18.06 Exam III Professor Strang May 4, 2015

Your PRINTED Name is: ______________________

Please CIRCLE your section:

R01	T10	26-302	Dmitry Vaintrob
R02	T10	26-322	Francesco Lin
R03	T11	26-302	Dmitry Vaintrob
R04	T11	26-322	Francesco Lin
R05	T11	26-328	Laszlo Lovasz
R06	T12	36-144	Michael Andrews
R07	T12	26-302	Netanel Blaier
R08	T12	26-328	Laszlo Lovasz
R09	T1pm	26-302	Sungyoon Kim
R10	T1pm	36-144	Tanya Khovanova
R11	T1pm	26-322	Jay Shah
R12	T2pm	36-144	Tanya Khovanova
R13	T2pm	26-322	Jay Shah
R14	T3pm	26-322	Carlos Sauer
ESG			Gabrielle Stoy

Grading 1:

2:

3:

____

1. (33 points)

(a) Suppose A has the eigenvalues $\lambda_1 = 1, \lambda_2 = 0, \lambda_3 = -1$ with eigenvectors $\mathbf{x}_1, \mathbf{x}_2, \mathbf{x}_3$ in the columns of this $S = \left[\ \mathbf{x}_1 \mid \mathbf{x}_2 \mid \mathbf{x}_3\ \right]$:

$$S = \begin{bmatrix} -1 & 1 & 1 \\ 0 & -1 & 1 \\ 0 & 0 & -1 \end{bmatrix}.$$

What are the eigenvalues and eigenvectors of the matrix $B = A^9 + I$?

(b) How could you find that matrix $B = A^9 + I$ using the eigenvectors in S and the eigenvalues $1, 0, -1$?

(c) Give a reason why the matrix B does have or doesn't have each of these properties:

i. B is invertible

ii. B is symmetric

iii. trace $= B_{11} + B_{22} + B_{33} = 3$.

2. (33 points)

(a) Show that $\lambda_1 = 0$ is an eigenvalue of A and find an eigenvector $\mathbf{x}_1$ with that zero eigenvalue:

$$A = \begin{bmatrix} -2 & 1 & 1 \\ 1 & -2 & 1 \\ 1 & 1 & -2 \end{bmatrix}$$

(b) Find the other eigenvalues λ_2 and λ_3 of this symmetric matrix. Does A have two more independent eigenvectors $\mathbf{x}_2$ and $\mathbf{x}_3$? Give a reason why or why not. (Not required to find $\mathbf{x}_2$ and $\mathbf{x}_3$.)

(c) Suppose $\dfrac{d\mathbf{u}}{dt} = A\mathbf{u}$ starts from $\mathbf{u}(0) = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$.

Explain why this $\mathbf{u}(t)$ approaches a steady state $\mathbf{u}(\infty)$ as $t \to \infty$. You can use the general formula $\mathbf{u}(t) = c_1 e^{\lambda_1 t}\mathbf{x}_1 + c_2 e^{\lambda_2 t}\mathbf{x}_2 + c_3 e^{\lambda_3 t}\mathbf{x}_3$ or $e^{At} = S e^{\Lambda t} S^{-1}$ without putting in all eigenvectors. **Find** that steady state $\mathbf{u}(\infty)$.

3. (34 points)

(a) If C is any symmetric matrix, show that e^C is a positive definite matrix. We can see that e^C is symmetric — which test will you use to show that e^C is positive definite?

(b) A is a 3 by 3 matrix. Suppose $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3$ are orthonormal eigenvectors (with eigenvalues $1, 2, 3$) of the symmetric matrix $A^T A$. Show that $A\mathbf{v}_1, A\mathbf{v}_2$, $A\mathbf{v}_3$ are orthogonal by rewriting and simplifying $(A\mathbf{v}_i)^T(A\mathbf{v}_j)$.

(c) For the 3 by 3 matrix A in part (b), find three matrices U, Σ, V that go into the Singular Value Decomposition $A = U\Sigma V^T$.

(d) TRUE or FALSE: If A is any symmetric 4 by 4 matrix and M is any invertible 4 by 4 matrix, then $B = M^{-1}AM$ is also symmetric. Give a reason for true or false.

Scrap Paper