18.06 Exam II Professor Strang April 10, 2015

Your PRINTED Name is: ______________________

Please CIRCLE your section:

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R02	T10	26-322	Francesco Lin
R03	T11	26-302	Dmitry Vaintrob
R04	T11	26-322	Francesco Lin
R05	T11	26-328	Laszlo Lovasz
R06	T12	36-144	Michael Andrews
R07	T12	26-302	Netanel Blaier
R08	T12	26-328	Laszlo Lovasz
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R10	T1pm	36-144	Tanya Khovanova
R11	T1pm	26-322	Jay Shah
R12	T2pm	36-144	Tanya Khovanova
R13	T2pm	26-322	Jay Shah
R14	T3pm	26-322	Carlos Sauer
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Grading 1:

2:

3:

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1. (33 points) Suppose we measure $b = 0, 0, 0, 1, 0, 0, 0$ at times $t = -3, -2, -1, 0, 1, 2, 3$.

(a) To fit these 7 measurements by a straight line $C + Dt$, what 7 equations $Ax = b$ would we want to solve?

(b) Find the least squares solution $\widehat{x} = (\widehat{C}, \widehat{D})$.

(c) The projection of that vector b in $\mathbf{R}^7$ onto the column space of A is what vector p?

2. (34 points) Suppose $q_1 = (c, d, e)$ and $q_2 = (f, g, h)$ are **orthonormal column vectors in $\mathbf{R}^3$**. They span a subspace S.

(a) Find the $(1, 1)$ entry in the projection matrix P that projects each vector in $\mathbf{R}^3$ onto that subspace S.

(b) For this projection matrix P, describe 3 independent eigenvectors (vectors for which Px is a number λ times x). What are the 3 eigenvalues of P? What is its determinant?

(c) For some vectors v and w in $\mathbf{R}^3$ the Gram-Schmidt orthonormalization process (applied to v and w) will produce those particular vectors q_1 and q_2. **Describe** the vectors v and w that lead to this q_1 and q_2.

3. (34 points)

(a) If q_1, q_2, q_3 are orthonormal vectors in $\mathbf{R}^3$, what are the possible determinants of this matrix A with columns $2q_1$ and $3q_2$ and $5q_3$? **Why?**

$$A = \begin{bmatrix} 2q_1 & 3q_2 & 5q_3 \end{bmatrix}$$

(b) For a matrix A, suppose the cofactor C_{11} of the first entry a_{11} is **zero**. What information does that give about A^{-1}? Can this inverse exist?

(c) Find the 3 eigenvalues of this matrix A and find all of its eigenvectors. Why is the diagonalization $S^{-1}AS = \Lambda$ not possible?

$$A = \begin{bmatrix} 2 & 1 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

Scrap Paper