setminus b$  in Matlab or Julia or the equivalent in another system?